



Supply Chain Emission Reduction and Financing Decisions under Carbon Cap Regulation with Government Subsidies

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Abstract. This paper studies a low-carbon supply chain with a capital-constrained manufacturer and a retailer. Under a carbon cap, we examine how credit and product subsidies affect emission reduction and financing decisions. Four game models are constructed and solved using KKT conditions for green credit and advance payment financing. The impacts of carbon cap, subsidies, and consumer low-carbon preference on supply chain decisions are analyzed. The main findings are as follows. (1) Two emission constraint scenarios exist: stringent (mandatory) and lenient (achievable via internal decisions). (2) Consumer low-carbon preference and subsidy levels are key factors affecting the carbon cap and emission reduction. (3) Both subsidies positively affect supply chain decisions and social welfare, with stronger effects under advance payment financing. (4) Advance payment financing is always the manufacturer's preferred strategy.

Keywords: Carbon cap policy, Capital constraint, Government subsidy, Carbon reduction.

1 Introduction

In recent years, the Chinese government has implemented various carbon emission policies, including cap-and-trade mechanisms, carbon taxes, and dual control policies. Among these, the cap-and-trade mechanism has been widely adopted by governments globally. Under this mechanism, the government imposes a carbon emission cap on enterprises. Firms that exceed or remain below the prescribed limit are allowed to trade emission allowances in the carbon trading market. The vibrancy of the carbon trading market, together with consumers' increasing demand for green and low-carbon products, is driving enterprises toward sustainable development. An increasing number of firms are investing in low-carbon product development and carbon abatement technologies. However, shortages of emission reduction funds occur frequently. As a result,

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seeking external financing has become a common strategy to alleviate financial pressure.

As the designer and enforcer of low-carbon policies and regulations, the government is also playing a role in assisting enterprises that face financial constraints in their emission reduction efforts. Designed to encourage sustainable green industrial upgrading, green credit requires compliance with environmental standards as a precondition for loan approval. This mechanism has effectively alleviated the financing constraints of enterprises that are willing to reduce emissions. Another financing mode considered in this paper is advance payment financing from the retailer.

To further alleviate the financing difficulties of emission-reducing enterprises, the government also provides policy support, among which government subsidies serve as a highly effective tool. In practice, governments design various forms of subsidy policies to assist enterprises in low-carbon production. For example, the Chinese government offers low-interest credit to banks, which then provide loans to eligible green and low-carbon enterprises. Another example is the "trade-in" subsidy for new energy vehicle consumers, aimed at promoting the adoption of new energy vehicles. It can be observed that in a capital-constrained supply chain, emission reduction production involves issues such as financing, subsidies, and product pricing. Thus, it is of considerable importance to investigate how capital-constrained firms determine their emission reduction and financing strategies in response to various government subsidy schemes, while simultaneously adhering to mandated carbon regulation policies.

2 Literature Review

There are three main literatures related to our research: sustainable operation low-carbon supply chains, Carbon control policies in the supply chain, and government subsidies.

First, the sustainable operation and financing issues of low-carbon supply chains have gained increasing attention. *chai et al.* [1] considered that control and trading of total carbon have a positive effect on the development of the supply chains. *Xia et al.* [2] compared and analyzed the game model between ordinary products and low-carbon products, and considered that carbon trading will increase the selling price of ordinary products and improve environmental benefits under certain circumstances. *xia et al.* [3] believe that from a long-term perspective, raising consumers' low-carbon awareness helps reduce carbon emissions and improve social welfare. *Cheng et al.* [4] used the differential game model to study cooperative and non-cooperational three-level low-carbon supply chain model, considered that the degree of willingness to cooperate would affect members' optimal operational decisions. Some scholars have further studied green technology investment and financing decisions in low-carbon supply chains.

Second, the main carbon control policies nowadays are carbon taxes, carbon quotas, and carbon total control and trading policies, these policies will all affect the operational decisions of enterprises [5]. *Cai et al.* [6] studied supply chain emission reduction decisions under carbon trading policies and low-carbon subsidy policies, and found that both policies can have positive emission reduction effects under certain conditions.

Zhang et al.[7] considered a non-cooperative game model of a low-carbon manufacturing supply chain to investigate the differential effects of carbon trading policies and government subsidy policies. They found that the carbon trading price can promote emission reduction among supply chain members, while the effect of subsidies is not significant.

Finally, government subsidies can promote the supply chain to achieve a positive low-carbon transformation. On the one hand, appropriate subsidies can stimulate enterprises' enthusiasm for emission reduction, and on the other hand, they can promote enterprises to achieve good and sustainable operations. Most of the existing research focuses on low-carbon supply chain[8,9], remanufacturing supply chains[10]and agricultural supply chains[11]. For instance, Zhao et al.[12] constructed a profit allocation model for closed-loop supply chain members under scenarios with and without financial subsidies and with different government subsidy recipients, and compared the pricing decisions of the closed-loop supply chain under different subsidy targets.

3 Problem Description and Model Assumptions

3.1 Problem Description

Under a carbon cap regime, this paper examines a two-echelon low-carbon supply chain comprising one manufacturer of low-carbon products and one retailer. The manufacturer faces capital constraints, whereas the retailer has sufficient funds. The manufacturer alleviates its capital shortage through two financing strategies: green credit and advance payment from the retailer. The manufacturer invests in emission reduction technology innovation to lower the carbon emission level of its products. The manufacturer sells the products to the retailer at a wholesale price w , and the retailer then sells them to the end market at a retail price p . To encourage enterprises to invest in emission reduction technology innovation, the government provides emission reduction subsidies to firms that engage in carbon abatement. Two forms of subsidies are offered: green credit subsidies and price subsidies. Let the green credit subsidy rate be denoted by $1 - \eta$ (where the manufacturer bears a proportion η of the principal and interest of the financing), and let the price subsidy for low-carbon products be denoted by θ . Notably, under the carbon cap policy, the manufacturer is required to ensure that its aggregate carbon emissions throughout the production process remain below the emission cap imposed by the government. The supply chain structure is depicted in Figure 1, and the principal notations employed in this paper are presented in Table 1.

Table 1. Notations and descriptions.

Notation	Description
q	market demand
w	wholesale price of the low-carbon product
e	emission reduction per unit of product
e_0	initial carbon emissions per unit of product

T	government-mandated emission reduction target
p	selling price of the low-carbon product
θ	subsidy rate for low-carbon products
η	proportion of green credit financing principal and interest borne by the manufacturer
c	production cost per unit of low-carbon product
β	consumer preference coefficient for low-carbon products
π_m^i, π_r^i	profit function of the manufacturer, the retailer
$i \in \{B, S, C, N\}$	B,S: scenario of bank financing under green credit subsidy with retailer advance payment. C,N: scenario of green credit under product subsidy with retailer advance payment.
r	Without loss of generality, the interest rates under both financing strategies are assumed to be equal, $0 < r < 1$
a	potential market size

3.2 Model Assumptions

The assumptions of this paper are as follows.

(1) The game model follows a Stackelberg structure, in which the manufacturer producing low-carbon products acts as the leader, and the retailer acts as the follower. Both players have complete and symmetric information, and their risk preferences are not considered.

(2) Following Reference [7], the cost of investment in carbon emission reduction technology innovation is set as $\frac{1}{2}ke^2$, where k denotes the cost coefficient of carbon emission reduction innovation investment.

(3) Following References [9,10], the demand function is assumed to be $q = a - bp + \beta e$. Moreover, for the convenience of model solving, it is assumed that the manufacturer's initial available funds are zero, which has been verified not to affect the conclusions.

(4) As described in the problem formulation, the carbon emissions of the supply chain are strictly monitored by the government. The actual emission level per unit of product after the manufacturer's emission reduction is denoted as $\frac{e_0 - e}{e_0}$. Without loss of generality, we assume $e_0 = 1$, so the corresponding carbon emission reduction level is $1 - e$. Therefore, based on the problem context, the manufacturer's emission reduction

level must satisfy the condition $1 - e \leq T$ to meet the government's carbon cap requirement.

(5) It is assumed that the potential market size a is sufficiently large, much larger than the other parameters in the model.

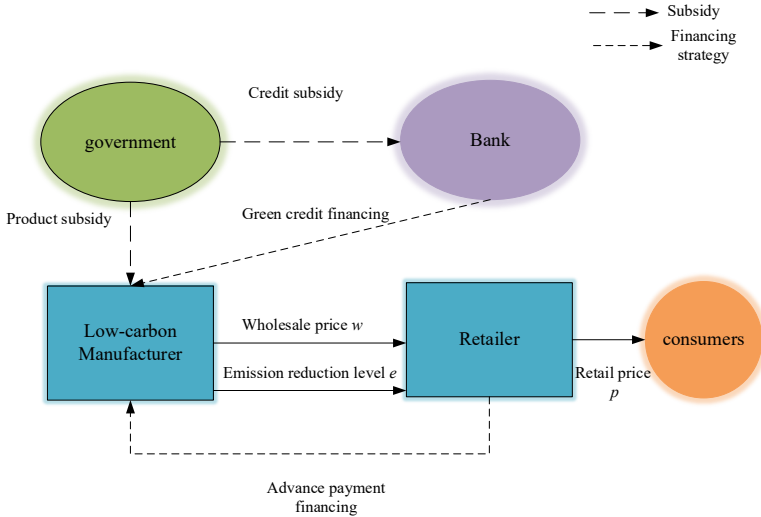


Fig. 1. Structure of emission reduction and financing in the low-carbon supply chain.

4 Model Analysis

4.1 Model of Emission Reduction and Financing Decisions under Credit Subsidy

Green credit financing (Model B). Under green credit financing, during the sales period, the bank provides the manufacturer with the required funds at an interest rate r . The objective functions of the manufacturer and the retailer are given as follows.

$$\pi_m^B = \max(w - c)q - \frac{1}{2}ke^2 - \eta r(\frac{1}{2}ke^2 + cq) \tag{1}$$

$$\pi_r^B = \max_p(p - w)(a - p + \beta e) \tag{2}$$

$$s.t. \quad 1 - e \leq T.$$

By solving the objective functions, the optimal decisions of the supply chain members under the credit subsidy scheme with the manufacturer selecting bank financing are given as follows.

$$\text{When } 0 \leq T \leq \bar{T}^B, \quad w_m^B = \frac{a + c(1 + \eta r) + \beta(1 - T)}{2}, \quad e^B = 1 - T, \quad p_r^B = \frac{3(a + \beta(1 - T)) + c(1 + \eta r)}{4},$$

$$q^B = \frac{a - c(1 + \eta r) + \beta(1 - T)}{4};$$

When $\bar{T}^B \leq T \leq 1$, $w_m^B = \frac{(1 + \eta r)(2k(a + c(1 + \eta r)) - c\beta^2)}{4k(1 + \eta r) - \beta^2}$, $e^B = \frac{\beta(a - c(1 + \eta r))}{4k(1 + \eta r) - \beta^2}$,

$$p_r^B = \frac{(1 + \eta r)(ck(1 + \eta r) - c\beta^2 + 3ak)}{4k(1 + \eta r) - \beta^2}, q^B = \frac{k(1 + \eta r)(a - c(1 + \eta r))}{4k(1 + \eta r) - \beta^2}.$$

where $\bar{T}^B = 1 - \frac{\beta(a - c(1 + \eta r))}{4k(1 + \eta r) - \beta^2}$.

Clearly, the solution results indicate that the optimal decisions of the supply chain correspond to two distinct levels of government-imposed carbon emission caps. When $0 \leq T \leq \bar{T}^B$, the manufacturer faces stringent carbon emission restrictions. The supply chain can only operate if it meets the government's cap; otherwise, severe penalties apply. When $\bar{T}^B \leq T \leq 1$, the emission restriction is lenient, and the supply chain can achieve internal equilibrium under the cap. These two cases are referred to as the stringent and lenient carbon emission conditions, respectively, consistent with the subsequent models.

