Monte Carlo Simulation-based Framework for Cryptocurrency Portfolio Risk Assessment

Zihang Qi*

China Agricultural University, Beijing, China

*2838268734@qq.com

Abstract. The cryptocurrency market has emerged as one of the most volatile and high-risk sectors in global finance, characterized by extreme volatility and speculative behavior. Digital assets like Bitcoin (BTC), Ethereum (ETH), and Binance Coin (BNB) exhibit distinct skewness, kurtosis, and fat-tail distributions-features that traditional Gaussian risk models struggle to capture. These statistical characteristics indicate that conventional risk models often underestimate the probability of extreme losses, making the development of effective risk measurement tools critical for investors and risk managers. This paper proposes a comprehensive Monte Carlo simulation framework that integrates fat-tail distributions with portfolio asset correlation structures to estimate Value at Risk (VaR) and Exponential Risk (ES) for cryptocurrency portfolios. Through Monte Carlo simulations using t-distributions and Gamma distributions, the study more accurately characterizes tail risks. The research demonstrates significant variations in risk estimates under different distribution assumptions, underscoring the importance of incorporating real market characteristics in risk management. This study aims to provide robust risk measurement tools for highly volatile digital asset markets and offer risk managers more reliable guidance.

Keywords: Cryptocurrency, portfolio investment, risk management, Value at Risk (VaR), Expected Loss (ES), Monte Carlo simulation, t-distribution, Gamma distribution

1 Introduction

Cryptocurrency markets have emerged as one of the most volatile and high-risk segments in global financial markets, driven by their unique trading dynamics and speculative behaviors [1]. Digital assets like Bitcoin (BTC), Ethereum (ETH), and Binance Coin (BNB) exhibit pronounced skewness, kurtosis, and fat-tail distributions-features that traditional Gaussian risk models struggle to capture [2]. These statistical characteristics indicate that conventional risk models often underestimate the probability of extreme losses. Consequently, developing effective risk measurement tools is critical for investors and risk managers, particularly when dealing with these highly volatile assets [3].

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Meanwhile, evolving international political dynamics and economic policies have further intensified uncertainties in financial markets, including the cryptocurrency sector. In these markets, regulatory shifts and macroeconomic developments frequently trigger sharp price swings and volatility spikes [4]. For instance, changes in regulatory frameworks or geopolitical risks have been shown to significantly amplify tail risks in digital asset returns, resulting in far more frequent and severe extreme events than observed in traditional markets [5]. Consequently, incorporating broader market and policy impacts into risk models has become an essential step for comprehensive risk assessment.

Value at Risk (VaR) and Expected Loss (ES) are fundamental risk measurement tools widely used in financial risk management to quantify potential portfolio losses under varying market conditions [6]. VaR defines the maximum potential loss at a specific confidence level, while ES captures the average loss exceeding the VaR threshold [7], providing sensitivity to the severity of tail events. However, the accuracy of these risk measures largely depends on the assumed return distribution. Empirical studies have shown that the standard normal distribution model is insufficient for assets with thick tails and volatility clustering characteristics, such as cryptocurrencies [8].

Recent research has explored alternative distribution assumptions and advanced modeling frameworks to better capture the fat-tailed and non-Gaussian characteristics of return distributions. For instance, stochastic volatility models with asymmetric heavy-tailed distributions outperform traditional norms in capturing tail risks and volatility dynamics in cryptocurrency returns [9]. Similarly, Copula-based methods for VaR and ES predictions can more flexibly model inter-asset dependencies, which is particularly crucial for portfolio risk assessment under extreme market conditions [10].

In practical applications for tail risk estimation, Monte Carlo simulation methods are widely used to generate numerous potential future return scenarios, enabling risk managers to estimate tail risk measures such as VaR and ES under complex distribution assumptions and market-dependent conditions [11]. Simulation-based risk estimation frameworks have proven highly effective in capturing effects like institutional spillovers and nonlinear correlations, which are challenging to model with traditional analytical methods [12]. Furthermore, historical research has proposed robust tail risk estimation techniques, such as block bootstrap filtered historical simulations, to address sequence dependence issues in residual returns and provide wider confidence intervals for extreme quantiles [13]. Meanwhile, the fat-tail behavior of financial returns reflects that extreme outcomes occur more frequently than predicted by normal distributions, which impacts the effectiveness of risk prediction and capital allocation strategies in highly volatile markets [14]. Combining fat-tail distribution modeling frameworks, such as generalized hyperbolic or skewed-t distributions, can enhance the accuracy of VaR and ES predictions, particularly at high confidence levels, which is crucial for stress testing and regulatory purposes.

