Design and Dynamics of Brake Systems: A Sensitivity Analysis Approach

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Abstract. To safely control any automobile, brake system plays a crucial role. Brake system performance defines the driver's confidence in handling the vehicle at different speeds on all terrains. So, it becomes important to understand the underlying dynamics to decide braking requirements of a vehicle. Brake performance is affected by several parameters. Performance is predicted by mathematical modelling using vehicle and brake parameters. Physical measurements of brake performance at vehicle level mostly yield values that vary from predicted. These variations are caused by multiple parameters like tolerances, tyre-road adhesion, environmental conditions and human reaction times. Literature study shows the brake calculations to finalize either front or rear brake sizes and thermal stress analysis on brake parts during braking. There are no studies to understand the discrepancies between calculations and the test results. This paper aims to provide a brake system design methodology and study the influence of brake part, vehicle parameter tolerances, and braking reaction time upon the vehicle brake performance. Attempt is made to establish tolerance bandwidth for major performance parameters like deceleration and stopping distance particularly for hydraulic brake systems. Further major contributing factors of brake performance variation are identified, and their percentage contribution is quantified. In the case of critical situations like homologation compliance, ensuring these major contributing factors at the nominal values yields results that are close to predicted.

Keywords: Brake system, brake performance, mathematical modelling, dynamics, parameters, deceleration, stopping distance.

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1. Introduction

Brakes are safety critical for any automotive vehicle. Ensuring performance in all the practically possible conditions is utmost important. For the same reason, Homologation norms [3] [4] [5] [6] [7] are also levied on this system. Along with meeting the government norms, customer specific requirements like braking confidence & feel also to be ensured while designing the system. There are various types of brake actuation systems like mechanical, hydraulic and air brakes. For each of the above brake types, performance is predicted by considering dynamic weight transfer of the vehicle and then calculating the required brake torques for corresponding decelerations.

To start with, the brake performance variation is studied for 3-wheeler hydraulic brakes. The same can be extended to all hydraulic brake systems. Similar process is followed for other types of brake actuation systems also. Currently 3-wheelers have hydraulic drum brakes on all three wheels. This is the simplest system as it doesn't involve disc brakes, boosters, valves, ABS, etc. So, evaluating brake performance for three-wheelers forms a base for all further studies. Reference three-wheeler brake layout is attached in Fig. 1.

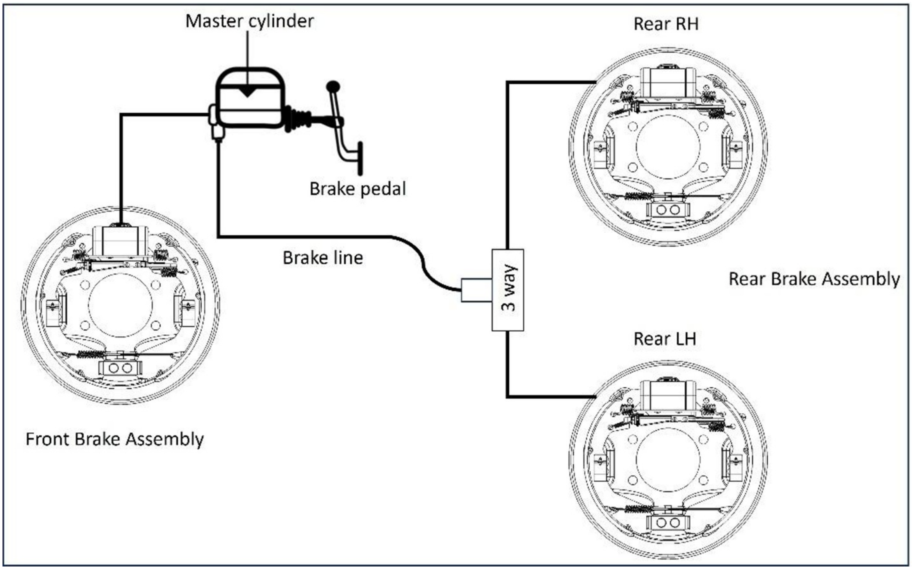

Fig. 1. – 3W Brake Layout

During braking, vehicle weight gets transferred from rear axle to front axle. This weight transfer varies with respect to vehicle decelerations. So, as a first step establishing the dynamic axle weights for corresponding decelerations is important to evaluate brake performance without wheel locking. Wheel locking is a phenomenon of wheel slipping instead of rolling as braking force is more than actual braking requirement of particular axle at a given deceleration. Wheel locking / skidding is safety critical as it causes loss of directional control and stability of the vehicle which can also lead to accidents.

