



# Numerical Study of Porous Medium Effects in Thermal Radiation Influenced MHD Flow Around Stretching Cylinders

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**Abstract.** This study focuses on the magnetohydrodynamics (MHD) fluid flow. Fluid taken is compressible, viscous and Newtonian. Fluid flow is taken over a continuously stretching, permeable horizontal cylinder in the presence of internal heat generation or absorption and thermal radiation. The governing partial differential equations, with the help of given boundary layer criteria and similarity transformations, are converted into system of non-linear ordinary differential equations. In order to determine the velocity and temperature distributions, these equations are solved numerically. Friction factor and heat determination rate is also determined. Tabular and graphical illustrations are used to present the impact of different parameters like magnetic parameter, radiation parameter, heat source/sink parameter on the fluid flow. It is the primary object of the research, to explore the influence of magnetic field, radiation parameter, and heat generation or absorption on the flow and temperature distribution along the stretching cylindrical surface. Fluid flows with all these parameters affect differently on cylindrical and flat geometries, which is also demonstrated in the results. This work has significant relevance to several engineering and industrial processes such as polymer extrusion, metal forming, and cooling fluids, where efficient transfer is essential. It also contributes to applications in aerospace thermal protection systems, nuclear energy, and porous catalytic reactors. Additionally, the study provides insight into fluid systems influenced by magnetic and radiative effects within permeable structures, including filtration units, enhanced oil recovery, biomedical fluid transport.

**Keywords:** Magnetic field-induced flow, Elongated cylinder, Volumetric heat production/extraction, Convective heat flow, Porous layer.

## 1 Introduction

Mixed and natural-convection within a fluid-saturated in a porous media causes heat transfer throughout the fluid. This heat transfer has very vast applications in engineering and industries, like petroleum extraction, plasma technology, magnetohydrodynamic (MHD) power generation, geothermal energy systems, heat insulation, drying processes, catalytic reactors, food processing, and solar energy collection. Due to these wide range of applications, this heat transfer is investigated extensively [1]. There are the systems where heat transfer occurs in a cylindrical geometry. These systems include electronic cooling, thermal building design, drilling operations, geothermal power generations, commercial refrigeration, and glass manufacturing. These systems increase the significance of studies related to the heat transfer around cylindrical surfaces. Recently, more special attention is paid to these kind of studies. Heat transfer over flat plates can't be neglected. There are many studies where hydrodynamic flow and heat transfer over stretching cylinders are examined. Carne [2] was the one, who analyzed the flow over a stretched plate. Two other scientists Lin and Shin [3, 4] studied the phenomena of laminar boundary-layer heat transfer. This was studied on both, vertical and horizontal, geometries of cylinder. Gupta [5] enhanced the criteria of these studies by making research on suction and blowing effects on stretched surfaces. Grubka and Bobba [6] investigated about the effects of surface temperature if it is modified periodically. Minkowycz and Cheng [7] added the effect of porous media and explored natural convection process in porous media surrounding vertical cylinders. Viscous dissipation in MHD Sisko fluid flow over a stretching cylinder was studied by Malik et al. [8]. Not neglecting the system with horizontal cylinders, Saeid [9] modeled natural convection from a horizontal cylinder. Not neglecting the possibility of the flow over shrinking or stretching cylinders, Ali et al. [10] studied the viscous flow through porous stretching or shrinking cylinders. Majeed et al. [11] studied heat transfer across a hyperbolically stretched cylinder, while Ali et al. [12] included Hall current effects in MHD mixed convection over a vertical plate. Veera Krishna and Chamkha [13] conducted several investigations involving ion slip, Hall current, and radiation absorption in MHD flows over porous surfaces.

In systems, like dissociation processes, renewable energy systems, and waste heat recovery, the process of heat generation and absorption is very important. Radiations also affect the fluid flow. In this regard, Abbas et al. [14] analyzed radiation effects on MHD flow over a porous stretching cylinder. The heat generation and absorption process in porous media is explored by Chauhan et al. [15]. Sharma et al. [16] studied MHD flow in on-Darcy porous media, and Abel et al. [17] examined upper-convected Maxwell fluid behavior. Further studies by Yasir et al. [18], Ghasemi et al. [19] and Mendal et al. [20] investigated nanofluid thermal performance, pressure drop in heat exchangers, and radiative MHD dusty flow around cylinders.

The main aim of the present study is to analyze the steady, viscous boundary-layer flow of an electrically conducting Newtonian fluid over a horizontal stretching cylinder under the influence of a transverse magnetic field and variable surface temperature, while considering heat source/sink effects. The investigation extends earlier works by incorporating magnetic diffusion and heat generation/absorption into the energy equation.

