Ship-Collision Causation Model of the Grand Canal Based on Formal Concept Analysis

Zhirong Tan[1,2], Lingchen Bao[*1], Tiancheng Wang[2]

[1]School of Navigation, Wuhan University of Technology, Wuhan 430063, Hubei, China
[2]State Key Laboratory of Maritime Technology and Safety, Wuhan 430063, Hubei, China

[*]Corresponding author: 352453@whut.edu.cn

Abstract. [Aim] It is essential to set up a simplified causation model in the Grand Canal so as to prevent ship-ship collision frequently. **[Data]** from year 2018 to 2022, 217 ship collision accidents retrieved from Suzhou Wujiang section. **[Method]** To apply a complex network model, a visibility graph algorithm could be given by converting time-difference series into a complex network, which aims to the features of the accident time series. The object set retrieved from the core concepts of two typical accident black spots, which the attribute set extracted from causation factors. Therefore, a causal concept lattice of ship collision accidents in the Grand Canal was set up. **[Conclusion]** A concept reduction analysis method is brought out to extract the basic reasons of two typical black spots. Thus, the strongest correlated cause are as followings: flotillas entering and leaving main route, risky overtaking, inconsistent intentions to give way, narrow navigable canal width, poor sight lines and negligence of lookout.

Keywords: Canal ship collision accident; Formal concept analysis; Attribute set; Concept reduction

1 Introduction

The Suzhou section of the Grand Grand Canal is a Grade III waterway with a total length of 81.5 km, where shipping activities are extremely busy. Taking the data from the Huguan Observation Point in 2022 as an example, the annual vessel traffic volume reached 349,754 vessel movements. However, the maritime traffic safety situation of this section remains severe. Statistics show that a total of 956 maritime traffic accidents occurred on the Suzhou section from 2018 to 2022, among which 602 were collision accidents. Preventing vessel collision accidents and putting forward targeted safety recommendations are of great significance to ensuring maritime navigation safety.

At present, numerous studies have identified the contributing factors of vessel accidents by adopting various models and methods. Langxiong Gan et al. [1] constructed a knowledge graph to determine the contributing factors of vessel collision accidents by collecting investigation reports of 241 collision incidents that took place

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from 2018 to 2021. Fault Tree Analysis (FTA) and Bayesian Network are proven to be effective methods for risk assessment and causal analysis of maritime traffic accidents [2].Ugurlu et al. [3] analyzed the causes of vessel collisions by combining Multiple Correspondence Analysis (MCA) on the basis of establishing a Fault Tree Analysis (FTA) model. Human factors are recognized as the most direct contributors to maritime accidents, and the Human Factors Analysis and Classification System (HFACS) has been widely applied in the analysis of human factors in maritime traffic accidents [4]. Laihao Ma et al. [5] identified human factors at different levels within the HFACS framework by using grounded theory, and determined the priority ranking of these factors via the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method. Ugurlu et al. [6] constructed a Bayesian Network based on the HFACS model to analyze the contributing factors of maritime traffic accidents occurring in the Black Sea over the past 20 years. The aforementioned studies mainly focus on the quantitative analysis of the correlation between human factors and accident causes. In contrast, the Suzhou section of the Grand Grand Canal features relatively stable vessel types, crew composition and water environment, which belongs to a linear steady-state water area. Therefore, the existing methods for accident cause analysis are not applicable to the investigation of vessel collision causes in the Suzhou section.

Formal Concept Analysis (FCA) is a data analysis theory proposed by Wille in 1982. Based on formal contexts, this theory extracts formal concepts, conducts formalized expression of these concepts, then constructs concept lattices to realize visual knowledge analysis [7]. Owing to its powerful capability in processing multi-type data such as text, FCA has been gradually applied in the field of accident analysis. Claudio et al. [8] proposed that FCA could be applied in the field of reliability analysis. Jihong Chen et al. analyzed the contributing factors of ship oil spill accidents [9] and port dangerous goods accidents [10] respectively by using the FCA method.Wang Shaojie et al.[11] employed Formal Concept Analysis (FCA) to demonstrate that vulnerabilities can be rapidly and accurately clustered by their common features, providing analysts with a visual lattice that reveals all inter-vulnerability relationships for further mining and patching efforts.Hao Fei et al.[12] employed Formal Concept Analysis (FCA) to demonstrate that constructing concept lattices from student-exercise and teacher-behavior data can effectively uncover high-quality teaching patterns, thereby guiding personalized instruction and enhancing students' autonomous learning ability.Li Jinhai et al.[13] employed multi-granularity real-valued Formal Concept Analysis (FCA) to demonstrate that concepts and decision rules can be systematically transferred between coarse- and fine-grained spaces, thereby extending classical FCA to interval-valued contexts and providing a theoretical basis for hierarchical knowledge discovery.Zhang Xiaohe [14] used an online rule fusion model based on formal concept analysis to prove that the regret bound of the classifier decreases with iterations, ensuring convergence to the optimal model.

To extract the key contributing factors of vessel collision accidents in the canal and formulate effective prevention and control measures, this study selects the Wujiang section of the Grand Canal—a water area with relatively stable navigation conditions and vessel traffic volume—as the typical research area, and takes 217 vessel collision accidents occurring in the Wujiang section from 2018 to 2022 as the data source. By introducing the FCA method, this study uses concept reduction theory to extract the core concepts of accident black spots and constructs a concept lattice for vessel collision accident causes. Subsequently, the discernibility attribute matrix method of formal contexts is applied to carry out attribute reduction of the concept lattice, so as to obtain the key contributing factors of collision accidents and conduct knowledge mining on the causal correlation of collision accidents.

2 Data and methods

2.1 Data Collection

Collisions represent the most prevalent type of incident occurring along the Suzhou section of the Grand Canal. Given the relative stability of vessels, crew, and aquatic environment within the Wujiang segment, this stretch may be regarded as a linear steady-state waterway. To conduct a causal analysis of canal vessel collisions, this study utilised 217 collision incidents occurring between January 2018 and December 2022 in the Wujiang section of the Grand Canal. Data was sourced from the Suzhou Municipal Transport Bureau. The temporal distribution of collisions in the Wujiang section is illustrated in Figure 1, while collision hotspots are detailed in Table 1.

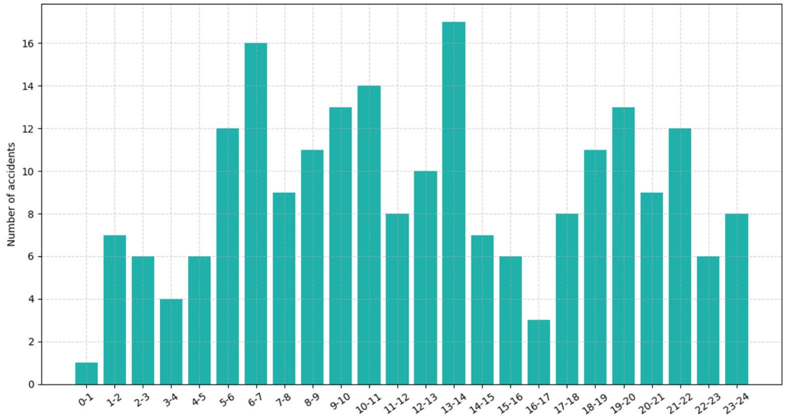

Fig. 1. Time distribution of ship collisions in Wujiang area of the Grand Canal

Table 1. Black spots of ship collisions in Wujiang area of the Grand Canal

Location	Number of accidents	Location	Number of accidents
Jiangling Bridge	11 cases	Bach Chapel waters	29 cases
Jiangxing Bridge	13 cases	Collin Bridge	9 cases
Yunli Bridge	10 cases	Wujiang waters at Watershed Pier	34 cases
Wujiang Bus Station waters	16 cases	Duck Dam waters	27 cases
Yunlong Bridge	13 cases	Tancho waters	13 cases

Based on statistics of high-incidence locations for vessel accidents along the Suzhou section of the Grand Canal, the frequency of occurrence at these accident hotspots is ranked to determine the relative risk levels of each accident site within this section.The frequency of incidents is ranked as shown in Table 2.

Table 2. Ranking of Accident Frequency Locations Along the Suzhou Section of the Grand Canal

Location	Frequency
Pingwang Anchorage	9
Pingwang Bridge	8
Fenshui Pier	5
Baodai Bridge	5
Wujiang Bus Station	4
Wangting Anchorage	4
Bacheng Anchorage	3
Yunli Bridge	3
Pingwang Xuehu Bridge	2
Shengze Anchorage	2

2.2 Formal Concept Analysis

2.2.1 Concept Lattice Construction.

Formal Concept Analysis (FCA) is an effective method for analyzing data and extracting knowledge. This method constructs a concept lattice based on a formal context, which is represented using a Hasse diagram. In the concept lattice, each node contains a concept together with its intension [15]. The extension of a concept represents a specific set of objects, which may be entities, things, or individuals from any domain, whereas the intension of a concept corresponds to the characteristics and attributes shared by these objects.

To apply the FCA method to accident causation analysis, it is necessary to first construct a causation formal context and then develop a causation concept lattice analysis model. Definitions 1–4 present the theoretical foundations for the construction of concept lattices and attribute-oriented concept lattices .

Definition 1:A triple ((U, A, I)) is defined as a *formal context*, where ($U = \{x_1, \ldots, x_n\}$) denotes the set of objects, and each x_i is referred to as an object; ($A = \{a_1, \ldots, a_n\}$) denotes the set of attributes, and each a_j is referred to as an attribute; and I is a binary relation between U and A.

In a formal context, if $(x,a) \in I$, then object x is said to possess attribute a, denoted as xIa. A value of 1 is used to indicate that $(x,a) \in I$, while a value of 0 indicates that $(x,a) \notin I$. On the object set and the attribute set, the following operations are defined, respectively:

$$X* = \{a | a \in A, \forall x \in X, xIa\} \; \forall X \subseteq U \tag{1}$$

$$B* = \{x|x \in U, \forall a \in B, xIa\}\ \forall B \subseteq A \tag{2}$$

where X* denotes the set of objects within the subset *X* of the object set *U* that share common attributes; B* denotes the set of attributes within the subset *B* of the attribute set *A* that share common objects.

Definition 2: Let (*U, A, I*) denote a formal context, wherein $X \subseteq U$ and $B \subseteq A$, with *X* and *B* being arbitrary subsets of the object set *U* and the attribute set *A* respectively. If *X** = *B* and *B** = *X*, then the ordered pair (*X, B)* is termed a formal concept. Concurrently, *X* constitutes the extension of the concept (*X, B*), whilst *B* constitutes its intension. The lattice structure formed by all concepts within the formal context (*U, A, I*) under the partial order relation is termed a concept lattice, denoted as *L*(*U, A, I*).

Definition 3: Let (*U, A, I*) denote a formal context, and suppose $X \subseteq U$ and $B \subseteq A$. Define the operators□, ◇as follows [17]:

$$X^{\square} = \{a \in A|a* \subseteq X\} \tag{3}$$

$$X^{\diamond} = \{a \in A|a* \cap X \neq \emptyset\} \tag{4}$$

$$B^{\square} = \{x \in U|x* \subseteq B\} \tag{5}$$

$$B^{\diamond} = \{x \in U|x* \cap B \neq \emptyset\} \tag{6}$$

where $X^{\square}$ denotes the set of attributes that objects in *X* necessarily possess, and $X^{\diamond}$denotes the set of attributes that objects in *X* may possess; denotes the set of attributes in *B* that necessarily possess objects, denotes the set of attributes in *B* that may possess objects.

Definition 4: Assuming (*U, A, I*) is a formal context where $X \subseteq U$ and $B \subseteq A$. If $X = B^{\square}$and $B = X^{\diamond}$, then the ordered pair (*X, B*) is termed a directional attribute concept. *X* constitutes the extension of the oriented attribute concept (*X, B*), whilst *B* constitutes its intension. The lattice structure formed by all oriented attribute concepts under the partial order relation is termed the lattice of oriented attribute concepts, denoted as $L_p(U, A, I)$.

In Formal Concept Analysis (FCA), the property-oriented concept lattice L_p(U, A, I) comprises ordered pairs (X, B) where X = B↑ and B = X↓, with X and B denoting the extent and intent, respectively. Redundancy reduction is achieved by constructing the POC representative concept matrix Λ and its minimal form Λ_{min}; the representative concept function f(Λ_{min}) is then reduced to minimal disjunctive normal form to yield all property-oriented reducts {F_i}. These reducts partition concepts into core (intersection of all reducts), relatively necessary (shared by some but not all), and absolutely unnecessary (absent from all reducts).

For attribute reduction in the standard concept lattice L(U, A, I), the discernibility matrix K is built from discernibility sets DIS((X_i, B_i), (X_j, B_j)) = ($B_i \setminus B_j$) ∪ ($B_j \setminus B_i$). The discernibility function is then defined as f(K) = $\Lambda_{\{(i \neq j)\}}$ DIS((X_i, B_i), (X_j, B_j)), which is simplified via disjunctive and conjunctive operations into its minimal disjunctive normal form; each conjunctive term therein constitutes an attribute reduct D_n . Accordingly, the attributes are partitioned into the necessary attribute set $\alpha =$

$\bigcap_{i\in N_+} D_i$, the relatively necessary attribute set $\beta = \bigcup_{i\in N_+} D_i - \bigcap_{i\in N_+} D_i$, and the absolutely unnecessary attribute set $\gamma = A - \bigcup_{i\in N_+} D_i$.

These procedures offer a rigorous framework for extracting essential knowledge and simplifying complex formal contexts[16].

3 FCA Modeling of Canal Ship

3.1 Canal Vessel Collision Accident Attribute Set

Based on collision incident reports from the Wujiang section and the physical context of accident black spots, combined with expert consultation and text mining [18], eight causal attributes for vessel collisions in the Wujiang section were ultimately identified: reckless overtaking (a_1), conflicting intentions to yield (a_2), vessel mechanical failure (a_3), fleet entry/exit from the main channel (a_4), high vessel traffic volume (a_5), relatively narrow navigable channel width (a_6), poor visibility (a_7), and negligent lookout (a_8). The collection of these attributes constitutes 'Attribute Set A'.

3.2 Object Set for Canal Ship Collision Accidents Based on Conceptual Reduction

3.2.1 Construction the Contextual Framework for the Root Causes of Ship Collision Black Spots.

The number of ship collision accidents in the Wujiang section of the Beijing–Hangzhou Grand Canal is relatively high, making it necessary to simplify a large volume of accident data in order to obtain a set of representative accident cases. Given that the Wujiang section involves complex traffic scenarios, such as ship locks and operational areas, it is not feasible to extract representative accident cases solely based on different accident categories.

Therefore, this study integrates concept reduction with accident black spot theory [19]. Two typical ship collision accident black spots in the Wujiang section are selected to construct accident causation attribute concept lattices, respectively. Through concept reduction, the core concept set for each accident black spot is extracted, thereby simplifying the overall ship collision accident dataset in the Wujiang section according to accident types and their influencing factors, and enabling the extraction of effective information. The core concept set obtained after reduction of the accident dataset is defined as the "object set G"

To extract the core concept sets of ship collision accident black spots, it is necessary to construct a causation attribute concept lattice for each collision accident black spot separately, and then obtain the corresponding concept reduction set through concept reduction. Taking the waters near Kelin Bridge and the Wujiang Bus Station as examples: The nine ship collision accidents that occurred near Kelin Bridge constitute the object set $U_1=\{x_1,x_2,...x_9\}$; Since no ship collision accidents in this area were caused by mechanical failure of vessels (a_3) or poor visibility (a_7) ,the attribute set for this area is $A1=\{a_1,a_2,a_4,a_5,a_6,a_8\}$. The sixteen ship collision accidents that occurred in the waters near the Wujiang Bus Station constitute the object set *U2={y_1,y_2,…,y_{16}}*, and the corresponding attribute set is *A={a_1,a_2,…a_8}*. Here, I denotes the binary relation between the object set and the attribute set. A value of 0 indicates that the corresponding causative attribute is not a contributing factor to the occurrence of the accident, whereas a value of 1 indicates that the attribute is one of the contributing factors leading to the accident. The formal causation context (U_1,A_1,I)for ship collision accidents at Kelin Bridge is shown in Table 3, and the formal causation context (U_2,A,I) for ship collision accidents near the Wujiang Bus Station is shown in Table 4.

Table 3. Causal formal context of ship collisions at Collin Bridge

U_1	a_1	a_2	a_4	a_5	a_6	a_8
x_1	0	0	1	0	1	0
x_2	1	0	0	1	0	1
x_3	1	0	0	1	0	1
x_4	0	0	1	1	1	0
x_5	0	0	1	0	1	0
x_6	1	0	1	1	0	0
x_7	1	0	0	0	0	1
x_8	1	0	1	1	0	0
x_9	0	1	0	0	1	1

Table 4. Causal formal context of ship collisions at Wujiang Bus Station

U_2	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
y_1	0	0	0	0	0	1	0	0
y_2	0	1	0	0	0	0	0	1
y_3	0	1	0	1	1	0	1	0
y_4	1	0	0	0	0	0	1	1
y_5	0	1	0	0	0	0	0	1
y_6	1	0	0	1	1	0	0	0
y_7	1	0	1	0	0	0	0	0
y_8	0	1	0	1	1	0	0	0
y_9	0	0	0	0	0	1	1	0
y_{10}	0	0	0	0	0	0	1	1
y_{11}	1	0	0	0	0	0	0	1
y_{12}	0	0	1	0	0	0	1	1
y_{13}	0	1	0	1	1	0	1	0
y_{14}	0	1	0	0	1	0	0	0
y_{15}	0	0	0	1	1	0	1	0
y_{16}	0	1	0	0	1	1	0	0

3.2.2 Construction for the Causal POC (Property-oriented Concept) of Ship Collision Accident Black Spots.

Based on the causal form background of the Collin Bridge vessel collision incident *(U_1, A_1, I)* and the causal form background of the Wujiang Bus Station vessel collision incident *(U_2, A, I)*, each aspect attribute concept is treated as a node within the causal attribute concept lattice. The inclusion relationship between each node is determined, and an edge is drawn between any two nodes sharing such an inclusion relationship. The causal attribute concept lattice for the Collin Bridge vessel collision accident is constructed as shown in Figure 2; the causal attribute concept lattice for the Wujiang Bus Station vessel collision accident is depicted in Figure 3. The node concepts and their complement concepts within the Wujiang Bus Station attribute concept lattice are listed in Table 5.

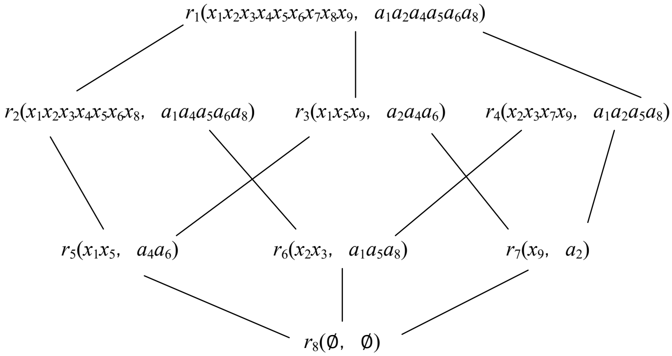

Fig. 2. Causal POC lattice of ship collision at Collin Bridge

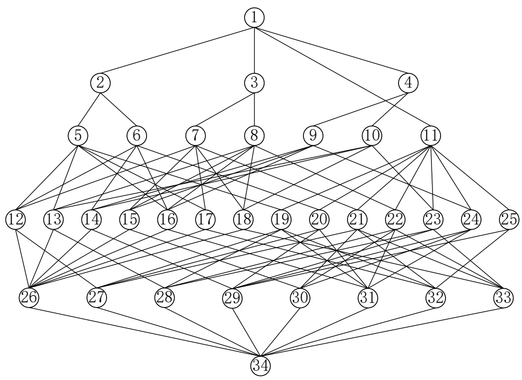

Fig. 3. Causal POC lattice of ship collision at Wujiang Bus Station

Table 5. Each node of Concepts and complement concept in POC lattice

ID	Concept	Complementary Concept
1	(U_2, A)	$(U_2, \emptyset)$
2	$(y_1y_2y_5y_6y_7y_{11}y_{14}y_{16}, a_1a_2a_3a_4a_5a_6a_8)$	$(y_1y_2y_5y_6y_7y_{11}y_{14}y_{16}, a_7)$
3	$(y_1y_2y_5y_6y_9y_{10}y_{11}y_{14}y_{15}y_{16}, a_1a_2a_4a_5a_6a_7a_8)$	$(y_1y_2y_5y_6y_9y_{10}y_{11}y_{14}y_{15}y_{16}, a_3)$
4	$(y_1y_6y_7y_9y_{10}y_{11}y_{15}, a_1a_3a_4a_5a_6a_7a_8)$	$(y_1y_6y_7y_9y_{10}y_{11}y_{15}, a_2)$
5	$(y_1y_2y_5y_6y_{11}y_{14}y_{16}, a_1a_2a_4a_5a_6a_8)$	$(y_1y_2y_5y_6y_{11}y_{14}y_{16}, a_3a_7)$
6	$(y_1y_2y_5y_7y_{11}, a_1a_2a_3a_6a_8)$	$(y_1y_2y_5y_7y_{11}, a_4a_5a_7)$
7	$(y_1y_2x_5y_9y_{10}y_{14}y_{15}y_{16}, a_2a_4a_5a_6a_7a_8)$	$(y_1y_2x_5y_9y_{10}y_{14}y_{15}y_{16}, a_1a_3)$
8	$(y_1y_6y_9y_{10}y_{11}y_{15}, a_1a_4a_5a_6a_7a_8)$	$(y_1y_6y_9y_{10}y_{11}y_{15}, a_2a_3)$

9	$(y_1y_7y_9y_{10}y_{11}, a_1a_3a_6a_7a_8)$	$(y_1y_7y_9y_{10}y_{11}, a_2a_4a_5)$
10	$(y_1y_6y_7y_9y_{15}, a_1a_3a_4a_5a_6a_7)$	$(y_1y_6y_7y_9y_{15}, a_2a_8)$
11	$(y_2y_3y_4y_5y_6y_7y_8y_{10}y_{11}y_{12}y_{13}y_{14}y_{15}, a_1a_2a_3a_4a_5a_7a_8)$	$(y_2y_3y_4y_5y_6y_7y_8y_{10}y_{11}y_{12}y_{13}y_{14}y_{15}, a_6)$
		. . . .
29	(y_7, a_1a_3)	$(y_7, a_2a_4a_5a_6a_7a_8)$
30	(y_{10}, a_7a_8)	$(y_{10}, a_1a_2a_3a_4a_5a_6)$
31	(y_{11}, a_1a_8)	$(y_{11}, a_2a_3a_4a_5a_6a_7)$
32	(y_{14}, a_2a_5)	$(y_{14}, a_1a_3a_4a_6a_7a_8)$
33	$(y_{15}, a_4a_5a_7)$	$(y_{15}, a_1a_2a_3a_6a_8)$
34	$(\emptyset, \emptyset)$	$(\emptyset, A)$

Each node within the causal attribute concept grid comprises a concept and its intension. The extension of the concept consists of accident cases occurring at collision black spots, while its intension comprises the causal factors of collision accidents.

3.2.3 Simplified Conceptual Construction for the Causal Attributes of Ship Collision Accident Lack Spots.

Due to the presence of numerous concepts within the causal attribute concept grid, many of which contain irrelevant or redundant information, this study employs a POC representative concept matrix to reduce the grid and extract its core concepts. First, the complement concepts for all concepts within the causal attribute concept lattice must be obtained. The complement concepts for each concept in Colin Bridge's causal attribute concept lattice are: $(U_1, \emptyset)$, $(x_1x_2x_3x_4x_5x_6x_8, a_2)$, $(x_1x_5x_9, a_1a_5a_8)$, $(x_2x_3x_7x_9, a_4a_6)$, $(x_1x_5, a_1a_2a_5a_8)$, $(x_2x_3, a_2a_4a_6)$, $(x_9, a_1a_4a_5a_6a_8)$, $(\emptyset, A_1)$. Then, select one object x_i and one attribute a_i from the object set U_1 and attribute set A_1 respectively, while traversing all complementary concepts. Taking (x_7, a_4) as an example: (x_7, a_4) belongs to $(x_2, x_3, x_7, x_9) \times (a_4, a_6)$, and no other complementary concepts form a similar relationship with (x_7, a_4). Therefore, concept $r_4(x_2x_3x_7x_9, a_1a_2a_5a_8)$ serves as the representative concept oriented towards attribute concepts. Similarly, the representative concepts for all attribute concepts form the POC representative concept matrix Λ. Furthermore, to obtain the attribute concept reduction set for the causal formative context (U_1, A_1, I) of the Collin Bridge vessel collision incident, the minimal POC representative concept matrix $\Lambda_{\min}$ must be constructed. Through disjunctive and conjunctive operations on $\Lambda_{\min}$, the representative concept function $f(\Lambda_{\min})$ is simplified to minimal disjunctive normal form. whose conjunctive expressions collectively represent the reduced attribute concepts of the causal formal context (U_1, A_1, I). The POC representative concept matrix Λ for the Collin Bridge incident is shown in Equation 7, while the minimal POC representative concept matrix $\Lambda_{\min}$ is presented in Equation 8.

$$\boldsymbol{\Lambda}=\begin{pmatrix} r_3r_5 & r_2r_5 & \emptyset & r_3r_5 & \emptyset & r_3r_5 \\ \emptyset & r_2r_6 & r_4r_6 & \emptyset & r_4r_6 & \emptyset \\ \emptyset & r_2r_6 & r_4r_6 & \emptyset & r_4r_6 & \emptyset \\ \emptyset & r_2 & \emptyset & \emptyset & \emptyset & \emptyset \\ r_3r_5 & r_2r_5 & \emptyset & r_3r_5 & \emptyset & r_3r_5 \\ \emptyset & r_2 & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & r_4 & \emptyset & r_4 & \emptyset \\ \emptyset & r_2 & \emptyset & \emptyset & \emptyset & \emptyset \\ r_3r_7 & \emptyset & r_4r_7 & r_3r_7 & r_4r_7 & r_3r_7 \end{pmatrix} \tag{7}$$

$$\boldsymbol{\Lambda}\begin{pmatrix} r_3r_5 & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & r_2 & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & r_4 & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ r_3r_7 & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}_{min} \tag{8}$$

$f(\boldsymbol{\Lambda}_{\text{min}})=r_2\wedge r_4\wedge(r_3\vee r_5)\wedge(r_3\vee r_7)=r_2\wedge r_4\wedge[r_3\vee(r_5\wedge r_7)]=(r_2\wedge r_3\wedge r_4)\vee(r_2\wedge r_4\wedge r_5\wedge r_7)$

The concept reduction sets for the attribute-oriented approach concerning the causal background of the Collin Bridge vessel collision incident comprise two elements: $V_1=\{r_2\wedge r_3\wedge r_4\}$ and $V_2=\{r_2\wedge r_4\wedge r_5\wedge r_7\}$.

To determine the relative importance of different attribute concepts, we may categorise them into three groups: core-oriented attribute concept sets, relatively necessary-oriented attribute concept sets, and absolutely unnecessary-oriented attribute concept sets. The core attribute concept set comprises concepts common to all attribute concept reduction sets; the relatively necessary attribute concept set comprises concepts shared by one or more attribute concept reduction sets, though not all concept reduction sets; the absolutely unnecessary attribute concept set comprises concepts appearing in the attribute concept lattice but absent from all attribute concept reduction sets.

Similarly, the complementary concepts for all concepts within the causal attribute concept grid of the Wujiang Bus Station waterway vessel collision incident are presented in Table 5. The POC represents the concept matrix Λ as shown in Equation 9, while the minimum POC denotes the concept matrix Λ_{min} as depicted in Equation 10.

$$\boldsymbol{\Lambda}=\begin{pmatrix} c_7c_{12}c_{15}c_{17}c_{18}c_{26} & c_4c_8c_9c_{10}c_{13}c_{14}c_{15}c_{16}c_{18}c_{26} & c_3c_5c_7c_8c_{12}c_{13}c_{15}c_{16}c_{17}c_{18}c_{26} & c_6c_9c_{12}c_{14}c_{15}c_{16}c_{17}c_{26} & c_6c_9c_{12}c_{14}c_{15}c_{16}c_{17}c_{26} & \emptyset & c_2c_5c_6c_{12}c_{13}c_{14}c_{16}c_{17}c_{26} & c_{10}c_{13}c_{14}c_{17}c_{18}c_{26} \\ c_7c_{12}c_{21}c_{27} & \emptyset & c_3c_5c_7c_{12}c_{19}c_{21}c_{27} & c_6c_{12}c_{20}c_{27} & c_6c_{12}c_{20}c_{27} & c_{11}c_{19}c_{20}c_{21}c_{27} & c_2c_5c_6c_{12}c_{19}c_{20}c_{27} & \emptyset \\ c_{21} & \emptyset & c_{21} & \emptyset & \emptyset & c_{11}c_{21} & \emptyset & \emptyset \\ \emptyset & c_{22} & c_{22} & \emptyset & \emptyset & c_{11}c_{22} & \emptyset & \emptyset \\ c_7c_{12}c_{21}c_{27} & \emptyset & c_3c_5c_7c_{12}c_{19}c_{21}c_{27} & c_6c_{12}c_{20}c_{27} & c_6c_{12}c_{20}c_{27} & c_{11}c_{19}c_{20}c_{21}c_{27} & c_2c_5c_6c_{12}c_{19}c_{20}c_{27} & \emptyset \\ \emptyset & c_4c_8c_{10}c_{13}c_{22}c_{23}c_{26} & c_3c_5c_7c_8c_{13}c_{19}c_{22}c_{26} & \emptyset & \emptyset & c_{11}c_{19}c_{22}c_{23}c_{28} & c_2c_5c_{13}c_{19}c_{28} & c_{10}c_{13}c_{23}c_{28} \\ \emptyset & c_4c_9c_{10}c_{14}c_{23}c_{24}c_{29} & \emptyset & c_6c_9c_{14}c_{20}c_{24}c_{25}c_{29} & c_6c_9c_{14}c_{20}c_{24}c_{29} & c_{11}c_{20}c_{23}c_{24}c_{25}c_{29} & c_2c_6c_{14}c_{20}c_{25}c_{29} & c_{10}c_{14}c_{23}c_{25}c_{29} \\ c_{21} & \emptyset & c_{19}c_{21} & \emptyset & \emptyset & c_{11}c_{19}c_{21} & c_{19} & \emptyset \\ c_7c_{15}c_{18} & c_4c_8c_9c_{10}c_{15}c_{18} & c_3c_7c_8c_{15}c_{18} & c_9c_{15} & c_9c_{15} & \emptyset & \emptyset & c_{10}c_{18} \\ c_7c_{15}c_{21}c_{30} & c_4c_8c_9c_{15}c_{22}c_{24}c_{30} & c_3c_7c_8c_{15}c_{21}c_{22}c_{30} & c_9c_{15}c_{24}c_{30} & c_9c_{15}c_{24}c_{30} & c_{11}c_{21}c_{22}c_{24}c_{30} & \emptyset & \emptyset \\ \emptyset & c_4c_8c_9c_{16}c_{22}c_{31} & c_3c_5c_8c_{16}c_{19}c_{22}c_{31} & c_6c_9c_{16}c_{20}c_{31} & c_6c_9c_{16}c_{20}c_{31} & c_{11}c_{19}c_{20}c_{22}c_{31} & c_2c_5c_6c_{16}c_{19}c_{20}c_{31} & \emptyset \\ \emptyset & c_{24} & \emptyset & c_{24} & c_{24} & c_{11}c_{24} & \emptyset & \emptyset \\ c_{21} & \emptyset & c_{21} & \emptyset & \emptyset & c_{11}c_{21} & \emptyset & \emptyset \\ c_7c_{17}c_{21}c_{32} & \emptyset & c_3c_5c_7c_{17}c_{19}c_{21}c_{32} & c_{17}c_{25}c_{32} & \emptyset & c_{11}c_{19}c_{21}c_{25}c_{32} & c_2c_5c_{17}c_{19}c_{25}c_{32} & c_{17}c_{25}c_{32} \\ c_7c_{18}c_{21}c_{33} & c_4c_8c_{10}c_{18}c_{22}c_{23}c_{33} & c_3c_7c_8c_{18}c_{21}c_{22}c_{33} & \emptyset & \emptyset & c_{11}c_{21}c_{22}c_{23}c_{33} & \emptyset & c_{10}c_{18}c_{23}c_{33} \\ c_7c_{17} & \emptyset & c_3c_5c_7c_{17} & c_{17} & \emptyset & \emptyset & c_2c_5c_{17} & c_{17} \end{pmatrix} \tag{9}$$

$$\boldsymbol{\Lambda}\begin{pmatrix} \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & c_6c_{12}c_{20}c_{27} & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & c_{22} & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & c_{10}c_{13}c_{23}c_{28} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & c_2c_6c_{14}c_{20}c_{25}c_{29} & c_{10}c_{14}c_{23}c_{25}c_{29} \\ c_{21} & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & c_{19} & \emptyset \\ c_7c_{15}c_{18} & \emptyset & \emptyset & c_9c_{15} & \emptyset & \emptyset & \emptyset & c_{10}c_{18} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & c_6c_9c_{16}c_{20}c_{31} & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & c_{24} & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & c_{17} & \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}_{min} \tag{10}$$

$$f(\boldsymbol{\Lambda}_{\min})=c_{17}\wedge c_{19}\wedge c_{21}\wedge c_{22}\wedge c_{24}\wedge(c_7\vee c_5\vee c_{18})\wedge(c_9\vee c_{15})\wedge(c_{10}\vee c_{18})\wedge(c_6\vee c_{12}\vee c_{20}\vee c_{27})\wedge(c_6\vee c_9\vee c_{16}\vee c_{20}\vee c_{31})\wedge(c_2\vee c_6\vee c_{14}\vee c_{20}\vee c_{25}\vee c_{29})\wedge(c_{10}\vee c_{13}\vee c_{23}\vee c_{28})\wedge(c_{10}\vee c_{14}\vee c_{23}\vee c_{25}\vee c_{29})$$

$$=c_{17}\wedge c_{19}\wedge c_{21}\wedge c_{22}\wedge c_{24}\wedge[(c_9\wedge c_{18})\vee(c_{10}\wedge c_{15})\vee(c_{15}\wedge c_{18})\vee(c_7\wedge c_9\wedge c_{10})]\wedge[(c_2\wedge c_{10})\vee(c_6\wedge c_{10})\vee(c_{10}\wedge c_{20})\vee(c_2\wedge c_{23})\vee(c_6\wedge c_{23})\vee(c_{20}\wedge c_{23})\vee(c_{10}\wedge c_{14}\wedge c_{25}\wedge c_{29})\vee(c_{13}\wedge c_{14}\wedge c_{25}\wedge c_{29})\vee(c_{14}\wedge c_{23}\wedge c_{25}\wedge c_{29})\vee(c_{14}\wedge c_{25}\wedge c_{28}\wedge c_{29})]\wedge[(c_6\wedge c_{20})\vee(c_9\wedge c_{12})\vee(c_{12}\wedge c_{16})\vee(c_{12}\wedge c_{31})\vee(c_9\wedge c_{27})\vee(c_{16}\wedge c_{27})\vee(c_{27}\wedge c_{31})]$$

Due to the excessive number of concept reduction sets for attribute concepts arising from the Wujiang Bus Station vessel collision incident, and given that deriving all concept reductions holds little practical significance, only partial concept reduction sets are presented. The attribute concept classification is shown in Table 6.

Table 6. POC classification of formal context of ship collision at Wujiang Bus Station

Types of property-oriented concept	Property-oriented concept
Core POC set ζ	c_{17}, c_{19}, c_{21}, c_{22}, c_{24}
Relatively necessary POC set τ	c_2, c_6, c_7, c_9, c_{10}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{18}, c_{20}, c_{23}, c_{25}, c_{27}, c_{28}, c_{29}, c_{31}
Absolutely unnecessary POC set Ψ	c_1, c_3, c_4, c_5, c_8, c_{11}, c_{26}, c_{30}, c_{32}, c_{33}, c_{34}

The collision incident involving vessels in the waters near Colin Bridge and Wujiang Bus Station encompasses seven core concepts. Given the complexity of the extensions of these seven concepts, their extensions are denoted as z_1, z_2, …, z_7 for simplified expression. The object set G = {z_1, z_2, …, z_7} of the FCA model for canal vessel collision incidents is presented in Table 7.

Table 7. Object set for the FCA model of canal ship collision accident

Extension	Connotation
z_1	$a_1a_4a_5a_6a_8$
z_2	$a_1a_2a_5a_8$
z_3	$a_2a_5a_6$
z_4	$a_1a_2a_4a_5a_8$

z_5	$a_2a_4a_5a_7a_8$
z_6	$a_1a_4a_5a_7a_8$
z_7	$a_1a_3a_7a_8$

3.3 Conceptual Construction for Canal Ship Collision Accidents

Based on the core concept set and causal attributes of two vessel collision black spots along the Suzhou Wujiang section of the Grand Canal, a causal formal background (*G, A, I*) for vessel collision incidents in this section is established. Here, the object set is $G = \{z_1, z_2, \ldots, z_8\}$, the attribute set is $A = \{a_1, a_2, \ldots, a_8\}$, and *I* denotes the binary relation between *G* and *A*. The constructed formal causal background (*G,A,I*) for vessel collision accidents in the Suzhou Wujiang section of the Grand Canal is presented in Table 8:

Table 8. Causes of Ship Collision Accidents in the Wujiang Section of the Grand Canal

G	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
z_1	1	0	0	1	1	1	0	1
z_2	1	1	0	0	1	0	0	1
z_3	0	1	0	0	1	1	0	0
z_4	1	1	0	1	1	0	0	1
z_5	0	1	0	1	1	0	1	1
z_6	1	0	0	1	1	0	1	1
z_7	1	0	1	0	0	0	1	1

Based on the causal form background (G, A, I) of vessel collision incidents in the Wujiang section, a conceptual framework for the causes of vessel collisions in the Suzhou-Wujiang section of the Grand Canal is constructed (see Figure 4). The concepts represented by each node within this causal conceptual framework are detailed in Table 9.

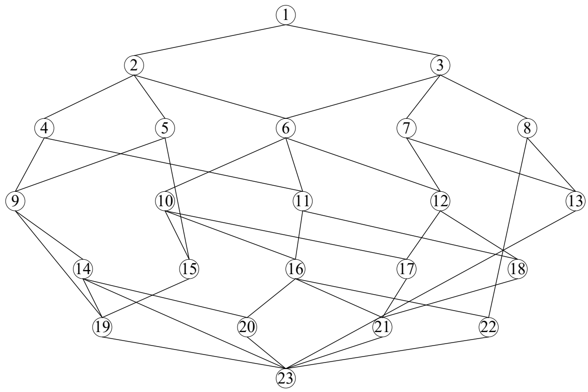

Fig. 4. Causal concept lattice of ship collisions in Suzhou–Wujiang section of Grand Canal

Table 9. Concepts of each node in causal concept lattice

ID	Concept	ID	Concept
1	$(G, \emptyset)$	13	$(z_3, a_2a_5a_6)$
2	$(z_1z_2z_4z_5z_6z_7, a_8)$	14	$(z_7, a_1a_3a_7a_8)$
3	$(z_1z_2z_3z_4z_5z_6, a_5)$	15	$(z_5z_6, a_4a_5a_7a_8)$
4	$(z_1z_2z_4z_6z_7, a_1)$	16	$(z_1z_4z_6, a_1a_4a_5a_8)$
5	$(z_5z_6z_7, a_7)$	17	$(z_4z_5, a_2a_4a_5a_8)$
6	$(z_1z_2z_4z_5z_6, a_5a_8)$	18	$(z_2z_4, a_1a_2a_5a_8)$
7	$(z_2z_3z_4z_5, a_2)$	19	$(z_6, a_1a_4a_5a_7a_8)$
8	(z_1z_3, a_6)	20	$(z_5, a_2a_4a_5a_7a_8)$
9	$(z_6z_7, a_1a_7a_8)$	21	$(z_4, a_1a_2a_4a_5a_8)$
10	$(z_1z_4z_5z_6, a_4a_5a_8)$	22	$(z_1, a_1a_4a_5a_6a_8)$
11	$(z_1z_2z_4z_6, a_1a_5a_8)$	23	$(\emptyset, A)$
12	$(z_2z_4z_5, a_2a_5a_8)$		

Due to the numerous nodes within the causal concept lattice, it proves challenging to extract key elements of the causal factors underlying vessel collision incidents along the Wujiang section of the Grand Canal. To enhance the accuracy of causal analysis, this study employs the recognisable attribute matrix method within the formal background framework to simplify the attributes within the causal concept lattice.

3.4 Attribute Reduction of Canal Ship Collisions Causal Concept Lattice

To construct an identifiable attribute matrix for causal form backgrounds, based on the causal form background *(G, A, I)* of vessel collision incidents in the Wujiang section, two objects are arbitrarily selected from the object set G to identify the differences in attributes they contain. If object z_1 contains attributes $a_1a_4a_5a_8$, and object z_6 contains attributes $a_1a_4a_5a_7a_8$, then the differential attribute between them is a_6a_7. Similarly, the discernible attribute matrix *K* constituting the formal causal background of incident causation is formed by the differential attributes among all objects in the object set *G*, as shown in Table 10.

Table 10. Discernible attribute matrix of ship collision causal formal context in Wujiang section of Grand Canal,Suzhou

	z_1	z_2	z_3	z_4	z_5	z_6	z_7
z_1	$\emptyset$						
z_2	$a_2a_4a_6$	$\emptyset$					
z_3	$a_1a_2a_4a_8$	$a_1a_6a_8$	$\emptyset$				
z_4	a_2a_6	a_4	$a_1a_4a_6a_8$	$\emptyset$			
z_5	$a_1a_2a_6a_7$	$a_1a_4a_7$	$a_4a_6a_7a_8$	a_1a_7	$\emptyset$		
z_6	a_6a_7	$a_2a_4a_7$	$a_1a_2a_4a_6a_7a_8$	a_2a_7	a_1a_2	$\emptyset$	
z_7	$a_3a_4a_5a_6a_7$	$a_2a_3a_5a_7$	$a_1a_2a_3a_5a_6a_7a_8$	$a_2a_3a_4a_5a_7$	$a_1a_2a_3a_4a_5$	$a_3a_4a_5$	$\emptyset$

By applying disjunctive and conjunctive operations to the recognisable attribute matrix $\boldsymbol{K}$, the recognisable function $f(\boldsymbol{K})$ is simplified into minimal disjunctive normal form, wherein all components constitute reductions of the recognisable matrix $\boldsymbol{K}$.

$$f(\boldsymbol{K})=(a_4)\wedge(a_2 \vee a_6)\wedge(a_6 \vee a_7)\wedge(a_1 \vee a_7)\wedge(a_2 \vee a_7)\wedge(a_1 \vee a_2)\wedge(a_1 \vee a_6 \vee a_8)$$
$$=(a_1\wedge a_2\wedge a_4\wedge a_7)\vee(a_2\wedge a_4\wedge a_7\wedge a_8)\vee(a_1\wedge a_2\wedge a_4\wedge a_6)\vee(a_1\wedge a_4\wedge a_6\wedge a_7)\vee(a_2\wedge a_4\wedge a_6\wedge a_7)$$

The five attribute reduction sets are as follows: $D_1=(a_1\ a_2\ a_4\ a_7)$, $D_2=(a_2\ a_4\ a_7\ a_8)$, $D_3=(a_1\ a_2\ a_4\ a_6)$, $D_4=(a_1\ a_4\ a_6\ a_7)$, $D_5=(a_2\ a_4\ a_6\ a_7)$

To ascertain the significance of different accident causative factors, we may categorise them into essential attribute sets, relatively essential attribute sets, and non-essential attribute sets. Essential attribute sets comprise all factors common to every attribute reduction set; relatively essential attribute sets include one or more factors shared by some but not all attribute reduction sets; while non-essential attribute sets encompass attributes appearing in the formal context yet absent from all attribute reduction sets.

Necessary attribute set $\alpha = \{a_4\}$; Relatively necessary attribute set $\beta = \{a_1, a_2, a_6, a_7, a_8\}$; Non-necessary attribute set $\gamma = \{a_3, a_5\}$;

4 Results and Discussion of the FCA Model

The causal factors of vessel collisions in the Wujiang section of the Grand Canal near Suzhou can be categorised into three groups. Essential attributes include fleets entering and exiting the main navigation channel (a_4); relatively essential attributes encompass reckless overtaking (a_1), conflicting intentions to yield (a_2), narrow navigable width (a_6), poor visibility (a_7), and negligent lookout (a_8); Non-essential attributes include vessel mechanical failure (a_3) and high vessel traffic volume (a_5). Both essential and relatively essential attributes influence vessel collision incidents; targeted improvements in these areas can effectively reduce such occurrences. Although non-essential attributes exert lesser influence on collision incidents, they maintain a causal relationship with essential attributes. Enhancing these factors also contributes to collision prevention.

Along sections of the Wujiang stretch of the Grand Canal, numerous company wharves and harbour basins flank the waterway, with fleets operating continuously. Due to their slow speeds, these fleets frequently cause channel closures or blockages when entering harbour basins. Moreover, their movements often obstruct the sightlines of other vessels, significantly disrupting navigational order. Furthermore, fleets routinely berth along the channel margins, occupying portions of the navigable width and thereby reducing the space available for vessel passage and passing manoeuvres. The poor navigational order in these sections creates complex encounter scenarios between vessels, fostering conditions conducive to risky overtaking manoeuvres and conflicting intentions, which can lead to collisions.

The Wujiang section of the Grand Canal features numerous wharves with high vessel berthing volumes. Most vessels berth at an angle with minimal spacing, further constricting navigational passages. Additionally, rapid siltation near certain wharves forces vessels unable to berth alongside to anchor along the channel edges, further reducing navigable width. Improper vessel mooring complicates navigation order in this section, constituting a significant factor in collision incidents. Moreover, wake waves generated by passing vessels can displace obliquely moored craft, causing collisions between vessels with minimal clearance.

Based on the temporal distribution of vessel collisions in the Wujiang section of the Grand Canal (Figure 1), the majority of incidents occur during breakfast, lunch, and dinner periods. Vessels navigating this canal are predominantly family-operated, typically manned by approximately three crew members. During meal times, insufficient personnel are assigned to lookout duties, increasing the likelihood of negligence in vigilance and thereby heightening collision risks.

The Wujiang section of the Grand Canal features several parks with dense vegetation. This terrestrial foliage readily obstructs vessels' sightlines, making it difficult for navigating craft to anticipate traffic flow and vessel movements within the channel. This significantly increases the likelihood of collisions due to delayed anticipation and evasive manoeuvres. Furthermore, various landscape lighting strips have been installed along the Suzhou urban section of the Grand Canal. During night-time navigation, these bright and conspicuous lighting strips adversely affect crew lookout, significantly impacting the safe operation of vessels.

5 Conclusion

This study analyses the causal factors of vessel collision incidents on the Wujiang section of the Grand Canal from 2018 to 2022 by constructing a FCA model based on accident data. The visible graph algorithm was employed to transform the accident time series into a complex network for analysing temporal characteristics of collision incidents. Subsequently, an attribute set was constructed through expert consultation and text mining. Concept reduction was employed to establish an object set using two typical collision black spots within the Wujiang section. This enabled the construction of a conceptual lattice for collision accident causation in the Wujiang section, followed by attribute reduction to determine the relative importance of various causal factors. The findings offer a more comprehensive analysis than existing methodologies, providing valuable guidance for maritime authorities in formulating preventive measures and enhancing waterborne traffic safety.

Findings indicate that the complex network constructed from collision accident time series in the Wujiang section exhibits scale-free and small-world properties, revealing core causal factors critical to accident occurrence in this waterway. The most critical factor contributing to canal collisions is the entry and exit of fleets into the main navigation channel. Secondary factors include reckless overtaking, conflicting intentions during passing manoeuvres, narrow navigable width, poor visibility, and negligence in lookout duties. Ship mechanical failures and high vessel traffic volumes exert relatively minor influences on accident occurrence.

This study proposes causal analysis of canal vessel collisions through FCA modelling, innovatively employing concept reduction theory to extract core concepts for constructing the FCA object set. Nevertheless, limitations remain. Firstly, core concepts were not extracted for all collision black spots in the Wujiang section; analysis was confined to two representative black spots. Moreover, while the study focused solely on vessel collisions, the FCA methodology could also analyse other types of vessel incidents, such as groundings and groundings. Consequently, it is necessary to collect data on other vessel accident types to conduct a more comprehensive analysis of the causes of vessel accidents in the Suzhou Wujiang section of the Grand Canal.

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