After evaluating dynamic vehicle weights at different decelerations, required braking torques are calculated. Thus, vehicle brake performance requirements (ideal) are available and to counteract the same, brake system needs to be proposed. An ideal brake performance curve is a parabola whereas actual brake system performance is linear. A reference graph is provided in Fig. 2.

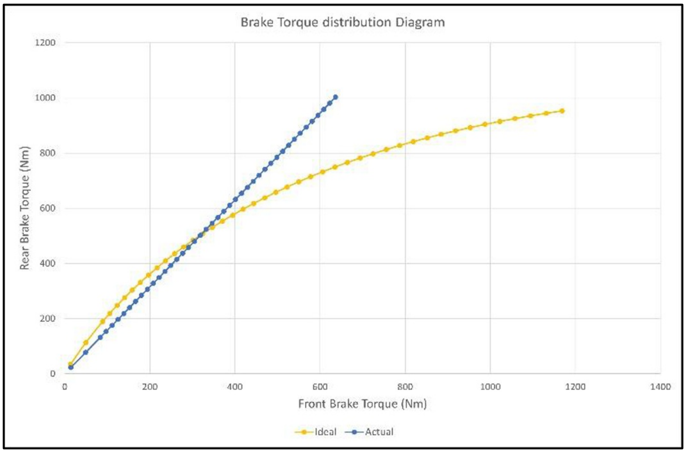

Fig. 2. – Ideal and actual brake performance curves

This linear brake performance curve is affected by various parameters & their tolerances as shown in Table 1. Hydraulic efficiencies are assumed to be 90% and mechanical efficiencies as 80% based on literature survey [2] and physical correlation to test results.

Table 1. Brake parameters

Sl No	Variable	Tolerance
1	Pedal Ratio	±0.1
2	Master cylinder diameter (mm)	±0.05
3	Tyre rolling radius (mm)	±3
4	Front Brake / Drum Diameter (mm)	±0.15
5	Front wheel cylinder diameter (mm)	±0.05
6	Front Brake Factor (%)	±10
7	Rear Brake / Drum Diameter (mm)	±0.15

8	Rear wheel cylinder diameter (mm)	±0.05
9	Rear Brake Factor (%)	±10

Considering all the above parameters at the USL (Upper Specification Limit), Mean and LSL (Lower Specification Limit), braking decelerations and stopping distances are calculated. Further to that taking one parameter at a time, brake performance is calculated at USL, Mean and LSL to understand the contribution of each parameter.

Influence of Rolling Resistant Coefficient (RRC) is also included as it needs to be optimized with the growing importance of fuel efficiencies without compromising braking performance.

Driver reaction time at different vehicle speeds and its influence on Brake Performance is also studied.

1 Detailed Study

1.1 Dynamic Axle Loads:

The forces that act on a moving vehicle are as shown below:

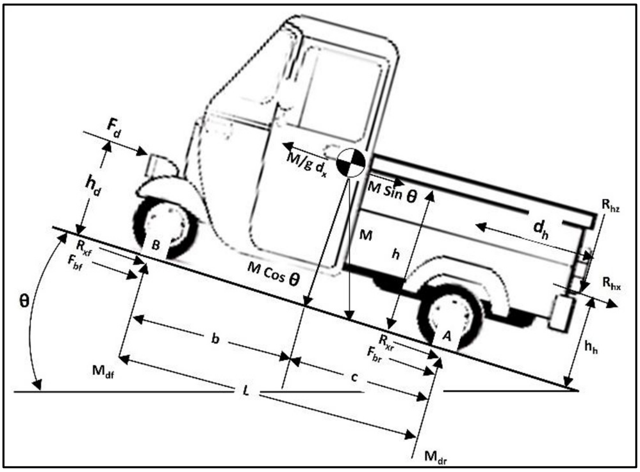

Fig. 3. – Forces acting on a vehicle [1]

