Study on Nonlinear Consolidation of Vacuum Preloading in Shallow Layer of New Dredger Fill Ultra-Soft Foundation

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Abstract. The ultra-soft soil foundation is widely distributed in coastal areas, and the vacuum preloading method without sand cushion is often used to reinforce the shallow surface layer. Based on the microscopic pore compression mechanism and non-Darcy seepage characteristics, a non-linear consolidation model of vacuum preloading without sand cushion considering the dynamic changes of permeability coefficient and void ratio with effective stress is established. The model takes the point source load as the boundary condition, and the analytical solution is obtained by the separation variable method and the impulse method. Based on the field test of the offshore wharf of Nansha Port in Guangzhou Port, the field verification is carried out. The results show that the error of the predicted consolidation degree of the model is only 18.09%, which is significantly lower than 39.53% of the Terzaghi theory, and the measured consolidation degree of 89 % and the residual settlement of 0.074 m meet the design requirements.It is proved that the model can effectively characterize the microstructure evolution and pore water pressure dissipation of ultra-soft soil during vacuum preloading. It has important theoretical value and engineering application prospect, and can be further verified in other dredger fill sites.

Keywords: Ultra-soft soil, no sand cushion vacuum preloading method, nonlinear consolidation theory.

1 Introduction

In the past ten years, China has made great progress in the field of super soft foundation reinforcement technology. As we all know, the traditional vacuum preloading method needs to lay medium-coarse sand with a mud content of no more than 5 % on the surface as a horizontal drainage cushion, and then carry out processes such as drainage plate insertion and filter pipe laying. However, due to the characteristics of ultra-soft soil foundation with high water content (water content higher than 85%), high compressi-

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bility and high void ratio (void ratio greater than 4.0), it is impossible to directly implement the conventional vacuum preloading method on the surface. Then the shallow vacuum preloading method developed by scholars has become one of the main methods to reinforce the ultra-soft soil foundation.[1]

The reinforcement principle of non-sand cushion vacuum preloading method is essentially the same as that of traditional vacuum preloading method. Both of them use atmospheric pressure to form surface negative pressure, which diffuses to the surrounding soil through the vertical drainage body in the foundation, reduces the pore water pressure, increases the effective stress and improves the soil strength, so as to achieve the purpose of strengthening the foundation.[1-4] The biggest difference between the shallow surface vacuum preloading method and the conventional vacuum preloading technology in the process is that the former does not set the horizontal drainage sand cushion, but promotes the surface negative pressure to be quickly transferred to the vertical drainage plate by encrypting the horizontal drainage filter pipe spacing and directly connecting with the drainage plate. In this way, the mode of sand cushion as a ' non-point source ' to transfer negative pressure will be transformed into a ' line source ' distribution mode of ' filter tube-drain plate '. The negative pressure under the membrane between the filter tubes is no longer a constant, but a function of the distance r between the filter tube and the negative pressure source, that is, the boundary conditions of the sand well foundation (containing vertical drainage bodies such as bagged sand wells, plastic drainage plates, etc.) model based on the classical Terzaghi 's one-dimensional seepage consolidation theory have changed[5]. In addition, the classical Terzaghi one-dimensional seepage consolidation theory assumes that the permeability coefficient and void ratio are constants.

However, in engineering practice, the permeability coefficient and void ratio of the ultra-soft soil foundation have changed greatly during the consolidation process[6-9]. The settlement calculation still follows the traditional vacuum preloading consolidation theory, resulting in the calculated settlement is less than the real settlement. After the reinforcement treatment, the project sometimes has a large post-construction settlement, causing the upper structure to crack. Based on this, in view of the above-mentioned key scientific problems, it is necessary to carry out theoretical research and analysis verification of vacuum preloading without sand cushion to reinforce soft soil foundation.

2 Microscopic Analysis of Compressibility and Permeability of Ultra-Soft Soil

The consolidation characteristics of ultra-soft soil are determined by its compression characteristics and permeability characteristics, and the microstructure characteristics and pore characteristics of ultra-soft soil affect and change its compressibility and permeability.