Advance payment strategy (Model S). When the manufacturer adopts the advance payment strategy (Model S), it seeks financial assistance from the retailer within the supply chain, agreeing to borrow funds at an interest rate r . The objective functions of the manufacturer, the retailer, and the government are given as follows.

$$\pi_m^S = \max_{w, e} (w - c)q - \frac{1}{2}ke^2 - \eta r(\frac{1}{2}ke^2 + cq) \tag{3}$$

$$\pi_r^S = \max_p (p - w)(a - p + \beta e) + \eta r(\frac{1}{2}ke^2 + cq) \tag{4}$$

s.t. $1 - e \leq T$.

Under credit subsidy with advance payment financing, the optimal supply chain decisions are as follows.

When $0 \leq T \leq \bar{T}^S$, $w_m^S = \frac{a + c + \beta(1 - T) + 2c\eta r}{2}$, $e^S = 1 - T$,

$$p_r^S = \frac{3a + c + 3\beta(1 - T)}{4},$$

$$q^S = \frac{a - c + \beta(1 - T)}{4};$$

When $\bar{T}^S \leq T \leq 1$, $w_m^S = \frac{(1 + \eta r)(2k(a + c + 2c\eta r) - c\beta^2)}{4k(1 + \eta r) - \beta^2}$, $e^S = \frac{\beta(a - c)}{4k(1 + \eta r) - \beta^2}$,

$$p_r^s = \frac{(1+nr)(k(c+3a)-c\beta^2)}{4k(1+\eta r)-\beta^2}, q^s = \frac{k(a-c(1+\eta r))}{4k(1+\eta r)-\beta^2}.$$

$$\text{where } \bar{T}^s = 1 - \frac{\beta(a-c)}{4k(1+\eta r)-\beta^2}.$$

4.2 Model of Emission Reduction and Financing Decisions under Product Subsidy

Under the product subsidy scheme, the manufacturer receives a government subsidy for emission reduction, which provides a certain amount of subsidy per unit of low-carbon product.

Green credit financing strategy (Model C). Under the product subsidy scheme, to encourage the manufacturer to invest in emission reduction, the government provides a product price subsidy of value θ per unit of low-carbon product. Accordingly, the profit functions of the supply chain members under the green credit scenario are given as follows.

$$\pi_m^c = \max_{w,e} (w-c+\theta)q - \frac{1}{2}ke^2 - r\left(\frac{1}{2}ke^2 + cq\right) \quad (5)$$

$$\pi_r^c = \max_p (p-w)(a-p+\beta e) + r\left(\frac{1}{2}ke^2 + cq\right) \quad (6)$$

$$\text{s.t. } 1-e \leq T.$$

The optimal decisions of the supply chain members are given as follows.

$$\text{When } 0 \leq T \leq \bar{T}^c, \quad w_m^c = \frac{a+c(1+r)-\theta+\beta(1-T)}{2}, \quad e^c = 1-T,$$

$$p_r^c = \frac{3a+c(1+r)-\theta+3\beta(1-T)}{4},$$

$$q^c = \frac{a-c(1+r)+\theta+\beta(1-T)}{4};$$

$$\text{When } \bar{T}^c \leq T \leq 1, \quad w_m^c = \frac{(1+r)(2k(a+c(1+r)-2\theta)-\beta^2c)+\beta^2\theta}{4k(1+r)-\beta^2},$$

$$e^c = \frac{\beta(a+\theta-c(1+r))}{4k(1+r)-\beta^2}, \quad p_r^c = \frac{(1+r)(3ak+k(1+r)-\beta^2c-\theta)+\beta^2\theta}{4k(1+r)-\beta^2},$$

$$q^c = \frac{k(1+r)(a-c(1+r)+\theta)}{4k(1+r)-\beta^2}.$$

$$\text{where } \bar{T}^c = 1 - \frac{\beta(a+\theta-c(1+r))}{4k(1+r)-\beta^2}.$$

Advance payment financing strategy (Model N). Under the product subsidy scheme, the profit functions of the supply chain members are given as follows.

$$\pi_m^N = \max_{w,e} (w - c + \theta)q - \frac{1}{2}ke^2 - r\left(\frac{1}{2}ke^2 + cq\right) \tag{7}$$

$$\pi_r^N = \max_p (p - w)(a - p + \beta e) + r\left(\frac{1}{2}ke^2 + cq\right) \tag{8}$$

$$s.t. \quad 1 - e \leq T.$$

The optimal supply chain decisions are as follows.

$$\begin{cases} w_m^N = \frac{a + c(1 + 2r) - \theta + \beta(1 - T)}{2} \\ e^N = 1 - T \\ p_r^N = \frac{3(a + \beta(1 - T)) + c - \theta}{4} \\ q^N = \frac{a - c + \theta + \beta(1 - T)}{4} \end{cases} \quad 0 \leq T \leq \bar{T}^N \tag{9}$$

$$\begin{cases} w_m^N = \frac{(1 + r)(2k(a + c(1 + r) - \theta) - \beta^2c) + \beta^2\theta}{4k(1 + r) - \beta^2} \\ e^N = \frac{\beta(a - c + \theta)}{4k(1 + r) - \beta^2} \\ p_r^N = \frac{(1 + r)k(3a + c - \theta) + \beta^2(\theta - c)}{4k(1 + r) - \beta^2} \\ q^N = \frac{k(1 + r)(a - c + \theta)}{4k(1 + r) - \beta^2} \end{cases} \quad \bar{T}^N \leq T \leq 1 \tag{10}$$

where $\bar{T}^N = 1 - \frac{\beta(a - c + \theta)}{4k(1 + r) - \beta^2}$.

5 Numerical analysis

In this section, we conduct a numerical analysis to investigate the impacts of relevant parameters and different subsidy schemes on the key decision variables. Drawing on related literature and real-world data. In November 2021, the People's Bank of China offered low-cost funds to banks to promote carbon reduction loans at an interest rate of 1.75%. According to China Times, domestic green hydrogen production costs range from 30~40 yuan/kg. Accordingly, we set $c = 35$ in this model. Reference [13] reports that each kilogram of green hydrogen from photovoltaic power generates 5.12 kg CO₂ over its full life cycle. In December 2020, the China Hydrogen Alliance issued

standards setting the emission threshold for clean and renewable hydrogen at 4.9 kg CO₂. Based on the above data, the required carbon emission reduction level for producing 1 kg of green hydrogen energy is obtained and is therefore set $T = \frac{5.12 - 4.9}{5.12} \approx 0.2$

as the value for the stringent carbon emission condition. According to the parameter requirements of our model, the remaining parameters are set as follows: $a = 100$, $k = 20$, $b = 10$, $\beta = 0.8$.

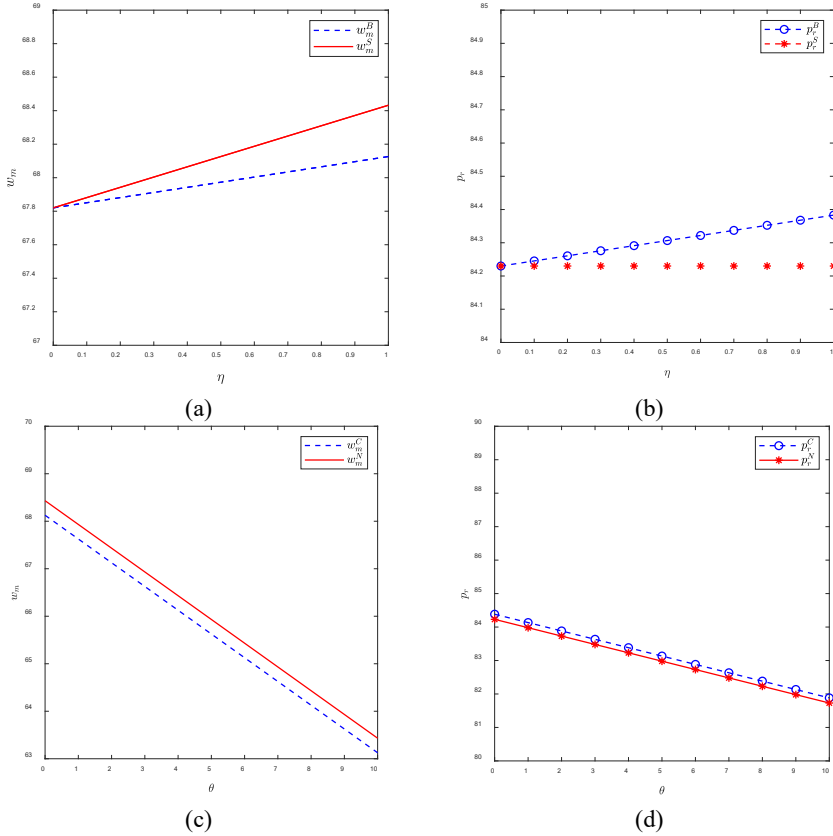


Fig. 2. The impact of η and θ on w_m and p_r under the SCE condition.

Figure 2 presents the effects of different government subsidy mechanisms on the wholesale and retail prices under the stringent carbon emission condition. As shown in Figure 2(a), (b) when the manufacturer bears a greater proportion of the financing cost (as η increases), both the wholesale and retail prices rise within a certain range. Notably, under the stringent emission condition and the advance payment strategy, the retail price exhibits an inverted U-shaped pattern, first increasing and then decreasing. This suggests that when the interest income from the advance payment is relatively low, the retailer raises the price to safeguard its profit margin. However, once the interest income reaches a certain threshold, the retailer reduces the retail price to stimulate

demand, and the marginal gain from increased demand offsets the loss from the price reduction.

Figure 2(c), (d) shows that as the product subsidy θ increases, both the wholesale and retail prices decline. Under the advance payment strategy, the wholesale price exceeds that under green credit financing, whereas the retail price is lower.

Figure 3 illustrates the impact of alternative subsidy mechanisms on the wholesale price, retail price, and carbon emission reduction level under the lenient carbon emission condition. As shown in Figure 3, under the government's relatively relaxed emission requirement, an increase in the subsidy parameter reduces both the wholesale and retail prices. Consistent with the numerical findings in Figure 2, at equivalent subsidy levels, the advance payment strategy consistently yields a lower retail price and a higher wholesale price. An increase in the product subsidy rate and a decrease in the manufacturer's financing cost share both enhance the carbon emission reduction level.

Under the credit subsidy scheme, as the manufacturer's share of financing costs declines, the emission reduction levels of the two financing strategies converge. Under the product subsidy scheme, the advance payment strategy always achieves a higher emission reduction level than the green credit financing strategy. From a long-term perspective, the government may consider promoting product subsidies to capture consumer attention, thereby leveraging consumer behavior to diffuse low-carbon products.

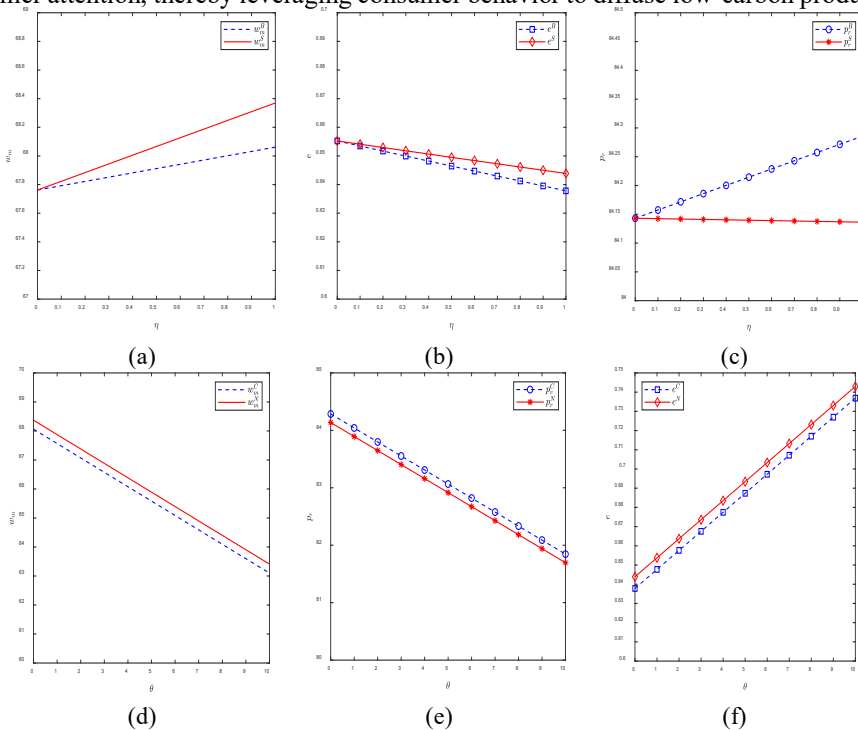


Fig. 3. The impact of η and θ on w_m , p_r , and e under the LCE condition.

Figure 4, where the superscripts 1 and 2 denote the lenient and stringent carbon emission conditions, respectively, compares the profits of supply chain members under the two different carbon emission conditions. Regardless of the carbon emission restriction condition, an increase in the subsidy rate leads to a corresponding increase in the profits of supply chain members. Under the lenient carbon emission condition, compared with the credit financing strategy, the advance payment strategy is more beneficial to both the manufacturer and the retailer at the same subsidy level for profit-maximizing supply chain members. This indicates that intra-supply chain financing has a significant advantage in enhancing the profit growth of supply chain members.

Therefore, when an emission-reducing enterprise faces capital constraints, choosing advance payment financing can lead to higher overall supply chain performance. The government should also consider providing subsidies to support enterprises, and in some extent, product subsidies can, help enterprises strengthen their emission reduction investments.

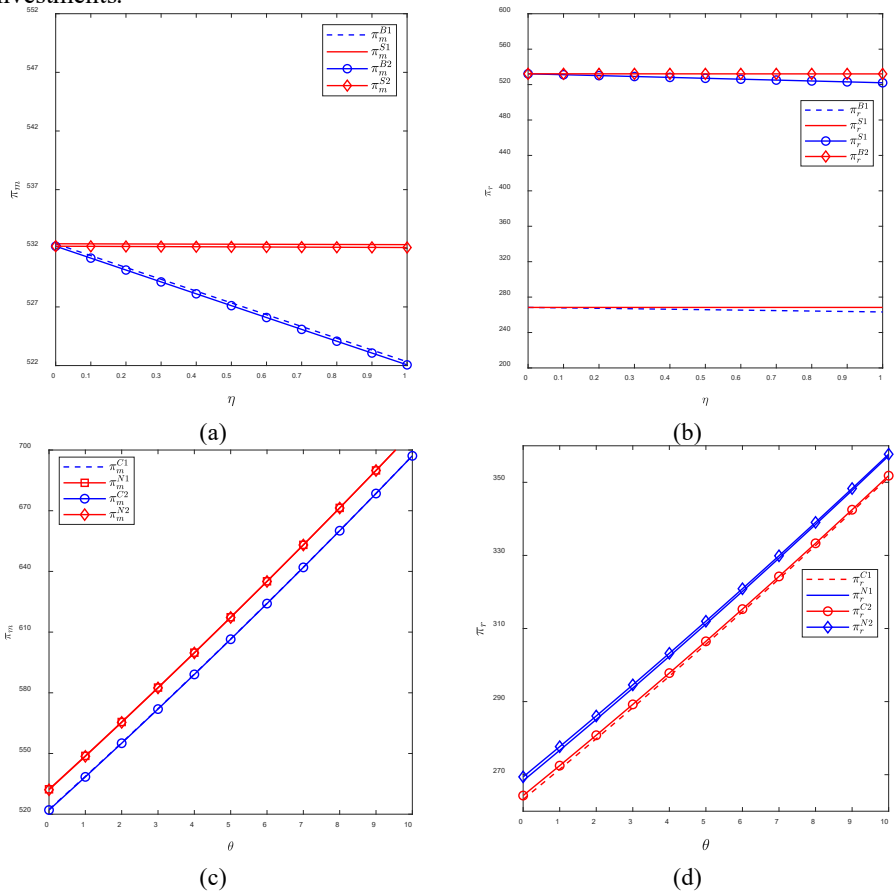


Figure 4. The impact of η and θ on π_m and π_r under both carbon emission conditions.

