Torque Sum Analysis Method for Slope Stability Based on Limit Equilibrium

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Abstract. The torque sum method is an analytical method proposed on the basis of the vector sum method by considering slope rotation. Aiming at the limitations that the torque sum method (TSM) fails to satisfy the mechanical equilibrium conditions under limit equilibrium state for slope stability analysis, a strict-equilibrium torque sum method (SE-TSM) is proposed in this paper.

Keywords: Slope; Stability Analysis; Limit Equilibrium State; Strict-Equilibrium Torque Sum Method

1 Introduction

Slopes are one of the common natural environments on which human beings depend for survival and development. Safety accidents caused by slope instability are numerous. The study and assurance of slope stability constitute an important field in geotechnical engineering[1]. In slope stability analysis, the most intuitive and primary criterion is the safety factor, which is used to judge whether a slope is stable. However, many analysis methods for the safety factor exist, and certain differences exist among the safety factor values obtained by different approaches[2]. At present, there are three main definitions of the slope stability safety factor in the engineering field: (a) The ratio of anti-sliding force (moment) to sliding force (moment) (or resistance-load ratio), which is adopted in planar sliding analysis and the Swedish slice method; (b) The ratio of shear strength to actual shear stress on the sliding surface, for which the strength reduction method[3] is commonly used, such as the Bishop method[4] and Janbu method[5]; (c) The overload definition, namely the load amplification definition, in which the external load is amplified until slope failure occurs, and the amplification factor is taken as the safety factor.

However, few existing analysis methods consider the vector characteristics of sliding, and many artificial assumptions are introduced in the process of analyzing the real state. To remedy this defect, Academician Ge Xiurun started from the basic concept of sliding, pointed out that sliding itself is a process of vector change, and obtained the sliding force and anti-sliding force on the sliding surface from the perspective of vectors, thereby deriving the safety factor of the slope. This method is called

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the vector sum method (VSM) [6,7], and its safety factor follows the first definition category.

The vector sum method is a slope stability analysis method proposed based on the vector characteristics of forces. After calculating the resultant force of the sliding mass, this method assumes the overall direction of the downward sliding trend and regards the sliding mass as moving translationally along this direction. Nevertheless, when a slope fails and slides, the sliding surface is often circular, polygonal, curved, or in other forms, indicating that the sliding process involves not only translation but also rotation of the slope mass. The vector sum method mainly considers the translational motion of the sliding mass. On this basis, Chen Wensheng et al. proposed a new slope analysis method for the rotational behavior of the slope mass: the torque sum method (TSM) [8], which also belongs to the first category of safety factor definitions. In this method, the slope stability safety factor is defined as the ratio of the sum of moments generated by the ultimate strength that the potential sliding surface can provide to resist the rotation of the slope mass around a certain point to the sum of moments caused by various loads (including self-weight) acting on the sliding mass and rotating around the same point.

The solution of the safety factor in the torque sum method depends on the determination of the real stress state of the slope, and the key of the torque sum method lies in the determination of the rotation center of the sliding mass. However, this method has theoretical limitations. Under the limit equilibrium state, the torque sum method only satisfies the moment equilibrium when the slope rotates around the assumed center. Since the torque sum method cannot simultaneously satisfy the strict force and moment equilibrium conditions under the limit equilibrium state, it is essentially a non-strict slope stability analysis method. To improve the existing method, based on the mechanical equilibrium requirements under the limit equilibrium state, this paper proposes a strict-equilibrium torque sum method, which establishes an approach to determine the unique rotation center of the sliding mass that satisfies strict equilibrium for the torque sum method.

2 Methodology

2.1 Basic Assumptions

For the strict-equilibrium torque sum method, the following basic assumptions are proposed:

(a) The stress distribution in the slope calculation domain can be obtained by stress analysis methods such as the finite element method and slice method. The basic physical and mechanical parameters of the slope, loads, and boundary conditions are determined through field investigation.

(b) Let Ω denote the computational domain of the slope, within which the potential sliding surface S of the slope is known.

(c) On the potential sliding surface of the slope, the material strength properties of the rock and soil mass are assumed to obey the Mohr-Coulomb criterion, and the shear strength is given by:

$$\tau_f = c + \sigma tan\varphi \tag{1}$$

(d) The safety factor of the strict-equilibrium torque sum method for rotation about a point is defined as follows:

Let the coordinates of the rotation point P be *(x,y)*.

