



# Time-Varying Hedge Ratios in Fertilizer Futures Markets: Evidence from an Ex-Ante MVHR Framework

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**Abstract.** This chapter examines time-varying hedging strategies for fertilizer prices using a non-homogeneous stochastic model with time-dependent parameters. The framework captures long-run dynamic and short-term volatility, allowing for heavy-tailed innovations. Ex ante minimum variance hedge ratios are estimated for long, short, and net positions, and the performance of futures contracts with different maturities is evaluated.

**Keywords:** MVHR, Hedging strategy, Stochastic modeling, Fertilizer prices, Futures contracts, Mean reversion.

## 1 Introduction

Fertilizers play a fundamental role in global food security by enhancing soil fertility and significantly increasing agricultural productivity. They provide essential nutrients such as nitrogen, phosphorus, and potassium, which are crucial for crop growth and higher yields. Without adequate fertilizer use, global food production would decline substantially, making it difficult to meet the nutritional needs of a growing population. Therefore, stable access to fertilizers is a key component in ensuring sustainable food systems and reducing the risk of food shortages worldwide ([7]). Furthermore, The fertilizer market is highly sensitive to geopolitical and energy shocks due to its strong dependence on natural gas and globally concentrated supply chains. Nitrogen fertilizer production relies heavily on natural gas, making prices particularly vulnerable to energy market fluctuations. In addition, geopolitical tensions, trade restrictions, and sanctions affecting major exporting countries can disrupt supply and sharply increase price volatility. As a result, fertilizer markets often experience significant instability during periods of global uncertainty, highlighting the importance of effective risk

management strategies ([1]).

Alongside that, the hedging against price risk constitute the most significant and widely acknowledged applications of derivatives instruments—particularly futures contracts. Hedging consists of taking a position in a derivative market that neutralizes potential losses in the underlying asset, thereby reducing overall exposure to adverse price movements. Furthermore, the effectiveness of this strategy has attracted substantial attention from both academics and market participants.

Over the past several decades, the subject of hedging using futures has been extensively investigated across various financial and commodity markets (i.e., in [10], [11], [12], [2], among of other). This is reflected in the vast corpus of literature devoted to the development of theoretical models, empirical methodologies, and performance evaluation metrics aimed to optimizing hedge strategies. Researchers have focused on several issues, such as optimal hedge ratios, risk minimization techniques, time-varying hedging effectiveness, and the impact of market conditions on hedging performance ([3]). The continued interest in this domain emphasize the importance of futures contracts as a key risk management tools, and highlights the current need for robust analytical frameworks able to capture the dynamic behavior of asset prices and the evolving structure of financial markets.

Furthermore, the financial crisis of 2008 significantly caused global collapse in commodity as well, introducing amplified volatility and uncertainty. It was only after the crisis, with the increasing financialization of commodity market— a phenomenon characterized by the growing involvement of financial institutions and investors in commodity trading - that fertilizer futures contracts began to experience more substantial increases. From 2015 therefore, these markets noticed a marked increases in liquidity and market participation, as institutional investors, hedge funds, and other financial entities began to engage more actively in fertilizer futures trading. This development reflects broader trends in the commodity and agricultural markets, driven by the intersection of traditional commodity trading and financial markets strategies.

Still in attempt to contribute to the increasing corpus of literature of futures hedging, the primary objective of this chapter is to develop a novel an comprehensive study focused on hedging strategies within fertilizer futures markets. By addressing the specific characteristics and risk dynamics of fertilizer commodities, this chapter aims to fill a gap in the existing research and provide new insights into the effectiveness of hedging mechanisms designed to these essential agricultural inputs. In particular, it seeks to answer the following research question:

**Does incorporating time-varying stochastic dynamics enhance hedging effectiveness in fertilizer price risk management?**

From a methodological perspective, the minimum variance hedging (MHV) approach is a classical and widely adopted technique for determining the optimal hedge ratio focused on minimizing the variance of the hedge portfolio's returns ([10]). In this framework, the hedge ratio is derived from a regression model in which historical changes in spot prices are regressed on corresponding changes

in futures prices. The estimated slope coefficient from this regression—typically obtained through ordinary least squares (OLS)—represents the optimal hedge ratio under the minimum variance criterion. Since in introduction by [10], this method is conceptually captivating due to its simplicity and analytical tractability. Specifically, the regression-based approach ensures that the variance of the hedged position is minimized with respect to the chosen hedging instrument. Moreover, the coefficient of determination ( $R^2$ ) from the regression serves as an in-sample measure of hedge effectiveness, quantifying the proportion of spot price variability that can be explained by futures prices movements. A higher  $R^2$  indicates a more effective hedge, as it reflects a greater alignment between the movement of spot and futures prices.