Given the complex statistical characteristics of cryptocurrency returns and the critical need for accurate risk measurement, this study proposes a comprehensive Monte Carlo simulation framework that integrates fat-tail distributions with portfolio correlation structures to estimate VaR and ES for cryptocurrency portfolios. By comparing results under different distribution assumptions and scenarios, the research provides an in-depth analysis of how distribution selection and market structure influence tail risks, offering guidance for robust risk measurement in highly dynamic financial environments [15].

2 Introduction to the Distribution Used

2.1 Normal Distribution

A normal distribution is a continuous probability distribution characterized by a bell-shaped curve with symmetrical distribution around its mean. Its key feature is that most observed values cluster near the mean, while the probability of occurrence decreases as values deviate further from the mean. This distribution is determined by two parameters: the mean parameter controls the distribution's position, and the variance parameter determines its dispersion. A larger variance results in a flatter distribution, whereas a smaller variance leads to a more concentrated distribution.

In financial and risk management, the normal distribution is commonly used as the baseline for asset return modeling to characterize typical market volatility. However, given that real financial data often exhibits fat-tailed and peak-heavy distributions, the normal distribution has inherent limitations in capturing extreme risks. This has led researchers to incorporate alternative distributions with stronger tail characteristics, such as the t-distribution, to address these shortcomings.

The standard normal distribution, a special case of the normal distribution with a mean of 0 and a variance of 1, is commonly used for variable normalization and probability calculations.

The probability density function of the standard normal distribution is:

$$f(z) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2}) \tag{1}$$

These z - standardized variables possess a mean of 0 and a variance of 1.

2.2 t distribution

The t-distribution, a continuous probability distribution, is widely used to estimate the difference between sample means and population means. While its shape resembles the standard normal distribution, it exhibits thicker tails, indicating a higher probability of extreme values. The distribution's shape is determined by its degrees of freedom (df): with fewer df, the tails become more pronounced, whereas as df increases, the t-distribution gradually approaches the standard normal distribution.

In financial and risk management models, particularly in Monte Carlo simulations, the t-distribution is commonly employed to simulate extreme market events. This approach is adopted because financial market data often exhibits fat-tailed and peak-heavy characteristics, meaning that significant volatility and abnormal losses occur more frequently than the normal distribution would predict. By utilizing the t-distribution, the impact of such extreme fluctuations on risk management can be more accurately captured.

The probability density function of the t-distribution is:

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \tag{2}$$

The degree of freedom is ν, while Γ is the gamma function.

2.3 Gamma Distribution

The Gamma distribution is a continuous probability distribution commonly used to model waiting times, queue systems, and risk modeling for financial products. Its probability density function depends on two parameters: the shape parameter α and the scale parameter θ. With right-skewed characteristics, the Gamma distribution can simulate various distribution forms as these parameters vary.

In financial risk modeling, the Gamma distribution is widely used to simulate scenarios such as waiting times and insurance risks. Its characteristics are particularly suited for capturing asymmetry and heteroscedasticity caused by multiple independent factors. Especially in risk calculations, the Gamma distribution can better model factors like market price volatility and transaction costs.

The probability density function of the Gamma distribution is:

$$f(x;\alpha,\theta)=\frac{x^{\alpha-1}e^{-\frac{x}{\theta}}}{\theta^{\alpha}\Gamma(\alpha)},\ x>0 \tag{3}$$

The shape parameter is α, the scale parameter is θ, and the gamma function is Γ.

3 Data Description and Data Pre-processing

3.1 Introduction to Data

The data utilized in this study originates from Yingwei Finance Platform, encompassing daily closing prices of three major cryptocurrencies (Bitcoin, Ethereum, and Binance Coin) from November 1, 2022 to the present. These data have undergone rigorous screening to ensure sample integrity and accuracy. The collection period spans three years, covering the following segments: November 1, 2022 to October 31,2023; November 1, 2023 to October 31,2024; and November 1, 2024 to October 31,2025. This multi-period analysis aims to capture the risk performance of these cryptocurrency portfolios across varying market conditions.