The safety factor is the ratio of the algebraic sum of moments about P from the resistive forces acting on the sliding surface (where shear forces take the strength values and normal resistances take the static equilibrium values) to the algebraic sum of moments about P from the calculated stresses on the sliding surface:

$$F(\mathrm{x}, y) = \frac{\sum M_r(x, y)}{\sum M_s(x, y)} \tag{2}$$

(e) The sliding surface is the interface where the sliding mass detaches from the parent body, which can be of arbitrary shape. For a rotating sliding mass, the sliding surface also acts as a constraint boundary, and the sliding mass is regarded as a rigid body.

(f) Under the plane limit equilibrium state, the sliding mass is required to globally satisfy the force equilibrium in the x- and y-directions (or any two mutually perpendicular directions in the plane) and the moment equilibrium about the rotation center.

(g) The coordinates of the rotation center of the sliding mass are uniquely determined by the requirements of force equilibrium and moment equilibrium. Thus, the proposed method is a strict-equilibrium analysis method.

2.2 Safety Factor Expression

For cases where slope rotation may occur, the safety factor of the strict-equilibrium torque sum method is defined by reference to the vector sum method as the ratio of the total ultimate anti-rotation moment about a certain rotation center provided by the potential sliding surface to the total sliding moment about the same rotation center from the calculated stresses acting on the sliding surface. The anti-sliding moment is provided by the shear strength and normal stress on the sliding surface, while the sliding moment is provided by the tangential stress and normal stress on the sliding surface.

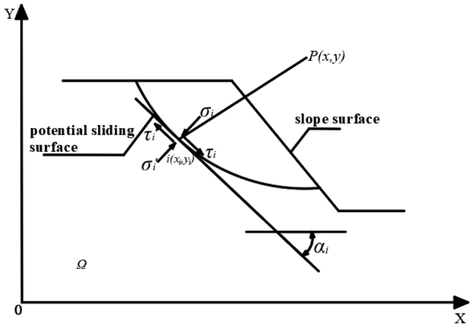

Fig. 1. Schematic diagram of the safety factor solution

Figure 1 shows the schematic diagram for solving the safety factor of the strict-equilibrium torque sum method (under the initial static equilibrium state), Let *Ω* denote the computational domain of the slope.

At any point on the potential sliding surface *S* of the slope, the cohesion is c_i and the angle of internal friction is φ_i.

α_i is the angle between the tangent to the sliding surface at point i and the positive direction of the x-axis. The normal stress σ_i and shear stress τ_i within any infinitesimal arc segment Δl_i on the sliding surface are generated by the external loads and self-weight of the sliding mass.

The sliding force at this point on the sliding surface is provided by σ_i and τ_i. The forces exerted by the bedrock on the sliding mass are σ_i' and τ_i', which form a pair of interaction forces with equal magnitudes and opposite directions. The integral (or summation) of the moments of the stresses on the sliding surface about the rotation point along the sliding surface is the denominator term in Equation (2). The coordinates of the rotation point P(x,y) in Figure 1 are to be determined.

The distance from the rotation point P(x,y) to the line where the shear stress at any point i on the sliding surface is located is:

$$d_{\tau i} = \frac{|\ k_{\tau i}x + (-1)y + b_i|}{\sqrt{k_{\tau i}^{\ 2} + (-1)^2}} \tag{3}$$

where $k_{\tau i} = \tan\alpha_i$, $b_i = y_i - k_{\tau i}x_i$, and (x_i, y_i) are the coordinates of point i.

The shear stress provides the rotational moment, and the sum of the moments is $\sum_{i=1}^{n} \tau_i \Delta l_i d_{\tau i}$.

The distance from the rotation point P(x,y) to the line where the normal stress at any point i on the sliding surface is located is:

$$d_{\sigma i} = \frac{| k_{\sigma i} x + (-1) y + B_i |}{\sqrt{k_{\sigma i}^2 + (-1)^2}} \tag{4}$$

where $k_{\sigma i} = -\frac{1}{k_{\tau i}} = -\frac{1}{\tan \alpha_i}$, $B_i = y_i - k_{\sigma i} x_i$.

The normal stress provides the rotational moment, and the sum of the moments is $\sum_{i=1}^{n} \sigma_i \Delta l_i d_{\sigma i}$.