## 2 Formulation

Consider a cylinder of radius  $a$ , embedded in a fluid-saturated porous medium. Fluid, in this case is Newtonian fluid. Fluid flow is caused by free convection. Fluid flow is steady, two-dimensional, and electrically conducting. Cylinder is sufficiently elongated in axial direction and stretches linearly with surface velocity  $u_w = U \frac{x}{l}$  and its surface temperature varies according to  $T_w(x) = T_\infty + T_0 \left(\frac{x}{l}\right)^n$ .

$x$ -axis is along the cylinder's axis while  $r$ -axis extends radially and  $r$ -axis extends radially outward from the surface with an effect of externally applied magnetic field  $B$  in its direction while induced magnetic field effect is neglected. The continuity, momentum, and thermal energy equations for this are expressed as follows

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{v}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{\sigma B^2}{\rho} u - \frac{v}{K_p} u, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{Q_0}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial r}, \tag{3}$$

### 2.1 Boundary conditions

At the cylinder surface ( $r = a$ )

$$u = u_w = U \frac{x}{l}, v = 0, T = T_w(x) = T_\infty + T_0 \left(\frac{x}{l}\right)^n \tag{4}$$

At( $r \rightarrow \infty$ )

$$T \rightarrow T_\infty, u \rightarrow 0 \tag{5}$$

The given system of PDE's (1) - (3) can be transformed in to a set of ODE's by introducing the following similarity variables:

$$\begin{aligned} \eta &= \frac{r^2 - a^2}{2a} \left( \frac{u_w}{vx} \right)^{\frac{1}{2}}, \psi = (vxu_w)^{\frac{1}{2}} af(\eta), u = \frac{1}{r} \left( \frac{\partial \psi}{\partial r} \right) \\ v &= -\frac{1}{r} \left( \frac{\partial \psi}{\partial r} \right), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned} \tag{6}$$

Where  $\eta$  represents the dimensionless similarity variable,  $\psi$  is the stream function defined in terms of the velocity component, and  $\theta$  denotes the dimensionless temperature.

### 2.2 Solution of the problem

By substituting the above-defined similarity transformations into equations (2) and (3) and applying the boundary conditions (4) and (5),

$$(1 + 2\beta\eta)f'''' - (f')^2 + ff'' + 2\beta f'' - M(f') - Kf' = 0 \tag{7}$$

$$(1 + 2\eta\beta)\theta'' + 2\beta\theta' + Prf\theta' + Pr.Q.\theta + \frac{4}{3}.Rd(\beta.\theta') = 0 \tag{8}$$

Together with the transformed boundary conditions:

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0 \tag{9}$$

Where prime indicates diff. w.r.t  $\eta$ .

The following dimensionless quantities characterize the flow and transfer rates:

$$C_f = \frac{2\tau_w}{\rho u_w^2}, Nu_x = \frac{xq_w}{K(T_w - T_\infty)} \tag{10}$$

Or equivalently, in non-dimensional form:

$$\frac{C_f}{Re^{\frac{1}{2}}} = -f''(0), \frac{Nu_x}{Re^{\frac{1}{2}}} = -\theta'(0) \tag{11}$$

Where  $Re$  is the local Reynolds number,  $\tau_w$  denotes the wall shear stress, and  $q_w$  represents the wall heat flux.

These are expressed as:

$$\tau_w = \mu \left( \frac{\partial u}{\partial r} \right)_{r=a}, q_w = -K \left( \frac{\partial T}{\partial r} \right)_{r=a}, Re = \frac{Ux}{\nu} \tag{12}$$

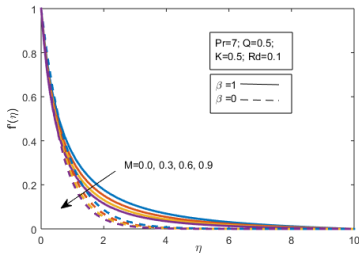
### 3 Results and Discussion

Fluid's velocity is effected by external magnetic field, presence of porous media, and the curvature of the cylinder. external magnetic field will generate Lorentz force, which will act as a resistance on the ionized fluid and will affect the fluid motion. Hence, due to the occurrence of magnetic field and existence of Lorentz force will decrease the fluid's velocity (Fig 1). Porous media will offer a resistance in the fluid flow. This resistance will cause a decline in the fluid's velocity (Fig 2). In contrast, the curvature parameter will enhance the fluid's velocity as, the curvature of the surface facilitates smoother flow motion and reduces the decelerating effects within the boundary layer