3.2 Calculate the Log Return

In this process, the first step involves calculating the logarithmic returns for each cryptocurrency sample. The procedure is as follows: First, we obtain historical price data for the three target cryptocurrencies from the previously mentioned sources, denoted as $P_{i,t}$, where represents each cryptocurrency i and represents the corresponding time period t. Then, we apply our logarithmic return calculation formula.

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \quad (4)$$

Calculate the logarithmic return for each trading day based on price data for every cryptocurrency.

3.3 Set Weights

Since the study focuses on a three-currency investment portfolio, weighting is required to integrate these assets. This process should be entirely based on investors 'subjective judgment, so there's no definitive consensus on weight selection. However, given that trading volume serves as a key indicator of capital inflows and outflows in the market--where higher volumes signify more participants and stronger market influence--the study adopts the average trading volume ratio of each currency among the three as the weighting reference. This approach ensures the weight distribution aligns with actual market conditions and accurately reflects each asset's contribution to the portfolio.

$$\text{Turnover}_i = \text{Volumn}_i \times \text{Price}_i \quad (5)$$

Turnover refers to the transaction value of assets, Volume indicates the trading volume, and Price represents the closing price. Subsequently, the portfolio weight is computed based on the average transaction value ratio, resulting in the example outcomes presented in Table 1.

Table 1. Average transaction volume share of BTC, BNB, and ETH

Assets	Weights
BTC	0.417366
BNB	0.485035
ETC	0.097599

We have made appropriate adjustments here, increasing the contribution of ETC to the portfolio, and therefore set the weight ratio to

$$w_{btc}=0.3,\ w_{eth}=0.2,\ w_{bnb}=0.5$$

3.4 Check Whether Data Fits Normal Distribution

To test whether the asset return data conform to a normal distribution, this study employed the Shapiro-Wilk normality test. The null hypothesis of this test posits that the data follows a normal distribution. If the p-value is less than the significance level (0.05 in this study), the null hypothesis is rejected, indicating that the data do not conform to a normal distribution.

When analyzing the return data of BTC, BNB, and ETH across three distinct time periods, the results are presented in Table 2 below:

Table 2. Shapiro-Wilk normality test results for BTC, BNB, and ETH returns across different time periods

Time_spread	BTC	BNB	ETH
2022/11/1-2023/10/31	8.0759e-16	1.7971e-17	1.6153e-16
2023/11/1-2024/10/31	1.4705e-5	1.0792e-9	1.1515e-9
2024/11/1-2025/10/31	1.3893e-8	2.4373e-10	2.3084e-9

Clearly, all p-values here are significantly lower than 0.05, confirming that they do not follow a normal distribution.

3.5 Basic Statistical Analysis of Data and Parameter Calculation

First, we calculated the average returns and sample kurtosis for BTC, BNB, and ETH to establish baseline data for subsequent simulations. Specifically, we derived the average returns for each cryptocurrency, and the results are as shown in Table 3.

Table 3. Overall Characteristics of BTC, BNB, and ETH Yield Series

property	average rate of return	sample kurtosis
BTC	0.0019	4.63
BNB	0.0025	6.51
ETH	0.0009	6.83

In addition to the mean, the sample size and the number of assets are calculated here to serve as the dimensions for the matrix to be used later.

Based on the kurtosis values derived from computational analysis, we confirm that the return distributions of BTC, BNB, and ETH exhibit significant fat-tail characteristics, indicating a higher probability of extreme market volatility events in these assets. To accurately model υυυthis statistical feature in Monte Carlo simulations, we employ the Student's t-distribution to simulate stochastic disturbances in returns. The degree of freedom parameter of the t-distribution directly determines the tail thickness: a smaller parameter results in thicker tails and a higher probability of extreme values, while an increasing parameter gradually approximates a normal distribution.

