



Research on a Mathematics Thinking Visualization Teaching Model Supported by Generative AI

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Abstract. Traditional teacher-led math teaching struggles with personalized learning and abstract concepts. Generative AI offers strong potential for personalization, real-time feedback, and problem generation. This study deeply integrates generative AI with mathematics thinking visualization to dynamically present abstract thought processes, while using AI to analyze student data and generate personalized learning paths. This model supports teaching according to aptitude and provides a replicable approach for integrating generative AI with math thinking visualization, advancing math education reform in the intelligent era.

Keywords: Generative AI; Mathematics Thinking Visualization; Personalized Learning; Teaching Model; Intelligent Education Reform

1 Introduction

The integration of generative artificial intelligence (AI) and digital resources is transforming mathematics education by overcoming the abstract nature of traditional instruction. AI effectively addresses resource and assessment limitations across all educational stages, as demonstrated by Kimi-based middle school support [1], cross-modal elementary frameworks [2], and GPT-assisted high school instructional design [3][4].

Concurrently, visualization tools concretize abstract concepts. Applications utilizing GeoGebra [5][6][7][8] and virtual manipulatives [7] successfully reduce cognitive load but often lack dynamic adaptability. While recent studies integrating AI and visualization, such as GPT-GeoGebra assistants [9], knowledge-augmented ChatGPT [10], AI-diagnostic feedback [11], and the AlphaGeometry system [12] show significant promise, they primarily present knowledge outcomes rather than the dynamic evolution of mathematical reasoning.

To address this gap, this study proposes a generative AI-supported mathematical thinking visualization model using a dual-engine inquiry approach. By combining GeoGebra for the real-time dynamic visualization of mathematical objects with AI-driven Socratic scaffolding for immediate cognitive feedback, this model achieves the dual visualization of both mathematical evolution and reasoning pathways. Ulti-

mately, it drives a pedagogical shift from outcome reception to process-oriented construction.

2 Research Methods

This experiment aims to verify the effectiveness of a generative AI-supported mathematics thinking visualization teaching model in teaching "Congruent Triangles" in middle school. Using AI tools to dynamically present the reasoning process of judging theorems, it attempts to solve the problems of abstract geometric concepts and invisible thinking processes, providing practical evidence for the deep integration of this model.

2.1 Research Subjects and Experimental Design

This quasi-experimental study involved 86 eighth-grade students from two parallel classes. The experimental group (N=43) received generative AI-supported thinking visualization instruction, while the control group (N=43) underwent traditional lecture-based teaching. Both groups were taught by the same instructor and showed no significant baseline differences in geometry pre-test scores ($p > 0.05$).

2.2 Experimental Intervention Procedure

The teaching intervention lasted for 2 class periods, generally following three cognitive stages: Situation Initiation, Dual-Engine Inquiry, Transfer & Evaluation (see Figure 1). This flowchart briefly illustrates the core differences between the experimental and control groups across the three stages: the experimental group achieved thinking visualization through AI-created situations, GeoGebra trial-and-error with AI questioning scaffolds, and AI-generated variations with process-based feedback; the control group relied on traditional lectures, blackboard demonstrations, and written homework.. There are significant differences in the specific operations of the experimental and control groups in each stage, as compared in Table 1.

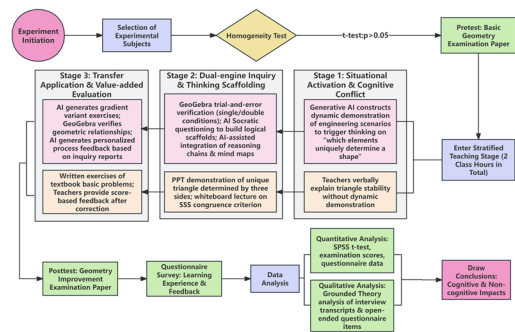


Fig. 1. Comparison of Teaching Processes between Experimental and Control Groups.

Table 1. Comparison of Teaching Processes in Each Stage between Experimental and Control Groups

Core Stage	Teaching	Control Class (Traditional Lecture)	Experimental Class (AI + GeoGebra Visualization)
Situation Initiation & Cognitive Conflict		Verbal explanation of triangle stability; no visual context.	AI creates dynamic real-world scenarios and uses core questions to trigger cognitive conflict, visualizing the starting point of thinking.
Dual-Engine Inquiry & Thinking Scaffold		Teacher presents the SSS theorem via PPT/blackboard; students passively receive knowledge.	Students use GeoGebra for trial-and-error. AI provides Socratic scaffolding to integrate findings into structured reasoning chains, externalizing thought.
Transfer Application & Value-Added Evaluation		Basic textbook exercises; strictly result-oriented, score-based evaluation.	AI generates tiered problems for students to verify via GeoGebra. AI delivers process-oriented, personalized feedback based on exploration data.

3 Results and Discussion

3.1 Analysis of Academic Performance

To evaluate the model's impact on academic achievement, a "Geometry Basics Pre-test" and a "Geometry Proofs Post-test" were administered before and after the experiment. An independent samples t-test was used to compare the scores between the two groups (see Table 2).

Table 2. Comparison of Academic Performance between Experimental and Control Groups (N = 86)

Group	N	Pre-test (SD)	t	Post-test (SD)	t	Effect Size
Experimental Group	43	72.50 (10.42)	0.29 (p = 0.77)	86.22 (8.11)	4.15**	0.88
Control Group	43	71.83 (11.17)		78.44 (9.53)	(p < 0.01)	-

Pre-test results showed comparable levels between the two groups ($p > 0.05$). Post-test results showed that the experimental group significantly outperformed the control group ($p < 0.01$), with a large effect size (Cohen's $d = 0.88$). This indicates that the teaching model can effectively break through the cognitive bottlenecks of traditional instruction and significantly enhance students' geometric proof abilities.

3.2 Analysis of Learning Attitude and Thinking Ability

To comprehensively evaluate the impact of the generative AI-supported mathematics thinking visualization teaching model on student learning outcomes, a questionnaire survey was administered to both the experimental (n=43) and control (n=43) classes after the experiment. The questionnaire was designed based on the three dependent variable dimensions (Learning Attitude, Thinking Ability, Knowledge Mastery), containing 3-4 measurement items per dimension, using a 5-point Likert scale (1 = Strongly Disagree, 5 = Strongly Agree). The specific dimension structure and detailed items are displayed in Figure 2, and the item classification is summarized in Table 1. The questionnaire was revised after pilot testing to form the final version, with an internal consistency Cronbach's α coefficient of 0.87, indicating good reliability.

A total of 86 questionnaires were distributed, and 86 valid questionnaires were returned, for a valid response rate of 100%. SPSS 26.0 was used for independent samples t-tests to compare the mean differences between the experimental and control classes on each dimension. Results are shown in Table 3. The experimental class scored significantly higher than the control class on all dimensions ($p < 0.001$). Notably, the experimental class scored highest on the Knowledge Understanding & Application dimension (4.36), indicating that the visualization scaffold effectively supported the concretization of abstract concepts.

Dimension	Item No.	Item Content
Learning Interest & Classroom Engagement	Q1	I am interested in the content of congruent triangles.
	Q2	I can actively participate in classroom discussions during lessons.
	Q3	I prefer the current classroom teaching method.
	Q4	I feel that class time passes quickly and the lessons are engaging.
Logical Reasoning & Problem-solving Ability	Q5	I can clearly comprehend the derivation process of congruent triangle criteria.
	Q6	When solving geometric problems, I can systematically analyze known conditions and conclusions.
	Q7	I can independently complete congruent triangle proof problems with clear logical thinking.
	Q8	I believe my geometric reasoning ability has been improved.
Knowledge Comprehension & Application	Q9	I can accurately memorize the congruence criteria for triangles.
	Q10	I can apply congruence criteria to solve practical problems.
	Q11	I can distinguish the applicable conditions of different congruence criteria.
	Q12	I have a good overall mastery of knowledge about congruent triangles.

Fig. 2. Questionnaire Dimensions and Items

Table 3. Comparison of Dimension Scores between Experimental and Control Classes

Dimension	Experimental Class (n=43)	Control Class (n=43)	t	p
Learning Interest & Classroom Participation	4.28 ± 0.51	3.21 ± 0.63	8.72	< 0.001
Logical Reasoning & Problem-Solving Ability	4.19 ± 0.55	3.05 ± 0.67	8.95	< 0.001
Knowledge Understanding & Application	4.36 ± 0.48	3.29 ± 0.59	9.31	< 0.001

Figure 3 visualizes the group differences using bar and radar charts. The bar chart (Fig. 3a) illustrates that the experimental class achieved significantly higher means across all three dimensions. The radar chart (Fig. 3b) further highlights this disparity: the experimental group displays a balanced, expansive profile with all scores exceeding 4.1, whereas the control group exhibits a significantly smaller area with scores ranging between 3.0 and 3.5, confirming the experimental group's overall superior performance.

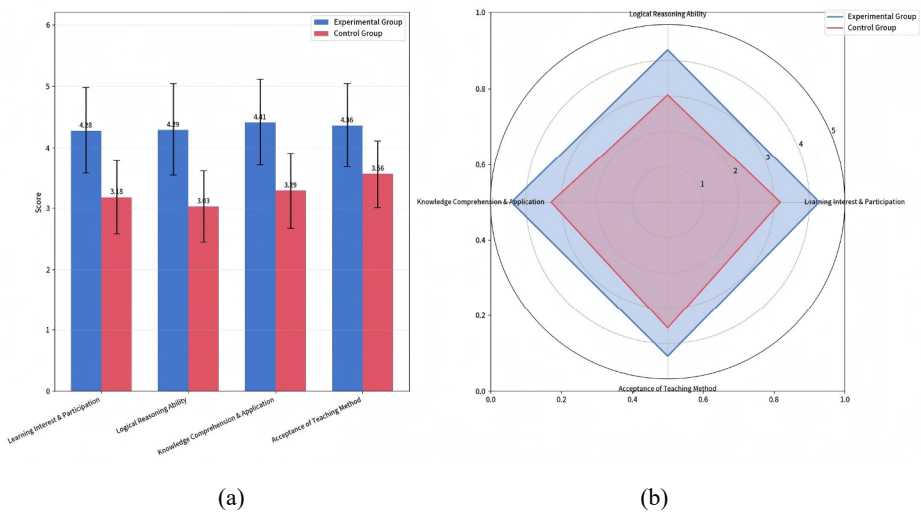


Fig. 3. Comparison of Learning Outcome Dimensions between Experimental and Control Groups, (a) Bar chart comparing means, (b) Radar chart comparing scores.

Corroborating the test scores, the questionnaire results attribute the experimental group's significant multi-dimensional improvement to three factors: AI-simulated contexts reducing cognitive load, dual-engine scaffolding (GeoGebra and AI) driving thought externalization, and personalized feedback enhancing knowledge transfer. Lacking these supports, the control group remained passive learners.

3.3 Qualitative Analysis of Cognitive Mechanisms

To further reveal the specific mechanisms of action of the generative AI-supported thinking visualization teaching model, this study employed grounded theory to analyze the experimental group students' exploration reports, human-AI dialogue records, and interview data, selecting typical cases for in-depth analysis of students' thinking breakthrough processes.

Grounded Theory Analysis.

Qualitative data from 43 experimental students (inquiry reports and AI logs) and semi-structured interviews with 6 students (about 23000 characters) were analyzed using NVivo 12. Through grounded theory coding, 75 initial open codes were clustered into four core categories: Scaffolding Effect, