The Mathematical Correlation between GDP and Macroeconomic Models

Jinxuan Yang

Nanjing Shi Ban Qiao Middle School, Nanjing, China
01C6698E19DA1969@outlook.com

Abstract. This study focuses on the research of the gross domestic product (GDP) accounting model, systematically sorts out the accounting framework of GDP, and mainly starts from the expenditure approach to deduce the GDP equilibrium conditions in the two-sector, three-sector and four-sector economic models, revealing the formation mechanism and policy implications of the multiplier effect. Meanwhile, by integrating the Aggregate Demand–Aggregate Supply (AD–AS) framework, the inherent linkage between real GDP fluctuations and the macroeconomy's general equilibrium was rigorously articulated; calculus, linear algebra, and dynamic optimization were then deployed to derive comparative-static results, simulate policy shocks, test stability conditions, and quantify welfare implications across multiple time horizons. Through the scenario analysis of fiscal, tax and international trade policies, the feasibility and application value of the model were verified. Study shows that scientific GDP accounting and model application help accurately measure the economic aggregate and provide a theoretical basis for the formulation of macroeconomic control policies.

Keywords: Mathematical correlation, Global GDP, Macroeconomic models.

1 Introduction

Gross domestic Product (GDP) is an important basis for measuring the economic development level and production capacity of a country or region. This study systematically sorts out the accounting framework of GDP, focusing on the expenditure approach, and successively deduces the GDP equilibrium conditions in the two-sector and above economic models, revealing the formation mechanism of the multiplier effect and its policy implications. This study further combines the aggregate demand - aggregate supply model to explain the intrinsic connection between GDP and the overall equilibrium of the macroeconomy and uses mathematical tools such as calculus and linear algebra to support the model. Finally, through the analysis of actual cases of fiscal policy, tax policy and international trade policy, the feasibility and application value of the model were verified. Research shows that scientific GDP accounting and model application not only help accurately measure the economic aggregate but also provide a theoretical basis for the formulation of macroeconomic control policies.

Gross domestic Product (GDP) is an internationally recognized core economic indicator used to measure the economic development level and production capacity of a

A. J. Moshayedi (ed.), *Proceedings of the 2025 International Conference on Hybrid Commerce, Human Capital, and Economic Dynamics (ICHCH 2025)*, Advances in Economics, Business and Management Research 374,
https://doi.org/10.2991/978-2-38476-585-0_60

country or region. With the development of global economic integration, research on GDP models has been continuously deepened, especially playing a significant role in studying the impact of economic policies and predicting economic trends.

This article aims to analyze GDP models under different economic structures through detailed mathematical derivations, study the impact of various economic variables on total output, and thereby provide analysis for macro policies such as fiscal, monetary, and trade.

This article mainly adopts mathematical derivation as the method, combined with calculus and linear algebra tools, to reflect the equilibrium conditions and practical applications of GDP. In the "Mathematical Derivation of GDP Models Based on Expenditure", for two-sector, three-sector, and four-sector economic models, by constructing consumption functions, equilibrium conditions, etc., the equilibrium expressions of GDP under each model are derived through algebraic operations. At the same time, the investment multiplier, government expenditure multiplier, tax multiplier, etc. are obtained to clarify the degree of influence of each economic variable on total output. Applied to the "Model Application and Expansion" section, through the scenario setting of fiscal policy, tax policy and international trade policy, the impact of different policy changes on GDP is analyzed to verify the feasibility and application value of the model.

The introduction introduces the significance of GDP and clarifies that the research topic of this paper is the mathematical derivation and application of the GDP accounting model. The basic framework of GDP accounting defines GDP as the market value of all final products and services produced within a country (or region) within a certain period. Introduce two main accounting methods, namely the expenditure approach and the income approach. The mathematical derivations of the GDP model based on the expenditure approach are conducted in detail for the economic models of two sectors (consumption and investment only), three sectors (including the government sector), and four sectors (including the foreign sector), to obtain the GDP equilibrium formulas under each model and analyze the corresponding multiplier effects. It is expounded that the composition of total demand (AD) in the AD-AS model is consistent with the accounting elements of the GDP expenditure method. It is explained that the GDP derivation model provides a mathematical basis for the movement of the AD curve and can be used to simulate the influence of various factors on total output and price levels [1].

