A Study on the Application of POE Teaching Strategy in Correcting Misconceptions of Function Limits

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Abstract. Function limit is a core abstract concept in Advanced Mathematics and serves as the foundation for students' understanding of subsequent content such as derivatives and integrals. However, its core connotations—including "infinite approach," "locality," and the "$\varepsilon - \delta$ definition"—are difficult to perceive intuitively, leading students to form various misconceptions during the learning process. These misconceptions are covert and persistent, seriously affecting teaching effectiveness and subsequent learning. The POE (Predict-Observe-Explain) teaching strategy, as an inquiry-based teaching method grounded in constructivist learning theory, effectively exposes students' misconceptions, stimulates cognitive conflict, and thereby achieves precise correction of misconceptions by guiding students to actively predict, observe and verify, and explain discrepancies. This paper focuses on common misconceptions in the teaching of function limit concepts, combines the core advantages of the POE strategy, constructs a teaching application model of the POE strategy in correcting misconceptions of function limits, explores the internal mechanism of the POE strategy in correcting misconceptions of function limits, offering theoretical references for the teaching reform of Advanced Mathematics.

Keywords: POE teaching strategy, function limit, misconception, advanced mathematics teaching.

1 Introduction

Function limit, as the core content of limit theory, is not only a bridge connecting elementary mathematics and advanced mathematics but also the key for students to master analytical methods in advanced mathematics and solve practical problems. The concept of function limit embodies characteristics such as "infinity," "locality," and "logicality," which are essentially different from the "finite and determinate" mathematical concepts students have previously encountered. This leads students to be susceptible to interference from existing knowledge and experience during the learning process, forming various misconceptions.

Current teaching of function limit concepts generally adopts "lecture-based" teaching, directly presenting the definition, properties, and calculation methods of function limits without diagnosing students' misconceptions or providing targeted correction. The long-term accumulation of misconceptions not only affects the learning

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of function limits themselves but also hinders the understanding and mastery of subsequent content such as derivatives and integrals. Therefore, exploring an efficient and feasible method for correcting misconceptions has become the key to improving the teaching quality of function limit concepts.

With the continuous deepening of higher education teaching reform, inquiry-based and interactive teaching models have gradually become important pathways for improving the teaching quality of Advanced Mathematics courses [1-6]. The POE teaching strategy originated in the 1980s, proposed by Gunstone and White [7]. This strategy focuses on students' active inquiry and cognitive experience, and highly aligns with the needs for correcting misconceptions of function limits: diagnosing misconceptions through the prediction stage, triggering cognitive conflict through the observation stage, correcting misconceptions through the explanation stage, and ultimately helping students accurately grasp the essential connotation of function limits. This paper explores the application pathway of the POE teaching strategy in correcting misconceptions of function limits.

2 Analysis of Common Misconceptions and Their Causes in Function Limit Concept Teaching

Based on teaching practice research, students' misconceptions in learning function limit concepts mainly concentrate in the following three aspects:

(1)Misconceptions about "x approaches x_0"

Most students confuse the relationship between "x approachesx_0" and "x equalsx_0," forming misconceptions such as "for the limit of a function as x approaches x_0 to exist, the function must be defined at x_0" and "if the function is undefined atx_0, then the limit definitely does not exist." For example, when learning the function $f(x)=(x^2-1)/(x-1)$, students generally believe that this function has no limit as $x\to1$ because the function is undefined at $\mathrm{x}=1$, neglecting the "locality" characteristic of limits—that is, the limit focuses on the changing trend of function values as x approachesx_0, regardless of whether the function is defined at x_0. This is a typical misconception formed based on the experience from elementary mathematics that "definition implies meaning."