M – Gross vehicle load (N), M CosΘ and M SinΘ are vertical and horizontal components of GVW acting at the center of gravity of height h (m)

M_{df} and M_{dr} - Dynamic front and rear axle loads (N)

F_{bf} and F_{br} – Front and rear braking forces (N)

R_{xf} and R_{xr} – Front and rear rolling resistance forces (N) – studied separately

F_d – Aerodynamic drag (N) acting at the height h_d (m)

R_{hx} and R_{hz} – Longitudinal and vertical hitch point forces (N) of a trailer, acting at distances h_h and d_h respectively (m)
d_x – deceleration (m/s^2)
h – Height of center of gravity (m)
L – Wheelbase (m)
b & c - are the distance from CG to front wheel & rear wheel respectively (m).
Considering the moments equilibrium at points A and B, dynamic axle loads are calculated using the below equations [1] [8].

$$M_{df} = (Mc \cos\Theta - R_{hx} h_h - R_{hz} d_h + M d_x h/g - F_d h_d - Mh \sin\Theta) / L \quad (1)$$

$$M_{dr} = (Mb \cos\Theta + R_{hx} h_h + R_{hz} (d_h+L) - M d_x h/g - F_d h_d - Mh \sin\Theta) / L \quad (2)$$

Considering the vehicle is decelerating on a flat surface $\Theta = 0$, there is no trailer attached here so both R_{hx} and R_{hz} are zero. Aerodynamic drag always aids braking and at low speeds, it can be assumed as zero. Neglecting it doesn't hamper brake performance. Dynamic axle load equations are reduced as below:

$$M_{df} = (Mc + M d_x h/g) / L \quad (3)$$

$$M_{dr} = (Mb - M d_x h/g) / L \quad (4)$$

Further, when the vehicle is stationary i.e, dx=0, equations are simplified as:

$$M_{sf} = Mc / L \quad (5)$$

$$M_{sr} = Mb / L \quad (6)$$

Where, M_{sf} and M_{sr} - Static front and rear axle loads (N)
The final equations for determining the dynamic front and rear axle loads are written as,

$$M_{df} = M_{sf} + M\, d\, h / L \quad (7)$$

$$M_{dr} = M_{sr} - M\, d\, h / L \quad (8)$$

d – deceleration (g units, g = 9.81m/s^2)
Decelerations in g units are convenient to use as it can be directly correlated with adhesion / traction coefficients.
Thus, above equations give us dynamic axle loads for decelerations starting from 0 to 1g. From dynamic axle loads, required braking force (F) / torque is found using tyre-road coefficient of friction (μ). So, the brake force (F) required is calculated using the below equation.

$$F = \mu M_d \quad (9)$$

M_d – Dynamic axle load (N)

1.2 Brake Force & Torque calculations:

Brake forces generated in a hydraulic circuit are calculated [2] from Pascal's law which states "Pressure remains constant throughout in a closed circuit for an incompressible fluid" as below.

$$P_{tmc} = P_{wc} \tag{10}$$

$$F_{tmc} A_{tmc} = F_{wc} A_{wc} \tag{11}$$

P_{tmc} and P_{wc} - Pressures at tandem master cylinder (TMC) and wheel cylinder (WC) respectively (bar)
A_{tmc} and A_{wc} – Areas of TMC and WC respectively (mm^2)
F_{tmc} – Input force to TMC (N)
F_{wc} – Output force from wheel cylinder (N)
Brake Torque is calculated using the following equation:

$$\text{Brake Torque (Nm)} = PE \times PR \times \eta_p \times A_{wc} \times \eta_{wc} \times BF \times R_{drum} / A_{tmc} \times \eta_{wc} \tag{12}$$

where,
PE - Pedal Effort (N)
PR – Pedal Ratio
η_p – Pedal Efficiency
η_{wc} – Wheel cylinder Efficiency
η_{tmc} – TMC Efficiency
BF – Brake Factor
R_{drum} – Drum radius (m)