Soil compressibility is the result of the interaction of pore structure and external load, which is mainly affected by pore characteristics (pore ratio, pore size distribution), pore water seepage type, microstructure and consolidation pressure. Consolidation pressure determines the compressibility of soil by changing its microstructure (including the type, size, shape and orientation of structural units) and pore characteristics (void ratio and pore size distribution)[10] . Therefore, the change of compressibility with pressure essentially reflects the response of pore structure evolution. It can be seen from Figure.1 and Figure.2 that the void ratio and compression coefficient decrease with the increase of consolidation pressure. The mechanism is that natural or low-pressure soil samples have high pore ratio, large equivalent pore size, and high proportion of large and medium pores. Such pores are easily compressed and annihilated or split, so they have high compressibility and large compression coefficient. With the increase of consolidation pressure, the macropores gradually annihilate, the equivalent pore size decreases, and the proportion of micropores and closed pores increases. These tiny pores are difficult to be further compressed, and the pore water is mainly bound water, which is not easy to be discharged, and the pore ratio changes slowly. At the same time, the strength of soil structure is enhanced, which leads to the decrease of compressibility and compression coefficient, and the a-p curve is gradually gentle and tends to be stable.

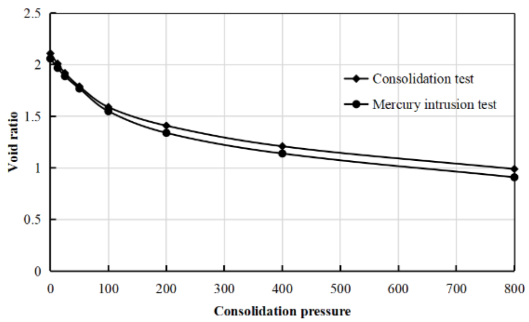

Fig. 1. The *e-p* curve of ultra-soft soil

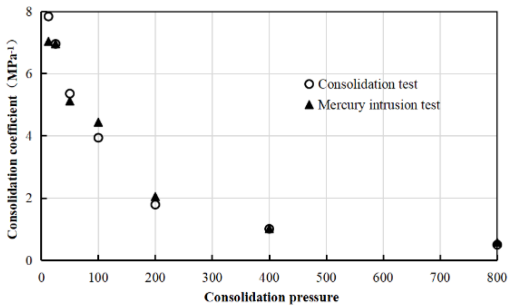

Fig. 2. The *a-p* curve of ultra-soft soil

The compressibility of natural soft soil depends on its initial structure, which is usually related to multi-parameters such as probability entropy of structural unit, void ratio, equivalent pore size and large and medium pore content. The consolidation pressure causes the change of structural characteristics, and then changes the compressibility.

The compressibility of soil after consolidation is still determined by the structural characteristics after consolidation. Since the structural evolution has a definite relationship with the consolidation pressure, the change of compressibility can be finally expressed as a function of pressure, as shown in the curves in Fig.1 and Fig.2. In summary, the compressibility of consolidated soil can be decomposed into natural structure part and consolidation change part. The former is related to the initial structural characteristics, and the latter is related to the structural evolution caused by consolidation, and can be quantitatively characterized by consolidation pressure.

The permeability of soil is affected by many factors, such as clay content, mineral composition and content, pore characteristics (pore size and existence form), soil structure and seepage fluid properties. When the other conditions are similar, the macroscopic seepage characteristics of soft soil depend especially on its pore size and existence form[11,12] . The permeability coefficient of natural soft soil is determined by its structural characteristics, including pore size, distribution, connectivity and tortuosity. Consolidation pressure can significantly change the pore structure (such as pore size and distribution characteristics) of soft soil, which in turn affects its permeability. It can be seen from Figure.3 that the permeability coefficient of soil samples decreases obviously with the increase of consolidation pressure. Natural undisturbed soil samples and low consolidation pressure soil samples have high void ratio and large equivalent pore size, mainly large and medium pores. Pore water flows in the form of free water, which is easy to flow and discharge, and the permeability coefficient is high. With the increase of consolidation pressure, the void ratio and equivalent pore size decrease, the proportion of large and medium pores decreases, and gradually changes to small, micro and ultra-micro pores. The pore water is dominated by bound water seepage, which is low in fluidity and difficult to discharge, and the permeability coefficient decreases significantly. The vertical permeability coefficient of the sample at the later stage of consolidation has dropped to a very small proportion of the natural soil sample.