When setting the degrees of freedom parameters, to prevent overfitting of sample statistics and enhance the robustness of model conclusions, We did not directly derive values from sample kurtosis. Instead, we established the settings based on the typical characteristics of financial time series, particularly the volatility patterns of cryptocurrency markets, and set υ= [3, 4, 4]. Specifically, BTC was assigned a lower degree of freedom to reflect its relatively high market volatility, while BNB and ETH were given slightly higher values to capture their distinct volatility patterns.

4 Methodology

4.1 Non-parametric Estimation

Nonparametric estimation is a distribution-free method widely used in financial risk analysis. It estimates Value at Risk (VaR) and Expected Loss (ES) using historical data quantiles, without requiring assumptions about normality or other known distributions.

4.1.1 Calculate VaR and ES Through Quantiles.

In the preceding process, we have consolidated the return data of three currencies into a single portfolio by weighting them proportionally. The next step involves directly calculating two risk metrics: VaR and ES.

Value at Risk (VaR) is a statistical measure that quantifies the maximum potential loss of an asset over a future period at a given confidence level. In this study, VaR at different confidence levels is used to assess the maximum potential loss of assets during a specified future period. Here, we directly calculate the sample quantiles of the data, with the specific quantiles determined by different confidence levels.

Next, we calculate the Expected Shortfall (ES), which measures the average loss size when losses exceed the VaR threshold. Compared to VaR, ES provides a more comprehensive assessment by accounting for potential losses in extreme scenarios.

We first calculate the Value at Risk (VaR) and Expected Loss (ES) at four confidence levels (95%, 97.5%, 99%, and 99.5%). Subsequently, calculations are carried out using data from three distinct time periods, and the results are presented in Table 4.

Table 4. Nonparametric Estimation Correspondence Table

Time_spread	risk measure	95%	97.5%	99%	99.5%
2022/11/1-2023/10/31	VaR	3511.38	5615.69	7787.15	18705.01
	ES	6044.59	8167.26	11678.28	18705.01
2023/11/1-2024/10/31	VaR	4056.79	5784.30	7783.46	9157.68
	ES	5926.80	7153.55	8297.39	9157.68
2024/11/1-2025/10/31	VaR	4144.44	5551.18	7883.47	11355.75
	ES	6111.51	7500.42	9693.80	11355.75

4.2 Monte Carlo Simulation Method（t distribution)

Given the limited historical data and the inherent unpredictability of non-parametric estimation, while our primary objective is to obtain more comprehensive market risk insights, this study also employs Monte Carlo simulation. As a stochastic sampling technique, Monte Carlo methods generate numerous random samples from known probability distributions to evaluate potential outcomes and derive their statistical properties. In this research, the t-distribution was specifically utilized for simulation purposes.

The primary rationale for adopting the t-distribution in this study stems from the pronounced fat-tail phenomenon observed in financial market data, where extreme values occur at significantly higher frequencies than normal distributions. Historical records consistently reveal extreme fluctuations far from average levels, which traditional normal distributions fail to adequately simulate. Consequently, conventional normal distributions may inadequately capture the true nature of market volatility in financial risk assessment.

In contrast, the t-distribution exhibits thicker tails than the normal distribution. This characteristic enables it to better capture the potential for extreme volatility in financial market data, particularly when sample sizes are limited, allowing the t-distribution to more accurately reflect actual market risk profiles. Given the high volatility and extreme risks inherent in financial markets, employing the t-distribution for Monte Carlo simulations proves to be a more appropriate approach. This methodology not only aligns with the distributional characteristics of real-world markets but also provides risk managers with more precise and reliable risk estimates. By leveraging the t-distribution to simulate market volatility, it becomes possible to better identify potential extreme scenarios in financial markets, thereby offering risk managers a more comprehensive risk assessment framework.