(2) Logical misconceptions about the $\varepsilon-\delta$ definition

The $\varepsilon-\delta$ definition is the rigorous mathematical definition of function limits, but its logical relationships are abstract and complex. Students easily form two major misconceptions: first, separating "for any $\varepsilon>0$" from "there exists $\delta>0$," believing that δ is a fixed value independent of ε, unable to understand the dependence of δ; second, being unable to understand the correspondence between "$0<|\mathrm{x}-\mathrm{x}_0|<\delta$" and "$|\mathrm{f}(\mathrm{x})-\mathrm{A}|<\varepsilon$," simply interpreting it as "when the distance between x and x_0 is less than δ, the distance between the function value and A is less thanε," neglecting the arbitrariness of ε and the logical progressive relationship, resulting in inability to accurately use the $\varepsilon-\delta$ definition to prove function limits. This is one of the most difficult misconceptions to correct in function limit teaching.

(3)Intuitive misconceptions about "infinite approach"

Students' understanding of the connotation of "infinite approach" remains superficial, forming misconceptions such as "when function values infinitely approach A, it means the function value will eventually equal A" and "the distance between function values and A can be infinitely small but must be greater than 0," unable to accurately grasp the dynamic process and mathematical connotation of "infinite approach." They equate the abstract "infinity" with intuitive "finite approach," leading to vague understanding of the essence of limits. This is the core cognitive obstacle for students learning function limits.

3 Construction of Teaching Application Model of POE Strategy in Correcting Misconceptions of Function Limit

Combining the teaching characteristics of function limit concepts and students' common misconceptions, this paper constructs a trinity teaching application model of "pre-class diagnosis—classroom implementation—post-class consolidation" for the POE strategy, focusing on precise diagnosis, effective correction, and consolidation of misconceptions. The specific implementation process is as follows:

3.1 Pre-class Diagnosis: Predicting Misconceptions and Clarifying Objectives

Pre-class diagnosis is the foundation of misconception correction. The core objective is to predict misconceptions students may encounter, clarify teaching priorities and correction targets, and prepare for classroom POE teaching implementation. Specific operational methods include:

(1)Teachers, combining teaching priorities of function limit concepts and students' cognitive characteristics, sort out students' common misconceptions, analyze the formation roots of various misconceptions, and design targeted preview tasks and predictive exercises.

This paper adopts a "two-tier" design for the diagnosis scale. The first tier consists of multiple-choice questions (testing surface cognition), and the second tier consists of multiple-choice questions (testing deep understanding and diagnosing misconception types).

Question 1, Tier 1 (Multiple Choice):

Regarding the limit of function $f(x) = (x^2 - 1)/(x - 1)$ as x approaches 1, your first judgment is ( )

A. The limit does not exist because when $x = 1$, the function is undefined (denominator is 0); no definition means no limit

B. The limit does not exist because as $x \to 1$, both numerator and denominator approach 0, making calculation impossible

C. The limit exists and the limit value is 2, regardless of whether the function is defined at $x = 1$

D. The limit exists and the limit value is 0, because as $x \to 1$, the numerator $x^2 - 1$ approaches 0 and the denominator approaches 0, and 0 divided by 0 equals 0

Misconception orientation: Options A/B/D correspond to different basic misconceptions (A: binding "function defined at a point" with "limit exists"; B: no cognition of "0/0 indeterminate form"; D: misunderstanding the calculation logic of "0/0 indeterminate form"). Option C represents correct cognition.

Question 1, Tier 2 (Multiple Choice):

If you believe the limit exists as $x \to 1$ (such as selecting C in Tier 1), or after further consideration, regarding the calculation logic of the limit, you agree with ( )

A. Directly substitute $x = 1$ into the function for calculation; both numerator and denominator are 0, therefore the limit is 0

B. First simplify the function to $f(x) = x + 1$ （$x \neq 1$）, then substitute $x = 1$ into the simplified function to obtain the limit value of 2, because the simplified function is completely identical to the original function

C. First simplify the function to $f(x) = x + 1$ （$x \neq 1$）, then find the limit as $x \to 1$, with value 2, because the limit focuses on the process of x approaching 1, unrelated to the function value and domain at $x = 1$

D. The limit value is 2, but the reason is that as $x \to 1$, both numerator $x^2 - 1$ and denominator $x - 1$ simultaneously approach 0, with the same approaching speed, therefore the limit is 1 + 1 = 2 (no simplification needed)

Misconception orientation: Options A/D correspond to deep misconceptions (A: incorrectly using "substitution method" to calculate indeterminate forms; D: vague cognition, not mastering the core logic of simplification). Option B: confusing "domain differences between original and simplified functions," mistakenly equating the two. Option C represents correct cognition.