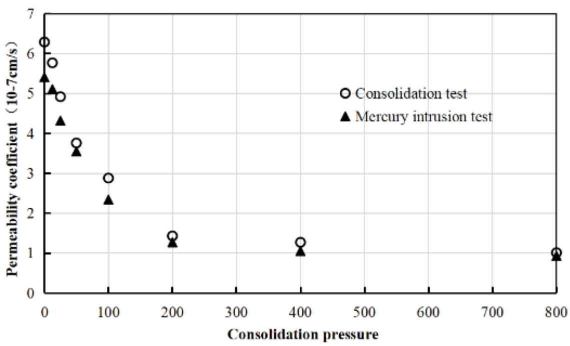

Fig. 3. The *k-p* curve of ultra-soft soil

In summary, the permeability of consolidated soil can be regarded as composed of natural structure part and consolidation change part. The former depends on the initial structural characteristics, especially the pore-related parameters ; the latter is related to the evolution of pore structure during consolidation and can be quantitatively characterized by the relationship with consolidation pressure.

3 Consolidation Basic Equation of Shallow Surface Point Source Vacuum Preloading Soft Soil

The compressibility of soft soil is related to the void ratio, pore size distribution, pore water seepage type, soil microstructure and applied external load. The consolidation pressure changes the microstructure type of the soil, the equivalent particle size and scale distribution of the structural unit body, the shape and orientation of the structural unit body, etc., thereby changing the compressibility of the soil. The consolidation pressure changes the pore ratio and pore scale distribution characteristics of the soil, which will also change the compression characteristics of the soil. Therefore, the variation characteristics of soil compressibility with pressure also reflect its characteristics with pore characteristics to a certain extent. The existing consolidation model of soft foundation reinforced by conventional vacuum preloading method generally does not consider the change of mechanical parameters of soil with consolidation, so it can be considered as linear consolidation theory. For the calculation of vacuum preloading consolidation of sand-free cushion, it is different from the nonlinear consolidation equation proposed by existing scholars. The main difference is that the traditional vacuum preloading method is to form a vacuum in the sand cushion, and then transfer it to the soil by the drainage plate. The negative pressure on the surface can be regarded as uniform load, while there is no uniform negative pressure on the surface of the vacuum preloading method without sand cushion, and the negative pressure is concentrated on the drainage plate, so the boundary conditions of consolidation are different. The consolidation problem of vacuum preloading method without sand cushion can be simplified as the point source vacuum preloading consolidation problem of axisymmetric sand drain foundation shown in Figure.4. The equivalent radius of sand drain foundation is R, the equivalent radius of drainage sand drain of plastic drainage plate is r_0, and the vacuum degree applied at the drainage plate is p_0.

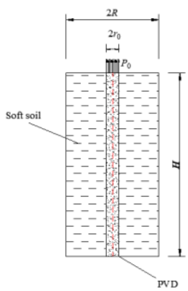

Fig. 4. Consolidation diagram of point source vacuum preloading

The basic equations and definite solution conditions for the consolidation of the above-mentioned point source vacuum preloading ultra-soft soil can be expressed as:

(1) Consolidation equation of ultra-soft soil

$$\frac{\partial u}{\partial t}-C_h\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-C_v\frac{\partial^2 u}{\partial z^2}=0 \tag{1}$$

In the formula, it is the pore water pressure in soft soil ; and are the horizontal and vertical consolidation coefficients of soft soil.

(2) Drainage continuity conditions in sand wells (considering the influence of well resistance)

$$2\pi r_0 dz k_h \left.\frac{\partial u}{\partial r}\right|_{r=r_0}-\pi r_0^2 k_s\left[\frac{\partial u_s(z)}{\partial z}-\frac{\partial u(z+dz)}{\partial z}\right]=0 \tag{2}$$

The above formula is simplified to:

$$-2k_h\left.\frac{\partial u}{\partial r}\right|_{r=r_0}=k_s r_0\frac{\partial^2 u_s}{\partial z^2} \tag{3}$$

In the formula, it is the water pressure in the sand well ; and are the horizontal permeability coefficient of soft soil and the equivalent permeability coefficient of sand well, respectively.