While the traditional MVH model assumes constant variance and linear relationships, recent advances in financial econometrics have explored extensions that allow for time-varying volatility, nonlinear dependencies, and structural breaks, which may enhance the robustness and effectiveness of hedging strategies in dynamic markets. For example [4] extend the classical mean-reversion framework to a continuous-time setting, by providing comprehensive method to optimal hedging when markets are incomplete. They derive explicit expressions for hedge strategies that minimize the expected squared hedging error over a finite horizon, under stochastic asset price dynamics. [5] investigate the effectiveness of dynamically adjusting hedge ratios over time to minimize the variance of hedged portfolio returns. [6] provide strong evidence that incorporating time-varying volatility and covariances significantly enhances hedging performance in foreign currency futures markets. By employing a multivariate GARCH model to estimate dynamic hedge ratios, they show that allowing for varying distributional properties of asset returns leads to more effective risk reduction compared to static approaches. Their results highlights the importance of modeling the dynamic nature of financial markets, demonstrating that optimal hedging strategies should adapt over time to reflect evolving risk conditions. [8] critically examine the traditional empirical approach to minimum-variance hedging and reveal that it can lead to biased and inefficient hedge ratio estimates due to statistical issues like measurement error and model misspecification. They propose a more robust estimation framework that accounts of hedge ratios and the evaluation of hedge effectiveness. Their findings highlight the importance of adopting sound econometric techniques when applying minimum-variance hedging in practice. [9] highlighted the importance of accounting for excess kurtosis in return distributions when estimating the Conditional Value at Risk (CVaR) for minimum variance hedging portfolios. The authors demonstrate that traditional methods assuming normality can significantly underestimate risk. By incorporating distributional Kurtosis into the CVaR estimation process, they provide a more accurate and reliable risk measure for hedging positions. The findings emphasize the necessity of using higher-moment-adjusted models in practical hedging strategies, particularly in markets characterized by fat-tailed return distributions.

The conventional approach for determining the MVHR has downsides despite its

elegance. It does not account for changes in the conditional information set from the ex-post to the ex-ante MVHR, this is because it is calculated from historical price changes. As a consequence, it is less effective in hedging future risks. In this chapter, we exploit the expectation hypothesis, in which futures prices are considered as the conditional expectation of the spot price under a risk-neutral measure, allowing for a consistent link between spot and futures dynamics within a stochastic framework. This study extends existing hedging literature by applying a time-varying ex-ante MVHR framework to fertilizer markets, which remain underexplored despite their strategic importance. The remainder of this chapter is organized as follows. Section 2 presents the specification of the futures contracts under consideration, outlines the traditional ex-post minimum variance hedge ratio (MVHR), and develops the ex-ante analytical expression of the hedge ratio derived from the proposed time-varying model. Finally, Section 3 provides a summary of the key results and concludes the chapter.

## 2 The traditional ex post MVHR

The vast majority of academic research relies on ([10]) variance minimization concept for building the most effective hedging strategy regarding a single or a portfolio of commodities. This fundamental methodology which was further developed by [11] and by [12] relies on decreasing the variance of the proposed hedged portfolio to the lowest possible degree.

As explained in several research studies, the objective of the MVHR is to minimize the variance of the return on a hedged portfolio, which combines a position in the spot asset with an offsetting position in a futures contracts. Specifically, the value of the hedged portfolio can be expressed as

$$V = S - hF, \quad (1)$$

where  $S$  is the value of the "spot" (which is the underlying asset),  $F$  represents the price of the futures contract, and  $h$  is the hedge ratio, i.e., the number of futures contracts per unit of the spot asset. The goal is to find the value of  $h$  that minimize the variance of changes in portfolio value  $\delta V$ :

$$\text{var}[\Delta V] = \text{var}[\Delta S - h\Delta F] = \text{var}[\Delta S] - 2hcov[\Delta F, \Delta S] + h^2\text{var}[\Delta F] \quad (2)$$

Taking the derivative of the variance with respect to  $h$  and setting it to zero yields the optimal hedge ratio:

$$h = \frac{\text{cov}[\delta F, \delta S]}{\text{var}[\delta F]} \quad (3)$$

This results also emerges naturally from an ordinary least squares (OLS) regression of changes in the spot price on changes in the futures prices:

$$\delta S_t = \alpha + \beta\delta F_t + e_t, \quad (4)$$

where  $\alpha$  and  $\beta$  are constant and  $e_t$  is the error term. The above traditional method for estimating the MVHR is discussed in many derivative texts, such as [13], and [14]; nevertheless, it provides a less effective hedge than the ex ante MVHRs.