4.2.1 Specific Simulation Methods and Detailed Steps.

Step1: Calculate the covariance matrix and the correlation coefficient matrix

To accurately model the yield relationships among different cryptocurrencies, we computed the covariance matrix and correlation coefficient matrix for BTC, BNB, and ETH. The covariance matrix reflects the volatility relationships between these assets, while the correlation coefficient matrix quantifies their interdependencies. The covariance matrix is shown as Table 5:

Table 5. Covariance Matrix of Three Cryptocurrencies

	BTC	BNB	ETH
BTC	0.0007		
BNB	0.0005	0.0009	
ETH	0.0007	0.0006	0.0010

The correlation coefficient matrix is shown as Table 6:

Table 6. Correlation Coefficient Matrix of Three Cryptocurrencies

	BTC	BNB	ETH
BTC	1	0.5832	0.7618
BNB	0.5832	1	0.5383
ETH	0.7618	0.5383	1

The covariance matrix and correlation coefficient matrix obtained by calculation can quantify the fluctuation relationship between different assets, which provides the key input for the simulation generation sample.

Step2: Cholesky decomposition and random sample generation

To generate random samples with the specified covariance structure, this step first performs ΣCholesky decomposition on the asset covariance matrix. The decomposition yields a lower triangular matrix L that transforms standard normal random variables into those with the desired covariance structure. Here, one of the calculated Cholesky decomposition matrices is shown in Table 7.

Table 7. Cholesky decomposition matrix

0.0267901	0	0
0.01854375	0.0237	0
0.02589314	0.0040	0.0185

Through this matrixL, we can transform independent standard normal distribution random samples Z_{rand} into correlated samples $Z_{corr}$with a specified covariance structure, namely

$$Z_{corr} = Z_{rand} \cdot L^T \tag{6}$$

After obtaining the relevant samples Z_{corr}, further adjustments are required to ensure the generated samples align with the actual market distribution characteristics. This involves mapping these samples to t-distribution-compliant distributions. First, we generated 100,000 independent standard normal random samples Z_{rand} and mapped them to random samples Z_{corr} with a specified covariance structure. Subsequently, we normalized these samples to conform to the standard normal distribution.

$$U_{sim} = \frac{Z_{corr}}{\sigma} \tag{7}$$

Here, σ denotes the standard deviation of each asset. Next, we map the standard normal samples to a t-distribution using its cumulative distribution function (CDF) and percentile point function (PPF). Specifically, the standard normal samplesU_{sim} are transformed into a t-distribution with degrees of freedom υ_j as follows:

$$T_{sim} = t_{dist}^{-1}\left(U_{sim},\upsilon_j\right) \tag{8}$$

Here, υ_j is the degree of freedom for each asset, and t_{dist}^{-1} is the inverse cumulative distribution function (quantile function) of the t-distribution. Then, we normalize the generated t-distribution samples to have unit variance:

$$T_{scaled} = T_{sim} / \left(\frac{\upsilon_j}{\upsilon_j\text{-}2}\right)^{\frac{1}{2}} \tag{9}$$

Finally, we fine-tune the mean and volatility to ensure the generated samples match each asset's market volatility and risk profile.

$$R_{sim}=\mu_j+\sigma_j \cdot T_{sim} \tag{10}$$

Where, μ_j and σ_j represent the historical mean and historical volatility of each asset. Through this process, the simulated samples that conform to the specified covariance structure and market risk characteristics are ultimately obtained.

Step3: Verify the accuracy of the model

Through the aforementioned steps, we obtained R_{sim} the final simulation samples that not only conform to the correlation structure among assets but also reflect the market volatility and extreme event characteristics of each asset. By calculating the correlation coefficient matrix of the simulation results, we ultimately validated the accuracy and stability of the model. Table 6 before shows the correlation coefficient matrix of the simulated data.

The correlation coefficient matrix of the original data is demonstrated in Table 8.

Table 8. Correlation Coefficient Matrix of Three Cryptocurrencies in Raw Data

	BTC	BNB	ETH
BTC	1	0.6164	0.8076
BNB	0.6164	1	0.5957
ETH	0.8076	0.5957	1

The two data correlation coefficient matrix is similar, so it can be verified as stable.

4.2.2 Calculate VaR and ES.

This step first utilizes the acquired simulation data to calculate the combined total return R_1 through weighted computation using predefined coefficients. Subsequently, VaR at different quantile points (e.g., 5%,2.5%,1%,0.5%) is calculated based on the return samples. VaR represents the maximum possible loss value under a given confidence level, meaning the probability of returns being lower than this value is 5%. Next, all samples with returns ≤ VaR are screened, and their average is calculated to derive the Expected Loss (ES). Similar to non-parametric estimation methods, the monetary values of VaR and ES are derived by multiplying the initial investment amount V_0. Following the same procedure, experiments were conducted using data from three distinct time periods, with 500 random numbers simulated for each period to obtain a reasonable numerical range (presented in the table as mean± standard deviation). The results were obtained and shown in Table 9.

Table 9. t-distribution correspondence table

Time_spread	Indicators	95%	97.5%	99%	99.5%
2022/11/1- 2023/10/31	VaR	3669.87 (64.69)	4782.21 (105.22)	6457.84 (200.38)	7912.35 (328.67)
	ES	5519.41 (137.70)	6884.91 (223.27)	8995.16 (426.91)	10885.10 (705.67)
2023/11/1- 2024/10/31	VaR	3518.30 (71.62)	4693.87 (108.16)	6523.92 (211.84)	8151.17 (361.11)
	ES	5549.84	7069.76	9501.63	11764.67

		(153.96)	(262.54)	(540.64)	(933.95)
2024/11/1-2025/10/31	VaR	3711.56	4846.80	6540.83	8005.97
		(69.22)	(103.73)	(203.29)	(327.69)
	ES	5574.47	6941.64	9031.17	10877.73
		(131.19)	(210.87)	(393.36)	(636.47)

4.3 Monte Carlo Simulation Method (Gamma Distribution)

Similar to the t-distribution, the Gamma distribution is widely used in financial risk management, particularly for simulating market volatility. With its flexible shape, it effectively captures the skewness and kurtosis observed in financial markets. Unlike the t-distribution, the Gamma distribution typically exhibits a right-skewed distribution, characterized by longer tails and a positive skew. This property makes it more suitable than the normal or t-distribution for describing extreme losses in asset returns.

In financial market data, the Gamma distribution is primarily applied in risk assessment and asset pricing modeling, particularly for modeling asset prices or volatility. By utilizing the Gamma distribution, we can simulate data constrained within positive ranges and exhibiting high volatility. Specifically, the two parameters of the Gamma distribution-the shape parameter $\alpha$and the scale parameter θ-determine its distribution characteristics. The shape parameter controls the skewness of the distribution, while the scale parameter regulates its spread.

Given the frequent occurrence of right-skewed distributions in financial markets, the Gamma distribution proves particularly effective in capturing extreme volatility and peak effects. This is especially true when data deviates significantly from normal distribution, as Gamma distribution delivers more accurate risk assessments. By employing Gamma distribution to simulate market data, we can more comprehensively reflect the true risk characteristics of markets, thereby providing a more reliable foundation for risk management.

4.3.1 Specific Simulation Methods and Detailed Steps.

Step 1: Calculate the covariance matrix and standardize it

The first step is the same as the previous transformation to a t-distribution，just yielding standardized random variables U_{sim}:

$$U_{sim} = \frac{Z_{corr}}{\sigma} \tag{11}$$

Step 2: Map to the Gamma distribution

In this step, we map standardized samples to Gamma distribution samples using the shape α and scale parameters θ of the Gamma distribution. Specifically, for each asset Xj, we apply the inverse cumulative distribution function (PPF) to transform standard normal samples U_{sim} into Gamma distribution samples:

$$X[:,j]=\gamma^{-1}(U_{sim}[:,j], \alpha=\alpha_1, \text{scale}=\theta) \tag{12}$$

γ^{-1}represents the inverse cumulative distribution function of the Gamma distribution, where α is the shape parameter and θ is the scale parameter.

Based on existing Gamma function images with varying parameters, we selected specific parameters $\alpha=8,\theta=0.5$ for simulation. The shape parameter α controls the distribution's tail characteristics and skewness: larger values create moderate skewness that effectively captures common market volatility without overemphasizing extreme risk events. The scale parameter θdetermines the distribution's spread, with smaller values ensuring generated samples align with actual market volatility ranges and reflect concentrated fluctuations. These parameter selections are grounded in market data characteristics, aiming to simulate real-world asset return volatility and risk profiles for effective risk assessment and asset pricing.