(3) Definite solution conditions

Initial value condition of vacuum preloading without sand cushion:

$$u(r,z,t=0)=0\,;\quad u_s(z,t=0)=0 \tag{4}$$

Point source load boundary conditions :

$$\frac{\partial u(r=R,z,t)}{\partial r}=0\,;\quad \frac{\partial u(r,H,t)}{\partial z}=0 \tag{5}$$

$$\frac{\partial u(r_0\le r\le R,0,t)}{\partial z}=0;\quad \frac{\partial u_s(H,t)}{\partial z}=0\,;\quad u_s(0,t)=-p_0 \tag{6}$$

4 Analytical Solution of Point Source Vacuum Preloading Soft Soil Consolidation

4.1 The Basic Equation of Equal Vertical Strain Condition

For the point source vacuum preloading soft soil problem shown in Fig.1, since the radial drainage path is much smaller than the vertical drainage path, the equal vertical strain condition can be used to solve the problem. For this reason, the two sides of Equation (1) are multiplied and integrated to obtain:

$$\int_0^{2\pi}\int_{r_0}^{R}\left[\frac{\partial u}{\partial t}-C_h\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-C_v\frac{\partial^2 u}{\partial z^2}\right]rdrd\theta=0 \tag{7}$$

$$\int_{r_0}^{R}\left[\frac{\partial u}{\partial t}-C_h\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-C_v\frac{\partial^2 u}{\partial z^2}\right]rdr=0 \tag{8}$$

By using the first form of continuous condition (3) and boundary condition (5), the following basic equations can be obtained.

$$\frac{\partial \bar{u}_z}{\partial t}+\frac{C_h}{n^2-1}\frac{k_s}{k_h}\frac{\partial^2 u_s}{\partial z^2}-C_v\frac{\partial^2 \bar{u}_z}{\partial z^2}=0 \tag{9}$$

In the formula, $\bar{u}_z=\dfrac{2}{R^2-r_0^2}\displaystyle\int_{r_0}^{R}urdr$; $n=R/r_0$ (10)

The initial value conditions are obtained by the same integral treatment of Equation (1) for the definite solution conditions (4) (5)and (6):

$$\bar{u}_z\big|_{t=0}=0;\quad u_s\big|_{t=0}=0 \tag{11}$$

The boundary conditions is:

$$\left.\frac{\partial \bar{u}_z}{\partial z}\right|_{z=0}=0;\quad \left.\frac{\partial \bar{u}_z}{\partial z}\right|_{z=H}=0 \tag{12}$$

$$\left.\frac{\partial u_s}{\partial z}\right|_{z=H}=0;\quad u_s\big|_{z=0}=-p_0 \tag{13}$$

4.2 Separation of Variables Method to Solve

The definite solution condition represented by Eq. (11~13) is transformed into.

$$\bar{u}_z\big|_{t=0}=0\quad u_{s0}\big|_{t=0}=p_0 \tag{14}$$

$$u_{s0}\big|_{z=0}=0;\quad \left.\frac{\partial u_{s0}}{\partial z}\right|_{z=H}=0 \tag{15}$$

$$\left.\frac{\partial \bar{u}_z}{\partial z}\right|_{z=0}=0;\quad \left.\frac{\partial \bar{u}_z}{\partial z}\right|_{z=H}=0 \tag{16}$$

The solutions of $\bar{u}_z(z,t)$ and $u_{s0}(z,t)$ are changed to:

$$u_{s0}(z,t)=\sum_{m=0}^{\infty}T_m(t)\sin\frac{(2m+1)\pi}{2H}z \tag{17}$$

$$\bar{u}_z(z,t)=\sum_{m=0}^{\infty}T_m(t)Z_m(z) \tag{18}$$

The formula (17) satisfies the boundary condition (15). Substituting Eq. (17) and Eq. (18) into Eq. (9), we obtain:

$$\sum_{m=0}^{\infty}[T'_m(t)Z_m(z)-C_vT_m(t)Z''_m(z)-\left(\frac{(2m+1)\pi}{2H}\right)^2\frac{C_h}{n^2-1}\frac{k_s}{k_h}T_m(t)\sin\frac{(2m+1)\pi}{2H}z]=0 \tag{19}$$

$$\sum_{m=0}^{\infty}\{\frac{T'_m(t)}{T_m(t)}-[C_v\frac{Z''_m(z)}{Z_m(z)}+\left(\frac{(2m+1)\pi}{2H}\right)^2\frac{C_h}{n^2-1}\frac{k_s}{k_h}\frac{\sin\frac{(2m+1)\pi}{2H}z}{Z_m(z)}]\}=0 \tag{20}$$