Indeed, while this method is basic and widely used in hedging literature, it comes with important assumptions and limitations. That is, it suppose that the relationship between spot and futures returns is linear, stationary, and time-invariant, which may not hold in real data, specially in markets like fertilizer where price volatility is affected by several external variables such as seasonality, policy shocks, and supply-chain disruptions as explained by [15], and [16]. Furthermore, under the minimum variance approach, the optimal hedging strategy is the one that simply offers the higher price risk reduction. This particular framework is less computationally demanding, which is also allows for easier interpretation, however, it emphasizes solely on the risk reduction and completely ignores the risk aversion and the expected return parameters for the optimal hedging planning such as in [17]. Therefore, on a minimum variance hedging model it is arbitrarily been assumed that all investor groups in energy market share infinite risk aversion, a hypothesis which is rather unrealistic even for the most conservative and modest organizations, as infinite risk aversion means that investors would reject investment opportunities which could offer significant potential returns for even the slightest amount of additional risk as explained in [18].

### 3 The realistic ex ante MVHR approach

Continuous-time, no-arbitrage modeling has become main stream in the derivatives literature. Indeed, spot and futures valuation models with stochastic spot asset have been incorporated by some authors as [19], using the Heath-Jarrow-Morton model, and by [20] how use a two-factor Gaussian model with jumps. [21] develop and compare three models of the stochastic behavior of commodity prices with mean-reversion criteria, and evaluate the futures contracts.

The authors in [22] used a two-factor model of commodity which permits mean-reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. He also evaluate futures contracts and risk premium for commodity prices. Furthermore, several researchers as [23] derives a three-factor model of commodity spot prices, convenience yields and interest rates, which nests many existing model specifications. A more recent effort by [24] explored well-known carry cost (CC) relation that links spot and futures prices to develop three increasingly realistic en ante MVHR models.

Building on the methodology proposed by [24] and by employing the continuous-time mean-reverting stochastic model developed previously, we derive an ex ante Minimum-Variance Hedge Ratio (MVHR) under the assumption of time-varying spot prices. Furthermore, The model assumes frictionless markets, continuous trading, and absence of arbitrage opportunities. To evaluate the effectiveness of this approach, we compare the performance of the en ante MVHR with that of

the traditionally estimated ex post MVHR.

Under risk-neutral measure, the future price is expressed as following:

$$F(t, T) = \mathbb{E}^{\mathbb{Q}}[X(T)|\mathcal{F}_t], \tag{5}$$

where  $F(t, T)$  is future price at time  $t$  with maturity  $T$ ,  $X(T)$  is spot price at time  $T$ . Let the spot price process (under risk-neutral space) be:

$$dX(t) = [h(t) - \phi(t)X(t) + \lambda(t)\sigma(t)X(t)]dt + \sigma(t)X(t)dW(t), \tag{6}$$

where  $h(t)$  is equilibrium level,  $\phi(t)$  is mean-reversion force,  $\lambda(t)$  is risk premium,  $\sigma(t)$ , and  $W(t)$  is the Wiener process under the risk-neutral measure. The spot price dynamics in (6) indicate that the time-varying long-run equilibrium price  $h(t)$  reflects structural economic forces such as production costs, global demand conditions, energy prices, and geopolitical influences, while the mean-reversion speed  $\phi(t)$  captures market adjustment mechanisms such as inventory rebalancing, international trade flows, and production responds to price shocks. The market price of risk  $\lambda(t)\sigma(t)X(t)$  represents investors' required compensation for bearing uncertainty in commodity markets, while the price-dependent volatility term  $\sigma(t)X(t)W(t)$  reflects realistic commodity behavior where higher price levels are usually associated with higher market uncertainty.

Thus, based on formula (5), we obtain:

$$F(t, T) = e^{-r(T-t)}[X(t)e^{\phi(T-t)} + \int_t^T h(s)e^{\phi(T-t)}]. \tag{7}$$

The dynamics of the futures price process can be derived as follow:

$$dF[t, T] = rF[t, T] + e^{-r(T-t)}e^{\phi(T-t)}[\phi(t)X(t) + \lambda(t)\sigma(t)X(t)]dt + e^{-r(T-t)}e^{\phi(T-t)}\sigma(t)X(t)dW(t). \tag{8}$$

From the equation (8), we note that futures price dynamics show that risk-free growth is adjusted by risk premiums and volatility transmission from spot markets, with the impact of uncertainty decreasing with maturity due to discounting and mean-reversion forces.