1.3 Traction Coefficients:

Based on brake torques, braking forces at the tyre road interface are calculated using tyre dynamic rolling radius. Traction coefficients are established at each deceleration values starting from 0.1g to 1g utilizing dynamic axle loads and the braking forces. Traction coefficient (μ_T) is the ratio of actual braking force (F_a) and the dynamic axle load (M_d) of the respective axle. Traction values are plotted against pedal effort. Refer Fig. 4.

Traction coefficients for front and rear axles are calculated as follows:

$$\mu_T = F_a / M_d \tag{13}$$

where,
μ_T = Traction coefficient
F_a = Actual Braking force (N)

M_d = Dynamic load of front (M_{df}) or Rear axle (M_{dr}) (N)

Having traction coefficients in place, maximum attainable vehicle decelerations without wheel lock can be found out. General road-tyre friction coefficient for tar road is between 0.7 to 0.8 [2]. For brake testing as per homologation standards [3] [4] [5] max available adhesion is 0.9. Wheels lock and vehicle slides if traction coefficient of any of the wheels exceeds max available adhesion of the road (without ABS).

For example, a three wheeled vehicle whose kerb weight is around 550 kgs, with wheelbase of 2m is considered for study. This vehicle uses front Ø200mm and rear Ø180mm drum brakes with Ø19.05mm Tandem Master Cylinder.

For the vehicle under study, brake calculations are done as per the above-mentioned load transfer and brake torque formulae. Results are plotted with pedal effort on X-axis and deceleration / adhesion on Y-axis. For all the graphs presented in this paper, pedal efforts start from 10 Kgs, ignoring threshold pressure and allowing the system to build at least 0.1g deceleration. For drum brakes to expand and generate brake torque, brake shoes must overcome return spring forces, which is termed as threshold force. As per the graph in Fig. 4, max deceleration that can be achieved is 6.02 m/s^2 at 270 N pedal effort, as the rear wheels lock above this deceleration with road adhesion value of 0.8. For all the practical purposes 0.8 road adhesion is true as most of the tar roads are in line with this.

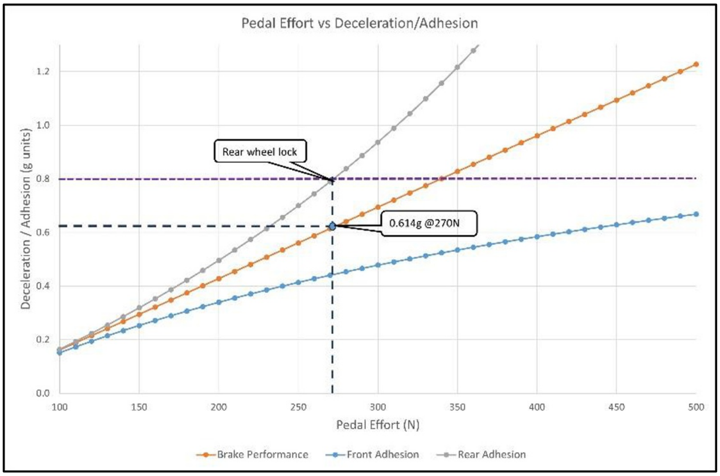

Fig. 4. – Pedal Effort vs Deceleration / Adhesion

1.4 Stopping distance:

From Fig. 4, vehicle decelerations at different pedal efforts are found. By using dynamic axle loads, brake torques and vehicle decelerations, stopping distances are calculated using equation of motion below:

$$v^2 - u^2 = 2as, \qquad => \qquad s = u^2 / 2a. \tag{14}$$

where, v = final velocity (m/s) – which is 0 for a complete stop,
u = initial velocity (m/s)
a = deceleration (m/s^2) (negative as vehicle is decelerating)
s = stopping distance (m)

1.5 Brake system parameters study:

First the calculations are run with all the parameters at lower specification limit, then at mean and at upper specification limit. This forms the worst case of all the tolerances, which may not be practically possible, but all other performance variations lie within the range specified between USL & LSL values.