Step 3: Adjust volatility and mean

To ensure the generated simulation samples match the actual market volatility (i.e., aligning the simulation with the mean and variance of the real original data), further adjustments to the sample's volatility and mean are required. First, calculate the mean and variance of the Gamma distribution:

$$\begin{gathered} E(X) = \alpha \cdot \theta \\ VaR(X) = \alpha \cdot \theta^2 \end{gathered} \tag{13}$$

Then, the simulated samples are adjusted according to these calculations to achieve the target volatility. The adjustment process is as follows:

$$A = \frac{\sigma}{\sqrt{VaR(X)}} \tag{14}$$

The updated simulated Xsample is:

$$X[:,j]=\mu_j+A \cdot X[:,j] \tag{15}$$

Here, $\mu_j$represents the historical mean of each asset, while the scaling factor A ensures the generated samples conform to the target volatility. Through these steps, we ultimately obtain Gamma-distributed simulation samples that align with market characteristics. These samples not only capture the correlation structure between assets but also effectively reflect market volatility patterns and extreme risk events. By leveraging these simulation samples, we can enhance financial risk assessment and asset pricing.

4.3.2 Calculate VaR and ES.

Similar to the previous t-distribution simulation process, the calculation of VaR and ES followed comparable steps. First, we computed portfolio returns using simulated R_1yield data and predefined weightings. Next, by determining quantiles of the return distribution, we obtained VaR values at different confidence levels, representing the maximum potential losses for the portfolio. Subsequently, we calculated the ES value by averaging all returns below the VaR threshold, which indicates the expected loss exceeding the VaR. Using data from three distinct time periods, we conducted experiments following this procedure, yielding the results in Table 10.

Table 10. Gamma Distribution Correspondence Table

Time_spread	Indicators	95%	97.5%	99%	99.5%
2022/11/1-2023/10/31	VaR	4448.70 (215.23)	5521.50 (291.23)	6830.47 (435.67)	7709.51 (607.01)
	ES	5950.44 (281.91)	6947.82 (384.97)	8167.00 (580.95)	9021.15 (779.41)
2023/11/1-2024/10/31	VaR	4469.06 (230.94)	5587.46 (301.12)	6947.92 (465.22)	7835.46 (630.23)
	ES	6027.17 (296.30)	7063.05 (404.45)	8328.29 (596.01)	9213.27 (775.50)
2024/11/1-2025/10/31	VaR	4486.82 (68.64)	5617.72 (92.19)	6984.20 (137.21)	7977.88 (186.27)
	ES	6036.10 (88.05)	7074.82 (120.73)	8368.19 (182.35)	9299.49 (257.21)

5 Results and Analysis

The empirical analysis in this study reveals extreme non-normality and structural instability in virtual crypto assets. Non-parametric estimation results demonstrate that crypto asset returns exhibit significant peak-thick-tail characteristics across all time dimensions, with substantial gaps between the 99.5% confidence level Value at Risk (VaR) and parameter model predictions. This risk endogeneity stems not from random fluctuations, but from macroeconomic policies' strong interventions through the "liquidity supply-regulatory premium-expectation management" chain. Specifically, when policy environments undergo drastic changes, market return distributions deviate from traditional smooth volatility assumptions, instead displaying highly nonlinear jump-diffusion characteristics. This implies that conventional risk measurement models often produce severe underestimations when facing policy-induced structural breaks (Structural Breaks), with such underestimations becoming particularly pronounced during market turbulence.

A longitudinal comparison of three observation periods clearly demonstrates the policy-driven risk evolution process, tracing a complete trajectory from "credit collapse" to "institutional repair" and then to "expectation game". During the crisis outbreak period (2022-2023), the Federal Reserve's unprecedented aggressive interest rate hikes to combat inflation caused extreme global liquidity shortages, triggering consecutive credit defaults among unregulated industry giants like Luna and FTX. This period saw the 99.5% VaR reach an extreme value of 18,705.01, far exceeding any parameter model's prediction range, reflecting a destructive one-sided downward risk under dual pressures of regulatory absence and macroeconomic tightening. Subsequently, during the regulatory repair phase (2023-2024), as Binance's compliance resolution and the anticipated approval of spot ETFs strengthened, the market entered the "institutional narrative" stage, with volatility significantly suppressed. The 99.5% VaR dropped to 9,157.68, reaching peak model fit. Entering the Trump New Deal era

(2024-2025), risk values rebounded to 11,355.75. Although his crypto-friendly statements boosted asset valuations, aggressive tariff policies and fiscal expansion intentions triggered strong "reinflation" expectations, hindering the Fed's rate-cutting rhythm. This binary game between industry benefits and macroeconomic headwinds once again demonstrated extreme market uncertainty, leading to a secondary surge in risk values.