Since the covariance of  $dt$  are zero, we need just to focus on the diffusion terms to calculate  $Cov[dX(t), dF(t, T)]$ . Indeed, the analytical formula of  $Cov[dX(t), dF(t, T)]$  is specified as follow:

$$cov[dX(t), dF(t, T)] = \sigma(t)^2 X(t)^2 e^{-r(T-t)} e^{\phi(T-t)} \nu dt \tag{9}$$

where  $\nu = Var[dS(t)]/dt$ , and

$$Var[dF(t, T)] = (e^{r(T-t)} e^{\phi(T-t)} \sigma(t)X(t))^2 \cdot Var[dW(t)] e^{2r(T-t)} e^{2\phi(T-t)} \sigma(t)^2 X(t)^2 \nu dt \tag{10}$$

The covariance and variance analyses further demonstrate that hedging effectiveness depends on price volatility, market uncertainty, price levels, and mean-reversion strength, with longer maturities generally exhibiting smoother price

movements due to market stabilization effects.

Accordingly, and using (9) and (10), the instantaneous hedge ratio  $h(t)$  is given by the following expression:

$$h(t) = \frac{Cov[dX(t), dF(t, T)]}{Var[dF(t, T)]} \quad (11)$$

$$h(t) = \frac{\sigma(t)^2 X(t)^2 e^{r(T-t)} e^{\phi(T-t)} \cdot v}{e^{2r(T-t)} e^{2\phi(T-t)} \sigma(t)^2 X(t)^2 \cdot v} \quad (12)$$

Cancel out common terms  $\sigma(t)^2 X(t)^2 \cdot v$ :

$$h(t) = \frac{e^{r(T-t)} e^{\phi(T-t)}}{e^{2r(T-t)} e^{2\phi(T-t)}} = e^{(r-\phi(t))(T-t)} \quad (13)$$

This result implies that hedging intensity depends primarily on mean reversion dynamics and contract maturity rather than historical covariance patterns. Equation (13) reveals that the instantaneous ex-ante hedge ratio is entirely determined by the mean-reversion intensity and the time to maturity of the futures contract at the hedging date.

We note that the traditional regression method hedge ratio (as in section 2) is calculated under the "real measure" (a.k.a. "physical measure" or "actuarial measure"). On the other hand, the hedge ratios calculated in sections 3 are calculated under the "risk-neutral measure".

While hedge ratios derived under the physical (real-world) and risk-neutral probability measures are not mathematically comparable, they Exhibit distinct economic interpretations. Indeed, the hedge ratio derived under the risk-neutral measure exhibit an ex ante, or forward-looking, hedge strategy, in contrast to the ex post nature of the traditional hedge ratio estimated under the real-world measure using historical data. This is similar to the black-Scholes option pricing model in which the hedge ratio (or "delta") of an option is forward-looking hedge ratio derived under the risk-neutral measure and that eliminate the instantaneous price risk of the option from the underlying asset. Our approach provide a comprehensive financial-economic interpretation of fertilizer price dynamics under a risk-neutral framework, showing that spot and futures prices are jointly driven by market fundamentals, risk compensation mechanisms, and stochastic uncertainty. In the futures pricing framework, futures prices represent the discounted expected future spot prices, where short-term contracts are mainly influenced by current price information, whereas long-term contracts are more sensitive to fundamental economic conditions embedded in the time-varying equilibrium price path.

Indeed, the optimal hedge ratio from (13) reflects the trade-off between interest rate effects and mean-reversion forces, implying that faster mean reversion improves hedging efficiency while longer maturities increase sensitivity to structural market fundamentals. As result, the model provides a dynamic framework for understanding fertilizer market behavior by integrating financial risk management considerations with economic fundamentals, supply chain constraints, energy cost transmission, and exposure to external shocks, thereby supporting the development of adaptive hedging strategies in volatile commodity markets.

## 4 Conclusion

This study has important implications for agricultural producers, input importers, policymakers, and food risk management. Dynamic hedging strategies can help producers stabilize input costs and protect profit margins against fertilizer price volatility, while enabling importers to better manage procurement timing and international price exposure. For policymakers, understanding time-varying price dynamics supports the design of more resilient and adaptive agricultural policies, thereby strengthening food production systems. However, the model does not incorporate transaction costs, liquidity constraints, or margin requirements, which may affect its practical implementation, and further empirical validation across different fertilizer markets remains necessary.

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