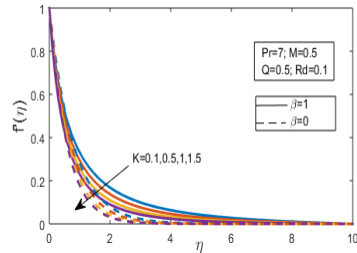
(Fig 3). Temperature profile is effected by magnetic field, porosity factor, curvature parameter, radiation parameter, heat source/sink parameter, and Prandtl number. Lorentz force generated by magnetic field will produce internal friction and ohmic heating within the fluid which will increase the fluid's temperature (Fig 9). Porosity factor will cause energy dissipation withi the fluid which will increase the fuid's temperature (Fig 8). The radiation parameter will also elevate the temperature field because radiative heat transfer adds additional thermal energy to the system, thickening the thermal boundary layer (Fig 6). Heat generation/absorption parameter will affect differently to the temperature profile. Heat generation will cause temperature to rise while heat absorption will decrease the fluid's temperature (Fig 7). Change in Prandtl number shows that higher Prandtl number possesses lower thermal diffusivity, which will restrict heat transfer. In result we will notice temperature reduction in the fluid flow (Fig 4). The effect of increasing curvature is to expand the boundary layer and increase the temperature adjacent to the surface. A more curved surface spreads the fluid rapidly, dropping the temperature grade at the wall and allowing down heat diffusion away from the surface. (Fig 4)

Note:  $\beta = 1$  ——— (cylindrical surface)  
 $\beta = 0$  - - - - - (flat surface)

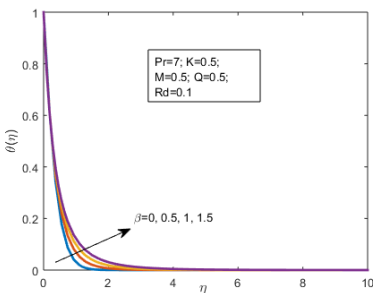
**Figures:**



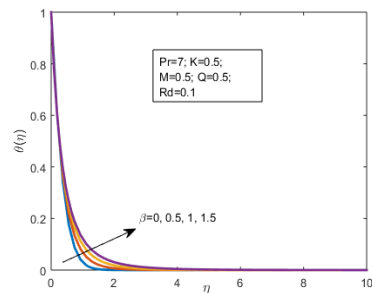
**Fig 1.** Variation of velocity profile ( $f'$ ) to changes in M



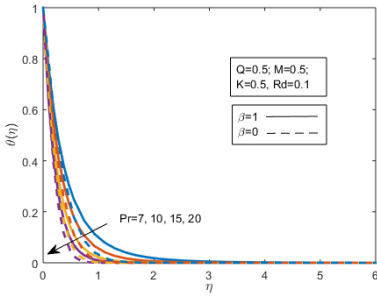
**Fig 2.** Variation of velocity profile ( $f'$ ) to changes in K



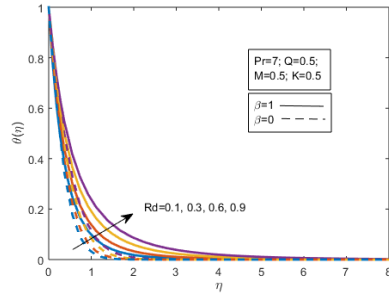
**Fig 3.** Variation of velocity profile ( $f'$ ) to changes in  $\beta$



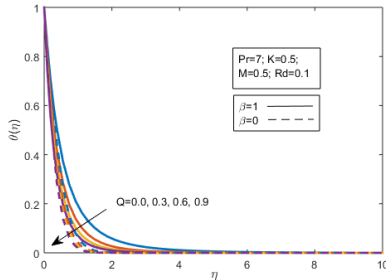
**Fig 4.** Variation of temperature profile ( $\theta$ ) to changes in  $\beta$



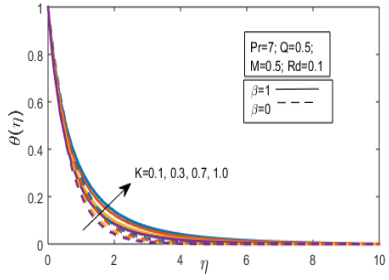
**Fig 5.** Variation of temperature profile ( $\theta$ ) to changes in Pr



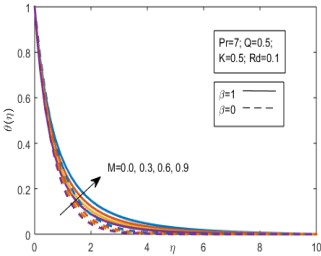
**Fig 6.** Variation of temperature profile ( $\theta$ ) to changes in Rd



**Fig 7.** Variation of temperature profile ( $\theta$ ) to changes in Q



**Fig 8.** Variation of temperature profile ( $\theta$ ) to changes in K



**Fig 9.** Variation of temperature profile ( $\theta$ ) to changes in M

### 4 Tables and Discussion

**Table 1.** Values of  $-f''(0)$  at the surface for several M when K=0 and  $\beta=0$

M	Present result	Reddy et al. [1]	Afify et al. [21]
0	-2.00002	-0.99994	-0.99996
0.2	-2.19089	-1.09544	-1.09544

From table 1 we can observe that as  $M$  increases the fluid’s velocity slows down. This because of the Lorentz force. The results of Reddy et al. [1] and Afify et al. [21] exhibit a similar decreasing trend but with smaller magnitudes, demonstrating that the present computations align closely with previous studies while showing slightly higher sensitivity to magnetic effects.