In evaluating the validity of simulation methods, the performance differences between t-distribution and Gamma distribution under varying risk thresholds reveal their distinct statistical limitations. Within the moderate-risk range at the 95% confidence level, the Gamma distribution (Alpha=8) demonstrates significant superiority. Its predicted value (4486.52 for 2024-2025) not only surpasses the t-distribution but also effectively covers the true risk of non-parametric estimation (4144.44). However, at the 99.5% extreme tail, the Gamma distribution's exponential decay characteristic causes its tail thickness to rapidly diminish at extreme depths, resulting in a lower predicted value compared to the t-distribution. Although the low-degree-of-freedom t-distribution exhibits stronger extreme-resistance due to its thick-tail power-law decay, both models still underestimate approximately 30% of the true risk when facing distribution jumps caused by policy shocks in 2025. In summary, the Gamma distribution is better suited for capturing asymmetric daily volatility risks, while for extreme shocks induced by major policy transitions, the lagged nature of parametric models must be recognized, and stress tests should incorporate extreme-value theory adjustments. The t-distribution, with its thick-tail power-law decay, is more effective in characterizing extreme tail risks of crypto assets at high confidence levels (e.g., 99%,99.5%), particularly providing more accurate stress scenario assessments through indicators like ES. When the market experiences sudden policy or institutional shocks that significantly increase the tail thickness, it is crucial to examine how the degree of freedom setting affects tail thickness. Comparing with non-parametric results helps minimize model specification bias.

Furthermore, the standard deviations in parentheses of Tables 9 and 10 reflect the numerical volatility of VaR and ES estimates across repeated Monte Carlo simulations. As shown, these standard deviations rise significantly with higher confidence levels across all time windows. This indicates that under policy shocks and heightened structural uncertainties, tail risk measures not only show substantial increases in magnitude but also experience expanded estimation uncertainties. In contrast, ES standard deviations generally exceed those of VaR, demonstrating that conditional tail mean measures are more sensitive to extreme loss events and face greater challenges in numerical stability at high confidence levels.

6 Conclusion

In this study, a risk measurement framework was developed for crypto asset portfolios to characterize tail risks in return distributions across different market phases. Using daily price data from BTC, BNB, and ETH, we first calculate logarithmic returns and assign portfolio weights consistent with the aforementioned non-parametric estimation.

Three risk assessment approaches are then implemented: 1) Directly extracting Value at Risk (VaR) and Exponential Risk (ES) from historical return distributions using non-parametric quantile methods; 2) Conducting Monte Carlo simulations with t-distributions to capture fat-tail characteristics through reduced degrees of freedom; 3) Introducing Gamma distributions to model return asymmetry. For parametric simulations, we construct asset correlations via Cholesky decomposition of covariance matrices and Gaussian copulas, then map correlated stochastic samples to target marginal distributions calibrated by historical means and volatilities, ultimately generating large-scale simulated portfolio return sequences for VaR and ES estimation.

The results demonstrate that crypto asset returns exhibit significant non-normality and tail risks, with distinct performance of distribution models in characterizing medium-risk intervals versus extreme tail scenarios: The Gamma distribution better captures asymmetric downside risks in daily volatility, while the t-distribution proves more sensitive to fat-tail shocks at high confidence levels, making it more suitable for tail risk scenarios. Longitudinal comparisons across three periods further reveal that shifts in market structure and policy narratives may alter tail thickness and correlation patterns, resulting in stage-specific variations in risk exposure.

This study holds significant potential for future expansion. For instance, we failed to account for the impact of specific time periods on VaR and did not conduct conditional VaR calculations. By incorporating more comprehensive macroeconomic policy variables, liquidity indicators, and on-chain data into the modeling framework, while allowing relevant structures to evolve over time, we could develop a time series model that would further enhance the explanatory power and predictive stability of extreme tail risks.

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