Pedal Effort vs Deceleration and Pedal Effort vs stopping distance are plotted. Refer Fig. 5 & Fig. 6.

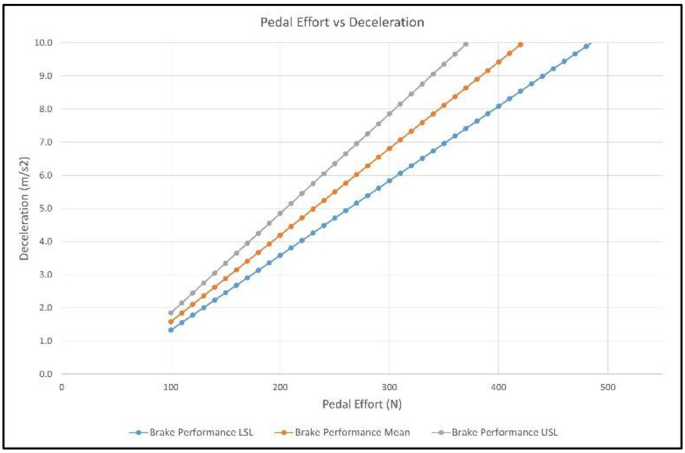

Fig. 5. – Pedal Effort vs Deceleration

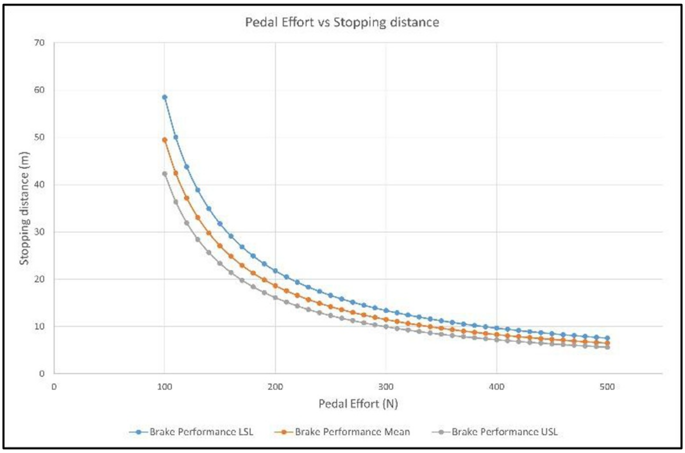

Fig. 6. – Pedal Effort vs Stopping Distance

Pedal effort vs Deceleration is a linear curve. From the graphs it is observed that for lower pedal effort the difference between USL & LSL with respect to mean is minimal. As the effort increases variations keep on increasing. Pedal Effort vs Stopping Distance curve is parabolic. Stopping distance variation with respect to mean is higher at lower pedal efforts and decreases as the effort increases. Vehicle decelerations found to vary between -14% and +16% whereas stopping distance varies between -16% and +14%.

Taking one parameter at a time, keeping all other parameters constant, calculations are run at USL, Mean and LSL for all the parameters. Pedal effort is kept constant at 20 Kgs. Speed at the time of braking is taken as 45kmph. Results are tabulated and are as follows. Refer Table 2.

Table 2. – Influence of Brake parameters

Sl No	Brake Parameter	Brake Performance Variation in %
0	Brake Performance	-16 +14
1	TMC Diameter (mm)	-0.66 +0.67
2	Pedal Ratio	-2.89 +2.73
3	Tyre Rolling Radius (mm)	-0.74 +1.67
4	Brake Diameter (mm)	-0.04 +0.04
5	Wheel Cylinder diameter (mm)	-0.15 +0.15
6	Brake Factor	-4.04 +3.74

Each parameter study shows that brake performance is influenced more by Brake Factor (44%), Pedal Ratio (32%) and tyre rolling radius (14%) compared to other parameters. Refer Fig. 7.