From the above formula (20), we can see that the first function in the formula and the function in the brackets are the functions of t and z respectively. So there must be:

$$\frac{T'_m(t)}{T_m(t)}=-\lambda_m \tag{21}$$

$$C_v\frac{Z''_m(z)}{Z_m(z)}+\left(\frac{(2m+1)\pi}{2H}\right)^2\frac{C_h}{n^2-1}\frac{k_s}{k_h}\frac{\sin\frac{(2m+1)\pi}{2H}z}{Z_m(z)}=-\lambda_m \tag{22}$$

That is:

$$T'_m(t)+\lambda_mT_m(t)=0 \tag{23}$$

$$Z''_m(z)+\frac{\lambda_m}{C_v}Z_m(z)=-\beta_m\sin\frac{(2m+1)\pi}{2H}z \tag{24}$$

where λ_m is a constant (eigenvalue), $\beta_m=\left(\frac{(2m+1)\pi}{2H}\right)^2\frac{C_h}{(n^2-1)C_v}\frac{k_s}{k_h}$

By the formula (23), the solution of $T_m(t)$ is obtained to be:

$$T_m(t)=A_m\exp(-\lambda_mt) \tag{25}$$

$$u_{s0}(z,t)=\sum_{m=0}^{\infty}A_m\sin\frac{(2m+1)\pi}{2H}z\exp(-\lambda_mt) \tag{26}$$

Substituting the above formula into the second formula of the initial value condition (14):

$$u_{s0}(z,0)=\sum_{m=0}^{\infty}A_m\sin\frac{(2m+1)\pi}{2H}z=p_0 \tag{27}$$

Integrate in the interval (1~*H*):

$$\sum_{m=0}^{\infty}A_m\int_0^H\sin\frac{(2m+1)\pi}{2H}z\sin\frac{(2m+1)\pi}{2H}z\,dz=p_0\int_0^H\sin\frac{(2m+1)\pi}{2H}zdz \tag{28}$$

The calculated solution is:

$$A_m=\frac{2p_0}{H}\int_0^H\sin\frac{(2m+1)\pi}{2H}zdz=-\frac{2p_0}{H}\frac{2H}{(2m+1)\pi}\cos\frac{(2m+1)\pi}{2H}z\bigg|_0^H=\frac{4p_0}{(2m+1)\pi} \tag{29}$$

Substituting the formula (29) into the formula (27), the analytical solution satisfying all the initial value conditions and boundary conditions is obtained:

$$u_s(z,t)=-p_0+\sum_{m=0}^{\infty}\frac{4p_0}{(2m+1)\pi}\sin\frac{(2m+1)\pi}{2H}z\exp(-\lambda_m t) \tag{30}$$

4.3 The Impulse Method Solves the Analytical Solution

It can be seen from Equation (9) that the basic equation for solving the pore water pressure u in soft soil is actually a non-homogeneous equation due to the emergence of the sand well water pressure term in the equation, and the corresponding boundary condition (13) is a homogeneous equation, so the impulse method can be used to solve the pore water pressure. The basic equation solved by the impulse method is :

$$\frac{\partial\overline{u}_{z0}}{\partial t}-C_v\frac{\partial^2\overline{u}_{z0}}{\partial z^2}=0 \tag{31}$$

The corresponding definite condition is converted to :

$$\left.\frac{\partial\overline{u}_{z0}(z,t)}{\partial z}\right|_{z=0}=0\ ;\ \left.\frac{\partial\overline{u}_{z0}(z,t)}{\partial z}\right|_{z=H}=0 \tag{32}$$

$$\overline{u}_{z0}(z,t)\big|_{t=\tau+0}=f(z,\tau) \tag{33}$$

In this equation,

$$f(z,\tau) = -\frac{C_h}{n^2-1}\frac{k_s}{k_h}\frac{\partial^2 u_s}{\partial z^2} = \frac{C_h}{n^2-1}\frac{k_s}{k_h}\sum_{m=0}^{\infty}\frac{4p_0}{(2m+1)\pi}\left[\frac{(2m+1)\pi}{2H}\right]^2 \sin\frac{(2m+1)\pi}{2H}z\exp(-\lambda_m\tau)$$

$$= \frac{C_h}{n^2-1}\frac{k_s}{k_h}\frac{\pi p_0}{H^2}\sum_{m=0}^{\infty}(2m+1)\sin\frac{(2m+1)\pi}{2H}z\exp(-\lambda_m\tau) \tag{34}$$

After $\bar{u}_{z0}(z,t;\tau)$ is obtained by solving the equations (31) ~ (34), $\bar{u}_z(z,t)$ is obtained according to the following formula:

$$\bar{u}_z(z,t) = \int_0^t \bar{u}_{z0}(z,t;\tau)d\tau \tag{35}$$

In this equation, $\bar{u}_{z0}(z,t;\tau)$ is the function of t in the solution of equation (31) ~ (33) after t is replaced by *t-τ*.