**Table 2.** Numerical result: friction factor  $-f''(0)$  and Nusselt number  $\theta'(0)$  for parametric study

						Present result					Reddy et al.[1]
Pr	Q	M	W	$\beta$	Rd	$-f''(0)$	$\theta'(0)$	$-f''(0)$	$\theta'(0)$	$-f''(0)$	$\theta'(0)$
20	0.5	0.5	0.5	0.5	0.5	3.25314	3.5275	1.6265	2.5697		
50	0.5	0.5	0.5	0.5	0.5	3.25314	5.6274	1.6265	4.6497		
100	0.5	0.5	0.5	0.5	0.5	3.25314	7.9982	1.6265	6.9879		
7	-0.5	0.5	0.5	0.5	0.5	3.25315	-0.493	1.6265	1.0967		
7	0	0.5	0.5	0.5	0.5	3.25317	1.3333	1.6265	1.9019		
7	0.5	0.5	0.5	0.5	0.5	3.25317	2.0609	1.6265	2.5224		
7	0.5	1	0.5	0.5	0.5	3.59872	2.0441	1.7993	0.9001		
7	0.5	1.5	0.5	0.5	0.5	3.90846	2.0299	1.9541	0.5644		
7	0.5	2	0.5	0.5	0.5	4.19177	2.0175	2.0958	0.8407		
7	0.5	0.5	0	0.5	0.5	2.85502	2.0812	1.4274	1.2531		
7	0.5	0.5	0.3	0.5	0.5	3.10162	2.0684	1.5507	1.1671		
7	0.5	0.5	0.7	0.5	0.5	3.39652	2.0538	1.6982	1.0248		
7	0.5	0.5	0.5	0.3	0.5	3.08852	2.0248	1.5442	1.2114		
7	0.5	0.5	0.5	0.7	0.5	3.41255	2.0969	1.7062	1.0250		
7	0.5	0.5	0.5	0.1	0.5	3.64368	2.1504	1.8218	0.9636		
7	0.5	0.5	0.5	0.5	0.1	3.25317	2.5916	3.1425	3.4125		
7	0.5	0.5	0.5	0.5	0.3	3.25317	2.2856	3.1425	1.6036		
7	0.5	0.5	0.5	0.5	0.6	3.25323	1.9689	3.1425	0.0713		

The validity and accuracy of the present numerical approach can be confirm by Reddy et al. [1]. Enhancement in  $\theta'(0)$  after increasing Prandtl number results in a compressed and elevated thermal transfer. Similarly, variations in other parameters like in  $M, Q, W, Rd$  show consistent physical trend.  $M$  suppresses the velocity gradient due to Lorentz force, while a rise in  $Rd$  tends to increase fluid temperature and reduce  $\theta'(0)$ . Minor numerical deviations between the two studies may result from differences in computational schemes or boundary approximations.

**Table 3.** Effect of Prandtl number on heat transfer rate  $-\theta'(0)$  at the wall for  $Q=M=\beta=K=Rd=0$ .

Pr	Present results	Elbashbeshy et al. [22]	M. Subhas et al. [23]
0.2	0.189662	0.16908	0.1691
0.7	0.428211	0.45391	0.4539
2	0.842426	0.91135	0.9114
7	1.66843	1.89540	1.8954
20	2.72877	3.35390	3.3539
70	4.38316	6.46219	6.4622

It is evident that the Nusselt number increases with rising Pr, indicating higher Pr fluids exhibit greater temperature gradients near the surface, thus enhancing heat transfer. The present results show excellent agreement with the previous studies, displaying the same increasing trend and only minor quantitative differences.

## 5 Concluding remarks

The investigation emphasizes how magnetic field strength, medium porosity, and radiation jointly affect the thermal flow behavior. Behavior of different parameters on the fluid flow is summarized as:

- Magnetic parameter and porosity factor significantly suppress the velocity field due to enhance resistive and drag forces.
- An increment in thermal radiation and heat generation parameter will rise up the temperature distribution.
- As the Prandtl number increases reduction in thermal boundary layer is noticed, which improves the rate of thermal energy flux at the surface.
- Degree of surface curvature affects both velocity and temperature of the fluid. Higher curvature intensifies the radial stretching effect, resulting in thinner momentum and thermal boundary layer.
- Variations in M, K and  $\beta$  reveals opposite trend for the heat and momentum transfer. The Nusselt number decreases, whereas the shear stress coefficient increases.

**Disclosure of Interests.** The authors have no competing interests to declare that are relevant to the content of this article.

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