6 Conclusions

Carbon emission reduction is an environmental protection action vigorously promoted by countries around the world, and government emission reduction subsidies serve as an important policy tool to incentivize enterprises to reduce emissions. However, there is limited research on the impact of government subsidies under stringent carbon cap constraints on supply chain members facing financial difficulties. Based on this, we investigate the emission reduction and financing problems of supply chain members when the government adopts different emission reduction subsidies, focusing on how they alleviate financial pressure through green credit financing and advance payment financing. The main findings and managerial implications are as follows.

First, under the carbon cap policy, the government imposes two types of carbon emission constraints on emission-reducing enterprises: the stringent carbon emission condition and the lenient carbon emission condition. Setting a stringent regulatory requirement at the initial stage of enterprise emission reduction can compel relevant firms to reduce carbon emissions, thereby meeting environmental protection needs.

Second, both government subsidy mechanisms enhance supply chain performance. Under both the stringent and lenient carbon emission constraints, an increase in either the credit subsidy rate or the product subsidy rate leads to a decrease in both the wholesale and retail prices, an increase in the carbon emission reduction level, and higher profits for both the manufacturer and the retailer.

Thirdly, compared with the green credit financing strategy, the advance payment strategy is a wise choice for supply chain members to obtain financing. Under this financing strategy, both the manufacturer and the retailer can achieve lower prices, a higher carbon emission reduction level, and higher profits under different carbon emission constraints and different subsidy schemes.

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Disclosure of Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix.

Proof of Model S

Solving the first-order condition of the retailer's objective function with respect to $p(w_m, e) = \frac{a + w + \beta e}{2}$ Substituting this into the manufacturer's profit function and applying the KKT conditions yields the following Lagrangian function.

$$L(w_m, e, \lambda) = (w_m - c) \frac{(a - w + \beta e)}{2} - \frac{1}{2} (1 + r\eta) k e^2 - r\eta \left(\frac{c(a - w_m + \beta e)}{2} \right) + \lambda (1 - e - T).$$

if the equation to be solvable, the following conditions must hold.

$$\begin{cases} \frac{\partial L}{\partial w_m} = a + c(1 + \eta r) + \beta e - 2w_m = 0 \\ \frac{\partial L}{\partial e} = \beta(w_m - c(1 + \eta r)) - 2ek(1 + \eta r) - 2\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 1 - e - T \\ \lambda(1 - e - T) = 0 \\ \lambda \geq 0 \end{cases}$$

the condition is satisfied only when $\lambda = 0$ or $1 - e - T = 0$. For the case when $\lambda = 0$, solving the simultaneous equations gives the following results:

$$w_m^B = \frac{(1 + \eta r)(2k(a + c(1 + \eta r)) - c\beta^2)}{4k(1 + \eta r) - \beta^2}, \quad e_m^B = \frac{\beta(a - c(1 + \eta r))}{4k(1 + \eta r) - \beta^2},$$

$$p_r^B = \frac{(1 + \eta r)(ck(1 + \eta r) - c\beta^2 + 3ak)}{4k(1 + \eta r) - \beta^2}, \quad q^B = \frac{k(1 + \eta r)(a - c(1 + \eta r))}{4k(1 + \eta r) - \beta^2}.$$

When $1 - e - T = 0$, We obtain $e^B = 1 - T$. Substituting this into the equations and solving the system by setting them equal to zero yields the following results:

$$w_m^B = \frac{a + c(1 + \eta r) + \beta(1 - T)}{2}, \quad p_r^B = \frac{3(a + \beta(1 - T)) + c(1 + \eta r)}{4},$$

$$q^B = \frac{a - c(1 + \eta r) + \beta(1 - T)}{4}.$$

The remaining models are solved similarly, and the details are omitted.

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