The equation (31) ~(34) is solved by the method of separation of variables in the form of (14).

$$\bar{u}_{z0}(z,t) = \sum_{m=0}^{\infty} T_m(t)Z_{m0}(z) \tag{36}$$

$$T'_m(t) + \lambda_m T_m(t) = 0 \tag{37}$$

$$Z''_{m0}(z) + \frac{\lambda_m}{C_v}Z_{m0}(z) = 0 \tag{38}$$

The above formula is the homogeneous equation of (23) and (24).

$$Z_{m0}(z) = a_m \sin\alpha_m z + b_m \cos\alpha_m z \tag{39}$$

Formula (36) is further written as:

$$\bar{u}_{z0}(z,t) = \sum_{m=0}^{\infty}(B_m \sin\alpha_m z + C_m \cos\alpha_m z)\exp(-\lambda_m t) \tag{40}$$

In this equation, $\alpha_m = \sqrt{\lambda_m / C_v}$, B_m and C_m are constants. From Eq. (32), we can get:

$$\sum_{m=0}^{\infty}\alpha_m(B_m \cos\alpha_m z - C_m \sin\alpha_m z)\exp(-\lambda_m t)\Big|_{z=0} = 0 \tag{41}$$

$$\sum_{m=0}^{\infty}\alpha_m(B_m\cos\alpha_m z - C_m\sin\alpha_m z)\exp(-\lambda_m t)\Big|_{z=H} = 0 \tag{42}$$

That is:

$$B_m = 0\,;\ \sin\alpha_m H = 0 \tag{43}$$

From the above equation, the eigenvalue can be obtained as :

$$\alpha_m = \frac{m\pi}{H}\,;\ m = 0,1,2,3,\ldots \tag{44}$$

$$\lambda_m = \left(\frac{m\pi}{H}\right)^2 C_v \tag{45}$$

The solution expressed by Eq. (40) is further expressed as:

$$\bar{u}_{z0}(z,t) = \sum_{m=0}^{\infty} C_m\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2 C_v t] \tag{46}$$

Substitute Eq. (46) into the initial value condition (33) :

$$\sum_{m=0}^{\infty} C_m\cos\frac{m\pi z}{H} = \phi\sum_{m=0}^{\infty}(2m+1)\sin\frac{(2m+1)\pi}{2H}z\exp[-(m\pi/H)^2 C_v\tau] \tag{47}$$

Integrate in the interval (1~H) to get :

$$\frac{H}{2}C_l = \phi\sum_{m=0}^{\infty}\int_0^H(2m+1)\exp[-(m\pi/H)^2 C_v\tau]\sin\frac{(2m+1)\pi}{2H}z\cos\frac{l\pi}{H}$$

$$= \phi\sum_{m=0}^{\infty}\int_0^H\frac{2m+1}{2}\exp[-(m\pi/H)^2 C_v\tau][\sin\frac{(2m+1-2l)\pi z}{2H} + \sin\frac{(2m+1+2l)\pi z}{2H}]dz$$

$$= -\phi\sum_{m=0}^{\infty}\frac{2m+1}{2}\frac{2H}{\pi}\exp[-(m\pi/H)^2 C_v\tau][\frac{1}{2(m-l)+1}\cos\frac{(2(m-l)+1)\pi z}{2H}$$

$$+\frac{1}{2(m+l)+1}\cos\frac{(2(m-l)+1)\pi z}{2H}]\Big|_0^H$$

$$= \phi\sum_{m=0}^{\infty}\frac{2m+1}{2}\frac{2H}{\pi}[\frac{1}{2(m-l)+1} + \frac{1}{2(m+l)+1}]\exp[-(m\pi/H)^2 C_v\tau]$$

$$=\phi\sum_{m=0}^{\infty}(2m+1)\frac{H}{\pi}[\frac{2(m-l)+1+2(m+l)+1}{(2m+2l+1)(2m-2l+1)}]\exp[-(m\pi/H)^2C_v\tau]$$

$$=\phi\frac{2H}{\pi}\sum_{m=0}^{\infty}\frac{(2m+1)^2}{(2m+2l+1)(2m-2l+1)}\exp[-(m\pi/H)^2C_v\tau]$$