2 Mathematical Derivation of the GDP Model Based on Expenditure

GDP refers to the market value of all final products and services produced by a country (or region) within a certain period. GDP is the core indicator for measuring economic growth and can determine whether an economy is expanding, contracting or stagnating. At the same time, the adjustments of fiscal policies (such as taxation and government spending) and monetary policies (such as interest rates and money supply) by the government and the central bank are often based on GDP data as an important reference. The total GDP is the most commonly used indicator by the international community to measure the scale of a country's economy. There is a positive correlation between GDP growth and the improvement of residents' income, employment opportunities, and public services (education, healthcare).

GDP=C(consumption)+I(investment)+G(government)+NX (net export) =X-M (1)

Private GDP= wage +profit +rent +indirect tax +interest +depreciation (2)

Two-sector economic model: Hypothesis：no government，no import and export, only consumption(C) and investment(I) [2].

• Consumption function:

$$C = C_0 + bY \tag{3}$$

(C_0 represents spontaneous consumption, b represents marginal propensity to consume, $0 < b < 1$)

Equilibrium condition:

$$Y = C + I \tag{4}$$

Derivation:

$$Y = C_0 + bY + I \tag{5}$$

$$Y - bY = C_0 + I \tag{6}$$

$$Y = \frac{C_0 + I}{1 - b} \tag{7}$$

Multiplier effect: The investment multiplier = $\frac{1}{1-b}$. For instance, if b=0.8, the multiplier =5, meaning that for every 1 unit increase in I, GDP increases by 5 units [3].

Three-sector economy(+government)

Consumption function:

$$C = C_0 + b(Y - T) \tag{8}$$

(T is the tax, assuming $T = T_0$ is the quantitative tax).

Equilibrium condition:

$$Y = C + I + G \tag{9}$$

derivation:

$$Y = C_0 + b(Y - T_0) + I + G \tag{10}$$

$$Y = C_0 + I + G - bT_0 \tag{11}$$

Government expenditure multiplier: $\frac{1}{1-b}$, tax multiplier = $\frac{-b}{1-b}$

Fourth-sector economy (+import and export): Net exports: $NX = X - M$, $M = M_0 + mY$(M_0 represents spontaneous imports, m represents marginal propensity to import) [4].

Equilibrium condition:

$$Y = C + I + G + (X - M) \tag{12}$$

derivation:

$$Y = C_0 + b(Y - T_0) + I + G + X - (M_0 + mY) \tag{13}$$

$$Y = \frac{C_0 + I + G + X - bT_0 - M_0}{1 - b + m} \tag{14}$$

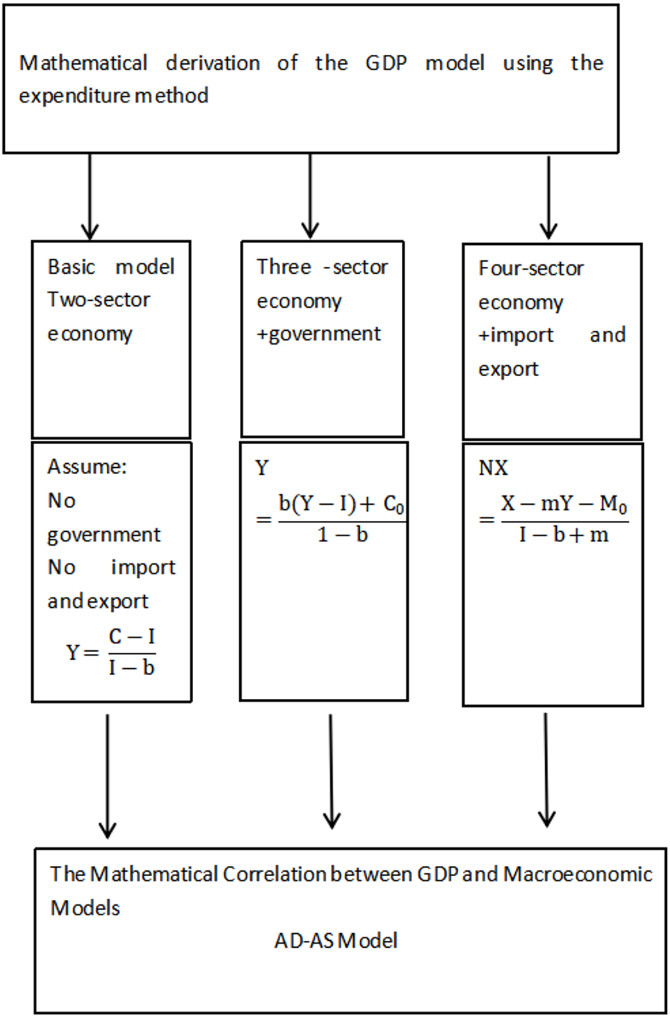

Fig. 1. Example of mathematical derivation.

The mathematical correlation between GDP and macroeconomic models [5]: AD (Aggregate Demand) =C + I + G + (NX).

AS (Aggregate Supply) reflects the supply capacity and price level of a company.

When AD intersects with AS, the equilibrium output and price level are determined. The movement of the AD curve is used to simulate the impact of factors such as fiscal policy, monetary policy, and external shocks on total output and price levels (Fig 1).

When the demand of this good increased， the AD curve will switched right. Then the intersection points of AD and AS will be higher. Show like the right paragraph, the equilibrium point is higher, the price is higher, and the quantity of goods increased (Fig 2).

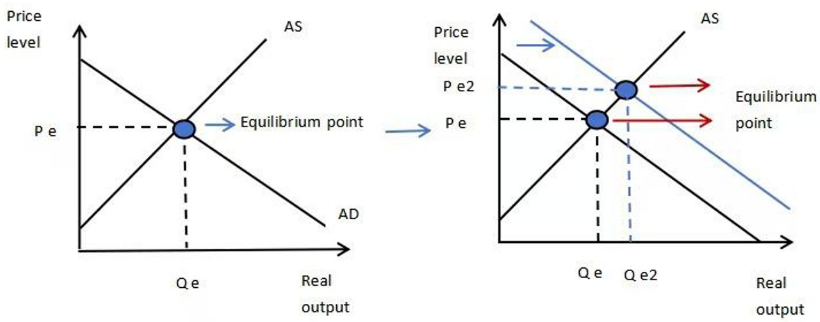

Fig. 2. AD and AS curve about GDP.

And also, the situation can be divided into Long-run AS and Short-run AS, the difference between two situations is the length of time. When the duration is short, we can draw the diagram like the chart above. But if the duration is long, the diagram of AS need to be changed like below (Fig 3) [6].

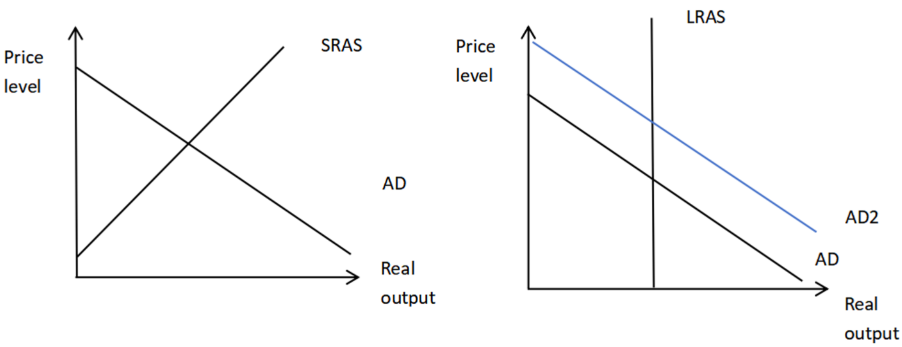

Fig. 3. Difference between LRAS and SRAS.

If the duration is long, when the curve AD switched right, only the price of good will increase, the real output will not change.