The total rotational moment of the sliding mass is:

$$\sum_{i=1}^{n} M_s(x,y) = \sum_{i=1}^{n} \tau_i \Delta l_i d_{\tau i} + \sum_{i=1}^{n} \sigma_i \Delta l_i d_{\sigma i} \tag{5}$$

Due to the different positions of the rotation point and the various shapes of the sliding surface, the anti-sliding moments provided by σ_i and τ_i of a certain infinitesimal segment can be positive or negative (rotation in the direction of the slope dip is defined as positive), which is mainly determined by the local geometric position. Regardless of positive or negative values, they shall all be included in the numerator term; the sliding moments provided by the interface stresses on the slope side may also be positive or negative, and shall all be included in the denominator term regardless of their signs. Since the sliding mass is an indivisible whole, this calculation method can reflect the integrity of the sliding mass.

The integral (or summation) of the moments about the rotation point generated by the tangential shear strength and normal reaction force on the sliding bed side of the sliding surface constitutes the total anti-rotation moment, which is the numerator term in Equation (2).

The line where τ_f is located is consistent with that of τ, so the distance from the rotation point (x,y) to the line where τ_f is located is also $d_{\tau i}$, and the total anti-rotation moment provided is $\sum_{i=1}^{n} \tau_{fi} \Delta l_i d_{\tau i}$.

The distance from the rotation point P(x,y) to the line where the normal reaction force is located is also $d_{\sigma i}$, and the total anti-rotation moment provided is $\sum_{i=1}^{n} \sigma'_i \Delta l_i d_{\tau i}$.

The total anti-rotation moment is:

$$\sum_{i=1}^{n} M_r(x,y) = \sum_{i=1}^{n} \tau_{fi} \Delta l_i d_{\tau i} + \sum_{i=1}^{n} \sigma'_i \Delta l_i d_{\sigma i} \tag{6}$$

Substituting Equations (5) and (6) into Equation (2), the calculation formula for the safety factor expression of the strict-equilibrium torque sum method is obtained:

$$F\left(x,y\right)=\frac{\sum_{i=1}^{n}\left(c_i+\sigma_i\tan\varphi_i\right)\Delta l_i d_{\tau i}+\sum_{i=1}^{n}\sigma'_i\Delta l_i d_{\sigma i}}{\sum_{i=1}^{n}\tau_i\Delta l_i d_{\tau i}+\sum_{i=1}^{n}\sigma_i\Delta l_i d_{\sigma i}} \tag{7}$$

3 Results

Calculating the safety factor of the strict-equilibrium torque sum method only relying on the definition of the safety factor by the original TSM cannot achieve strict mechanical equilibrium. The strict-equilibrium torque sum method needs to satisfy both force equilibrium and moment equilibrium conditions, i.e., on the basis of the TSM to ensure moment equilibrium, it is also necessary to consider the force equilibrium under the limit equilibrium state.

To establish the force equilibrium equations, starting from the definition of the safety factor, the equilibrium between the anti-sliding force and the sliding force under the limit equilibrium state should be considered. From the safety factor expression of the TSM:

$$\begin{aligned}&\sum\frac{\left(c_i+\sigma_i\tan\varphi_i\right)\Delta l_i d_{\tau i}}{F\left(x,y\right)}+\sum\frac{\sigma'_i\Delta l_i d_{\sigma i}}{F\left(x,y\right)}=\\&\sum\tau_i\Delta l_i d_{\tau i}+\sum\sigma_i\Delta l_i d_{\sigma i}\end{aligned} \tag{8}$$

It can be seen that when the sliding mass rotates around the rotation center P(x,y), under the limit equilibrium state, the anti-rotational force of the sliding mass is provided by the resistance on the sliding bed side of the sliding interface, and the rotational force is provided by the sliding force on the sliding mass side that can represent the entire sliding mass and loads. The stress diagram can be referred to Figure 1. The anti-rotation moment generated by the anti-rotational force and the rotational moment generated by the rotational force reach equilibrium.

To make the sliding mass reach force equilibrium under the limit state, the force equilibrium equations in the horizontal and vertical directions are established for the sliding mass as a whole as follows:

$$\begin{aligned}&\sum_{i=1}^{n}\frac{\left(c_i+\sigma_i\tan\varphi_i\right)\triangle l_i\cos\alpha_i}{F(x,y)}-\sum_{i=1}^{n}\frac{\sigma'_i\triangle l_i\sin\alpha_i}{F(x,y)}=\\&\sum_{i=1}^{n}\tau_i\triangle l_i\cos\alpha_i-\sum_{i=1}^{n}\sigma_i\triangle l_i\sin\alpha_i\end{aligned} \tag{9}$$

$$\sum_{i=1}^{n}\frac{\left(c_i+\sigma_i\tan\varphi_i\right)\Delta l_i\sin\alpha_i}{F(x,y)}+\sum_{i=1}^{n}\frac{\sigma'_i\Delta l_i\cos\alpha_i}{F(x,y)}= \sum_{i=1}^{n}\tau_i\Delta l_i\sin\alpha_i+\sum_{i=1}^{n}\sigma_i\Delta l_i\cos\alpha_i \tag{10}$$

By solving Equations (9) and (10) simultaneously, the coordinates of the rotation center can be obtained, and then substituting the coordinates of the rotation center into Equation (7) to get the safety factor F (the F obtained at this time is the safety factor of the TSM under strict equilibrium). That is, the coordinates of the rotation center and the safety factor are obtained by solving these three equations.