The equation is solved to obtain :

$$C_l=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\frac{(2m+1)^2}{(2m+2l+1)(2m-2l+1)}\exp[-(m\pi/H)^2C_v\tau]\,(l=0,1,2,3,...) \tag{48}$$

$$C_m=\frac{4\phi}{\pi}\sum_{l=0}^{\infty}\frac{(2l+1)^2}{(2l+2m+1)(2l-2m+1)}\exp[-(l\pi/H)^2C_v\tau]\,(m=0,1,2,3,...) \tag{49}$$

Substituting the above equation into the equation (46), we obtain :

$$\bar{u}_{z0}(z,t)=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\{\sum_{l=0}^{\infty}\frac{(2l+1)^2}{(2l+2m+1)(2l-2m+1)}\exp[-(l\pi/H)^2C_v\tau]\}\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_vt]$$

$$=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}P_{ml}(\tau)\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_vt] \tag{50}$$

In equation(50) , $P_{ml}(\tau)=T_{ml}\exp[-(l\pi/H)^2C_v\tau]$, $T_{ml}=\dfrac{(2l+1)^2}{(2l+2m+1)(2l-2m+1)}$.

Thus,

$$\bar{u}_{z0}(z,t;\tau)==\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}P_{ml}(\tau)\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_v(t-\tau)] \tag{51}$$

Substitute the formula (51) into the formula (35), and get :

$$\bar{u}_z(z,t)=\int_0^t\bar{u}_{z0}(z,t;\tau)d\tau=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\int_0^tP_{ml}(\tau)\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_v(t-\tau)]d\tau$$

$$=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}T_{ml}\cos\frac{m\pi z}{H}\int_0^t\exp[-(l\pi/H)^2C_v\tau]\exp[-(m\pi/H)^2C_v(t-\tau)]d\tau$$

$$=\frac{4\phi}{\pi}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}T_{ml}\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_vt]\int_0^t\exp[-(\pi/H)^2C_v(l^2-m^2)\tau]d\tau$$

$$=\frac{4\phi}{\pi(\pi/H)^2C_v}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{ml}}{(l^2-m^2)}\cos\frac{m\pi z}{H}\exp[-(m\pi/H)^2C_vt]\cdot\{1-\exp[-(\pi/H)^2C_v(l^2-m^2)t]\}$$

$$=\frac{4\phi}{\pi(\pi/H)^2 C_v}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{ml}}{(l^2-m^2)}\cos\frac{m\pi z}{H}\{\exp[-(m\pi/H)^2 C_v t]-\exp[-(\pi l/H)^2 C_v t]\} \quad (52)$$

The solutions (formula 30) and (formula 52) obtained above satisfy all the basic equations, boundary conditions and initial value conditions, and are the analytical solutions of the strict non-linear consolidation model of vacuum preloading without sand cushion.

Therefore, the consolidation degree of a certain point of super soft soil foundation reinforced by vacuum preloading method without sand cushion in shallow surface layer is shown as formula (53):

$$U=\frac{4\phi}{p\pi(\pi/H)^2 C_v}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{ml}}{(l^2-m^2)}\cos\frac{m\pi z}{H}\{\exp[-(m\pi/H)^2 C_v t]-\exp[-(\pi l/H)^2 C_v t]\} \quad (53)$$

5 Engineering Case Analysis

The Nansha District of Guangzhou is located in the front edge of the alluvial plain of the Pearl River Delta. It is widely deposited with soft soil of coastal facies and delta facies. The deep soft soil in this area has typical high water content, large void ratio, high compressibility, low strength, strong thixotropy, low permeability and significant rheological properties. The consolidation deformation lasts for a long time and has significant geological representation. The Nansha Port Offshore Wharf Project of Guangzhou Port is located in the southern peninsula of Shazai Island, Huangge Town, Nansha District, Guangzhou City. The total area of foundation treatment of this project is 255,000 square meters. Among them,A1~A5 area adopts shallow surface vacuum preloading reinforcement technology to deal with soft foundation.