3 Model Application and Case Analysis

To test the model's prediction results on actual policy changes, this paper sets up a scenario simulation of fiscal expenditure [7,8]. Suppose the economy is in a three-sector economic system (including the government sector), the basic consumption function is:

$$C=C\_0+b(Y-T\_0) \quad (15)$$

First, suppose the initial value parameters are as follows：

Autonomous consumption C_0=300

Marginal propensity to consume b=0.8

Taxes T_0=200

Investment I=500

Government output G=400

Then, according to the economic balance conditions of the three sectors:

Y= (C_0-b+T_0+I+G)/(1-b)

Y= (300-0.8×200+500+400)/ (1-0.8)

Y= (300-160+500+400)/0.2

Y= 1040/0.2

Y= 5200

At this time, the total output of the economy Y = 5200

Scenario of changes in government spending

If the output of government increased into △G=100,then:

$$\triangle Y=(\triangle G)/(1-b)=100/0.2=500 \quad (16)$$

And the new GDP turn into:

$$Y\_2=Y+\triangle Y=5200+500=5700 \quad (17)$$

Comparison of tax change scenarios:

When make the taxes decrease △T=-100,then the effect of GDP is：

$$\triangle Y=(-b\triangle T)/(1-b)=-0.8\times(-100)\div 0.2=400 \quad (18)$$

The new GDP turn into Y2=5200+400=5600

Scientific literature has been seemingly enriched by various theoretical and empirical studies analysing the relationship between stock market returns and macroeconomic forces during the last few decades. It is often argued that stock prices are determined by some fundamental macroeconomic variables [9]. Interpretation of result: Under the same increment (100 unit), the pull effect of increased government spending on GDP (500) is greater than the effect of tax reduction (400), which is consistent with the conclusion that "the expenditure multiplier is greater than the tax multiplier in theory. The simulation verifies the amplification effect of the multiplier effect, and policymakers can estimate the macro impact of fiscal policy based on it [10].

In order to explain the concepts mentioned above more clearly and intuitively, we can design a table for explanation，like the table 1，by changing the quantities of different components of GDP to describe the turn of GDP.

Table 1: The GDP turn

Scenario	Government output G	Tax revenue T	Balanced GDP Y	variation △Y
Basic scenario	400	200	5200	/
Increase spending scenario	500	200	5700	+500
Decrease tax scenario	400	100	5600	+400

4 Conclusion

This paper systematically deduces the conditions for GDP equilibrium under different sectoral situations, reveals the influences of consumption, investment, government expenditure and net exports on total output in the expenditure-based GDP model, and further explains the transmission mechanism of macroeconomic policies on GDP in combination with the AD-AS model. However, the model assumption is rather idealized, ignoring complex variables such as price elasticity, inflation, and international financial factors, and does not involve dynamic adjustment and nonlinear effects. In the future, the Dynamic Stochastic General Equilibrium (DSGE) model can be introduced, combined with factors such as behavioral economics and financial market shocks, to enrich the dynamics and applicability of th4e GDP model.

This study conducts an in-depth discussion on the GDP accounting model, with a focus on deriving the GDP equilibrium conditions in different sectoral economic models based on the expenditure approach. Clarify the influence of mechanisms of consumption, investment, government expenditure and net exports on total output in each model and clearly reveal the formation process and policy significance of the multiplier effect, such as the regulatory role of investment multiplier, government expenditure multiplier and tax multiplier on the economy under different economic structures. Combining the GDP derivation model with the AD-AS model, the close intrinsic connection between GDP and the overall equilibrium of the macroeconomy is profoundly expounded. Through mathematical derivation, a theoretical basis is provided for the movement of the AD curve, thereby effectively simulating the influence of factors such as finance, currency and external shocks on total output and price levels and improving the macroeconomic analysis system. At the same time, mathematical tools such as calculus and linear algebra are employed to provide strong support for the GDP model. Regarding suggestions for future GDP development, I believe that we should deepen empirical research, apply cutting-edge technologies such as big data and machine learning, widely collect data on the behaviors of microeconomic entities, enhance the accuracy of parameter estimation, and thereby improve the precision of models in predicting economic trends, providing a more reliable basis for policymaking. Expand dynamic and comprehensive modeling: Encourage the implementation of cross-departmental dynamic modeling to the isolated analysis mode among various economic sectors in traditional models. By integrating factors such as behavioral economics and financial market shocks, the dynamics and applicability of GDP models are enriched to provide more effective analytical tools for responding to complex and volatile economic situations.

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