In Reference [8], the TSM considers that the sliding mass rotates around a certain rotation center, which is obtained by the commonly used Grid search method. The rotation point is assumed to be located in a set rectangular search area, and grid nodes are evenly distributed in the rectangular area as the trial rotation points. The safety factor is calculated only once for each node, and the node corresponding to the minimum safety factor in the area is taken as the rotation center of the sliding mass. The sliding mass with this rotation center only satisfies the moment equilibrium but not necessarily the force equilibrium, so it is a non-strict equilibrium method that only satisfies the moment equilibrium under the limit equilibrium state.

According to the basic assumptions of the strict-equilibrium torque sum method, Equations (9) and (10) are established, i.e., the force equilibrium equations in the horizontal and vertical directions of the sliding mass under the limit equilibrium state. When the sliding mass satisfies these two equations, it simultaneously satisfies the force and moment equilibrium, i.e., strict equilibrium. By solving Equations (9) and (10) simultaneously, the unique coordinates P(x,y) of the rotation center that satisfy the strict equilibrium of the sliding mass under the limit equilibrium state can be directly obtained.

In the strictly balanced moment sum method, solving the rotation center coordinates involves a system of three nonlinear summation equations, which is too difficult for manual calculation and thus requires programming.

The specific solution process is: first, obtain the sliding surface stress state via methods like the finite element method and input relevant parameters; then assume an initial rotation center P(x,y), calculate physical quantities and verify its feasibility using the least squares method; if not feasible, update the coordinate iteratively via the Trust Region Reflective (TRF) algorithm until a valid solution is obtained.

The least squares method, an efficient iterative algorithm, achieves optimal solutions by minimizing the sum of squared residuals, enabling efficient solution of the rotation center under complex equilibrium. Based on this process, a C++ program was developed, including three core modules: parameter input, calculation and analysis, and result output.

Finally, substitute the coordinates of the rotation center obtained by the above program into Equation (7) to calculate the safety factor of the Strict-Equilibrium Torque Sum Method. The Strict-Equilibrium Torque Sum Method is improved on the basis of the original torque sum method, and compared with the original method, it has obvious advantages: first, it strictly satisfies the equilibrium of forces and moments under the limit equilibrium state, making the method more theoretically reasonable; second, the method ensures the uniqueness of the rotation center through strict mechanical equilibrium requirements, so that the determination of its rotation center does not require setting a search area, the rotation center can be found quickly, and the calculation amount is greatly reduced.

4 Conclusion

(a) Based on the original TSM, a strict-equilibrium torque sum method satisfying force and moment equilibrium under limit equilibrium is proposed. In the plane, it takes the sliding mass's rotation center coordinates as unknowns, establishes horizontal and vertical force equilibrium equations between sliding interface resistance and sliding force, and combines with the TSM safety factor equation. Iterative solution yields the unique rotation center coordinates and corresponding safety factor meeting strict equilibrium conditions, which is more theoretically perfect than the original TSM that only satisfies moment equilibrium.

(b) The original TSM's rotation center [8] is determined by the common grid search method, targeting the minimum safety factor. While this achieves the minimum safety factor, it involves large calculation workload, which increases exponentially when sliding surface search is needed. Additionally, the original method is non-strict equilibrium, only satisfying moment equilibrium. In contrast, the strict-equilibrium torque sum method strictly meets force and moment equilibrium of the sliding mass in the limit state, greatly reduces the calculation workload for determining the rotation center, and is of great significance for the future popularization and application of TSM.

(c) The strict-equilibrium torque sum method is capable of analyzing any sliding surface. The sliding surface corresponding to the sliding mass merely serves as its boundary condition. The shape of this sliding surface does not directly exert an influence on the displacement trend of the sliding mass as a rigid body; instead, it affects the subsequent displacement trajectory of the sliding mass by exerting reaction forces via the boundary.

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