In the vacuum preloading test of shallow surface without sand cushion, except for the measuring points at the boundary of the test area, the other measuring points are not affected by the boundary effect. Therefore, the sand well model with equal strain can be used to calculate and verify the vacuum preloading effect on site.

According to the engineering geological conditions of Nansha Port offshore wharf project in Guangzhou Port, the accuracy of the model and solution method is verified by nonlinear model and Terzaghi model.

The nonlinear mathematical model of consolidation parameters of vacuum preloading model is expressed as formula (54)~(56):

$$e=1.97-0.394\lg\frac{p'}{p'_0} \quad (54)$$

$$e=0.7856+0.0533\lg k \quad (55)$$

$$a_{1-2}=1.487e-1.052 \quad (56)$$

In the formula : e denotes the connected void ratio at any time; p'_0 represents the average effective principal stress ; p_0 is the initial average effective principal stress ; k is the permeability coefficient.

Through calculation and comparison, the time history curve of the average consolidation degree of the foundation is shown in Figure 5 :

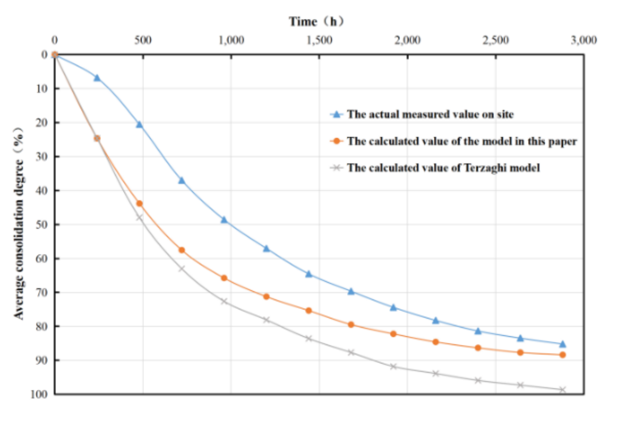

Fig. 5. Measure and calculate the average consolidation degree duration line of foundation

The measured value of consolidation degree is calculated by the measured settlement on site. The calculated value of consolidation degree is 89 %. According to the measured settlement, the residual settlement is calculated to be 0.074 m. The degree of consolidation and residual settlement meet the design requirements. Compared with the calculated value of consolidation degree, the final consolidation degree error calculated by the nonlinear model is 18.09 %, and the consolidation degree error calculated by the traditional Terzaghi theory is 39.53 %. It can be seen that the method in this paper fully considers the parameter changes in the process of vacuum preloading consolidation. Compared with the traditional Terzaghi theory, the nonlinear consolidation mathematical model of sand-free cushion is closer to the measured value.

6 Conclusion

In this paper, a theoretical model of non-linear consolidation of vacuum preloading without sand cushion based on pore compression law and non-Darcy seepage is established from a microscopic point of view, and a non-linear consolidation model of vacuum preloading without sand cushion is established. The model takes the point source load boundary as the fixed solution condition, and the equal vertical strain condition is used to solve the model. Under the condition that the permeability coefficient k and the void ratio e change with the consolidation effective stress, the nonlinear model is solved by the separation variable method and the impulse method, and the analytical solution of the model satisfying the boundary conditions is obtained.

Based on the offshore wharf project of Nansha Port in Guangzhou Port, the field test of shallow vacuum preloading was carried out, and the nonlinear model solution based on the field test was carried out. The consolidation degree of the offshore wharf project of Nansha Port in Guangzhou Port was 89 %, and the residual settlement was 0.074 m. The degree of consolidation and residual settlement meet the design requirements. Compared with the calculated value of consolidation degree, the final error of consolidation degree calculated by this method is 18.09 %, and the final error of consolidation degree calculated by traditional Terzaghi theory is 39.53 %. This method fully considers the parameter changes in the process of vacuum preloading consolidation. Compared with the traditional Terzaghi theory, the mathematical model of shallow surface nonlinear consolidation has a higher degree of agreement.

In this paper, the consolidation model is based on the microscopic nature of the consolidation of ultra-soft soil. Considering the microstructure changes, compaction effect and point source boundary conditions during the consolidation process, it is expected to play a good role in predicting the dissipation of pore water pressure in the vacuum preloading of ultra-soft soil foundation without sand cushion. It can be further verified in other dredger fill sites.

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