(2)Students complete preview tasks and predictive exercises, actively recording their questions and understanding. Teachers, through grading preview assignments, online feedback, pre-class quizzes, and other methods, precisely diagnose the types, distribution, and roots of students' misconceptions, clarifying the misconceptions and teaching difficulties that need to be focused on in classroom teaching.

(3)Based on pre-class diagnosis results, teachers adjust teaching plans, design POE inquiry activities tailored to students' misconceptions, clarify teaching objectives and operational methods for each stage, ensure the targeting and effectiveness of classroom teaching, and focus on core misconceptions for correction teaching.

3.2 Classroom Implementation: Three-step POE Advancement for Precise Misconception Correction

Classroom implementation is the core stage of misconception correction, strictly following the three-step process of "Predict—Observe—Explain." Combined with specific teaching content of function limits, misconceptions are corrected in a targeted manner. The specific operation takes "function limits as x approaches x_0" as an example:

(1) Prediction Stage: Exposing Misconceptions and Clarifying Conflict Points

Teachers, combining pre-class diagnosis results, clarify the conflict points for subsequent observation stages, stimulate students' desire for inquiry, and prepare for the generation of cognitive conflict.

(2) Observation Stage: Intuitive Demonstration Triggering Cognitive Conflict

Teachers dynamically display the graph of function $f(x) = (x^2 - 1)/(x - 1)$ and the changing process of function values as x approaches 1 through multimedia, allowing students to intuitively observe that "when x infinitely approaches 1, the function value infinitely approaches 2, but the function is undefined at$x = 1$," creating strong cognitive conflict with students' misconception that "no definition means no limit." Meanwhile, combining the geometric meaning of the $\varepsilon - \delta$ definition, dynamically demonstrate the correspondence between "arbitrariness of ε" and "existence of δ," allowing students to intuitively feel that "for any given ε, a corresponding δ can always be found such that when $0 < |x - 1| < \delta, |f(x) - 2| < \varepsilon$," correcting students' logical misconceptions about the $\varepsilon - \delta$ definition.

(3) Explanation Stage: Analyzing Roots and Correcting Misconceptions

Teachers guide students to analyze the reasons for discrepancies between prediction results and actual results based on observation results, analyze the roots of misconceptions in a targeted manner, gradually correct misconceptions, and construct correct function limit concepts. For example, for the misconception that "no definition means no limit," teachers, using the example of function$f(x) = (x^2 - 1)/(x - 1)$, explain that "the essence of function limits is the changing trend of function values as x approaches x_0, unrelated to whether the function is defined at x_0," emphasizing the "locality" characteristic of limits and breaking the negative transfer influence of elementary mathematics knowledge experience. For misconceptions about the $\varepsilon - \delta$ definition, through specific ε values (such as $\varepsilon = 0.1, \varepsilon = 0.01$), guide students to find corresponding δ, gradually understanding "arbitrariness of ε," "dependence of δ," and "correspondence between the two inequalities," dismantling abstract logic and helping students establish correct logical cognition.

3.3 Post-class Consolidation: Strengthening Application and Consolidating Correction Effects

Post-class consolidation is the extension of misconception correction. The core objective is to strengthen students' understanding of correct function limit concepts, consolidate misconception correction effects, prevent misconception rebound, and achieve internalization and application of scientific concepts. Specific operational methods include:

(1) Assign tiered consolidation exercises, focusing on students' common misconceptions.

(2) Establish a post-class feedback mechanism. After students complete exercises, teachers promptly grade and analyze students' error situations.

(3) Guide students to write learning reflections, cultivating reflection habits, and preventing misconception rebound.

4 Conclusion

This paper focuses on common misconceptions in function limit concept teaching, systematically exploring the application pathway of the POE teaching strategy in correcting misconceptions of function limits, constructing a trinity teaching application model of "pre-class diagnosis—classroom implementation—post-class consolidation." Through the three stages of "Predict—Observe—Explain," it precisely exposes students' misconceptions, stimulates cognitive conflict, effectively improves misconception correction rates, and significantly enhances students' concept understanding ability, logical reasoning ability, and learning enthusiasm. It solves problems in traditional function limit concept teaching such as difficulty in correcting misconceptions and poor teaching effectiveness, providing new practical pathways and theoretical references for misconception correction teaching of function limits in Advanced Mathematics.

References

1. Li Xiuyan. Exploration of Student-centered Teaching Reform Paths in Advanced Mathematics [J]. Technology Wind, 2025, (35): 68-70+135. https://link.cnki.net/doi/10.19392/j.cnki.1671-7341.202535022.
2. Xuan Guolong. Research on Knowledge Graph-enabled Advanced Mathematics Curriculum Reform [J]. Theory and Practice of Innovation and Entrepreneurship, 2026, 9(01): 52-54. https://kns.cnki.net/kcms2/article/abstract?v=nTS8fe6ew4orCyvZN7YaKv71-8d71JEPMm5AmNYaK4V8IT_XfXk12nSne5J6-Zl1MeNGD28CAyYAb3lX7Z6fVd97FaQq5JVBPA4t5AEDyFHLGZJrx26LRrD08qXupsv-bQe1B7JPeIn-YzfKvkzqcRURlull3jkCYOQKJUTmdCa9eKNbq61lQA==&uniplatform=NZKPT&language=CHS.
3. Xu Suzhen. Exploration and Practice of "Advanced Mathematics" Curriculum Teaching Reform Inspired by Mathematical Modeling Competitions—Taking Industrial Robot Technology Major as an Example [J]. Technology Wind, 2025, (34): 75-77. https://link.cnki.net/doi/10.19392/j.cnki.1671-7341.202534025.
4. Wang Fenling. Discussion on Advanced Mathematics Curriculum Teaching Reform and Application-oriented Talent Cultivation [J]. Education and Vocation, 2013, (14): 141-142. https://link.cnki.net/doi/10.13615/j.cnki.1004-3985.2013.14.002.

5. Chen Ping, Huang Zhixin, Yao Changfen, et al. From "Misconception" to "Scientific Concept": POE Model-based Sensor-assisted Inquiry Teaching of Buoyancy Concepts [J]. Physics Teaching Discussion, 2026, 44(01): 1-6. https://kns.cnki.net/kcms2/article/abstract?v=nTS8fe6ew4od9JSwq2rJopi0GSD9gOprisc7WOq5ABYGheiirRCRUEKvTF_mFuYJOENRjDEFtDRXzFtJ5bPAqS5DxP0drh2nsd4m5giCDsY8D6u3h00wSoz3HoYAqAMD8jBZd9pP-lFg_TnKP69n9mrxestRpwpVYo2ELgrpl5pYI157NGYXFA==&uniplatform=NZKPT&language=CHS.
6. Zheng Yusi, Zhang Jia, Wang Long. Exploration of POE Teaching Strategy Application in Correcting Biological Misconceptions [J]. Biology Teaching, 2025, 50(12): 26-29. https://kns.cnki.net/kcms2/article/abstract?v=nTS8fe6ew4qAjtho8TPGLb8ofPYlcqxSm9kJr-QdkTIxgTCFp5-7AnLghimksVN-SIUH-XzjMrIEnu7vUsu-7CrhkiK_tyRWhUcTrB_UqhrNvan2KAKYiYm3BuDBRpa29tzOL5LfRorQmgeGCWqg84CUtMMRGE4N5GR8JBbXfngKmJdbqHfr0w==&uniplatform=NZKPT&language=CHS.
7. White, R. T., & Gunstone, R. F. Probing Understanding [M]. London: The Falmer Press, 1992. https://book.douban.com/subject/2740112/.