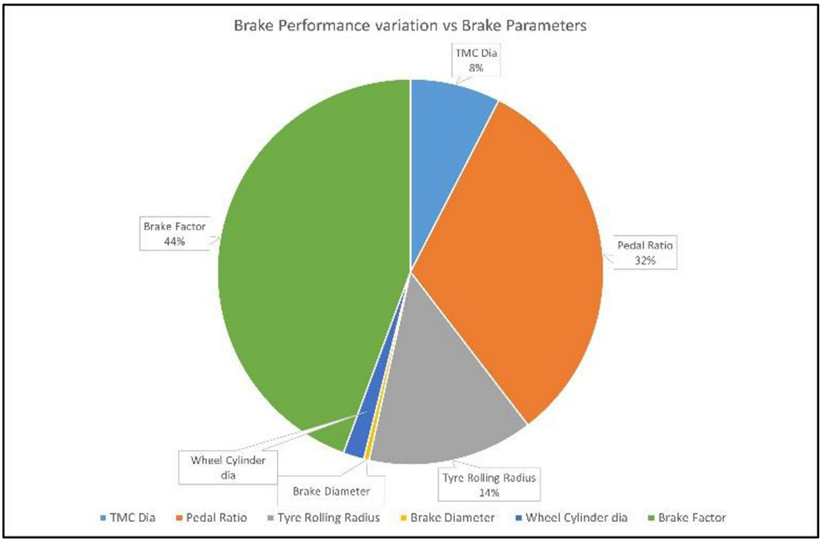

Fig. 7. – Brake Performance variations

1.6 Tyre Rolling Resistance and Driver reaction time influence:

Along with the above brake parameters, influence of tyre rolling resistance [1] coefficient (RRC) and driver reaction time [2] are also included below.

With the rising importance of increased mileage / range for automobiles, rolling resistance of tyres acquired prominence. Having low rolling resistance (LRR) tyres in the vehicle is beneficial in terms of mileage / range, but it has a negative effect on braking. So, it needs to be optimized to balance both the requirements. For evaluation, three RRC values (0.005, 0.01, 0.015) are considered and found that there is a reduction of 2.5% in brake performance changing the RRC from 0.015 to 0.005. Vehicle braking speed is 45kmph and pedal effort is 20kgs. Refer to Table 3.

Table 3. Influence of tyre rolling resistance

Sl No	Parameter	RRC Value (No units)	Brake Performance Variation in %
1	Tyre rolling resistance coefficient	0.005	-1.3
		0.01	
		0.015	1.2

Reaction time in brake performance testing is defined as how fast the driver presses the pedal to initiate braking, i.e., the time between driver starts pressing the pedal and starting of the vehicle deceleration. The way driver applies brakes impacts the stopping distance of the vehicle largely and the same is studied for different reaction times considering various vehicle speeds. Refer Table 4 & Fig. 8.

From the relation,

$$S = V \times t \tag{15}$$

S = Stopping distance (m), V = Vehicle speed (m/s), t = Reaction time (s)

Table 4. Influence of reaction time

Reaction time (sec)	Stopping distance due to reaction time (m)		
	Speed = 45 kmph	Speed = 50 kmph	Speed = 55 kmph
0.1	1.25	1.39	1.53
0.2	2.50	2.78	3.06
0.3	3.75	4.17	4.58
0.4	5.00	5.56	6.11
0.5	6.25	6.94	7.64

The above table shows that with the increase in reaction time, stopping distance increases. This is an additional distance travelled by vehicle before actual braking deceleration starts. With higher vehicle speeds the distance is further increased.

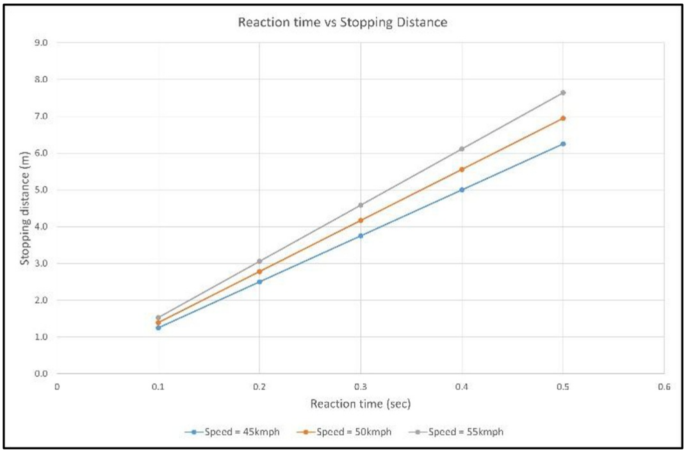

Fig. 8. – Reaction time vs Stopping distance

1.7 Influence of vehicle dimensions tolerance on Brake Performance:

The variations in vehicle parameters like front axle load, rear axle load, height of center of gravity and wheelbases do not directly affect the brake performance, but it restricts the maximum deceleration that can be achieved without wheel lock. The tolerance for vehicle parameters is considered as 3%. Speed at the time of braking is taken as

45kmph. Road adhesion is at 0.8. Maximum achievable deceleration without wheel look is the brake performance parameter tabulated as below. Refer to Table 5.

Table 5. Influence of vehicle parameters

Sl No	Vehicle Parameter	Brake Performance Variation in %
1	Front Axle Weight (N)	-1.2 +1.2
2	Rear Axle Weight (N)	-1.9 +2.0
3	Height of CG (mm)	-1.1 +1.0
4	Wheelbase (mm)	-1.1 +1.2

For the vehicle under study, front wheel doesn't lock till the maximum road adhesion availability of 0.9. Since the locking tendency of rear wheels is higher and there are two wheels at rear unlike single front wheel, the sensitivity of rear axle load tolerances is little high. For all the other vehicle parameters the variations are found to be 1.2 percent maximum.

Beyond the above-mentioned brake system and vehicle level parameters, other factors contribute very minimal to the brake performance and can be comfortably neglected.

1.8 Test Results:

Physical tested results at vehicle level yielded stopping distance of 18.78m, 19.12m & 20.23m at 20 Kgs pedal effort in 3 different vehicles, which are within the proposed tolerance limits.

Brake test results on a same vehicle with two different drivers yielded stopping distance variation of 1.3 % & 2.4 % with respect to predicted values. Reaction times for two drivers are observed to be 0.2 seconds & 0.3 seconds respectively.

2 Summary

This paper considers various forces acting on a moving vehicle and provides a methodology to design brake system for a vehicle. It establishes relationship between various brake performance parameters and studies the influence of tolerance variations of these parameters on performance, which in turn gives direction to control sensitive parameters for achieving better brake performance.

3 Conclusions

- Maximum brake performance variation is found to be -16% / +14% in terms of stopping distance & -14% / +16% in terms of deceleration.
- Major brake parameters whose tolerance influences the brake performance are Brake Factor (44%), Pedal Ratio (32%) and tyre rolling radius (14%).
- The influence of vehicle dimensional tolerance on brake performance is maximum 1.2 %. For the rear axle the variation is 2% max.
- To realize designed brake performance during homologation trails, Brake factor, Pedal ratio & Tyre rolling radius are to be ensured at the mean values.
- To yield better decelerations and stopping distance, reaction times should be maintained < 0.3 seconds.

Disclaimer: Exact vehicle details are not disclosed for calculations. To evaluate brake performance at vehicle level, modifications are done on the vehicle to match considered parameters as per calculations.

4 Abbreviations

ABS – Antilock Brake System
TMC – Tandem Master Cylinder
RR – Rolling Radius
USL – Upper Specification Limit
LSL – Lower Specification Limit
RRC – Rolling Resistance coefficient
CG – Center of Gravity
WC – Wheel Cylinder
PE – Pedal Effort
SD – Stopping Distance
BF – Brake Factor
PR – Pedal Ratio

LRR – Low Rolling Resistance
FAW – Front Axle Weight
RAW – Rear Axle Weight
GVW – Gross Vehicle Weight

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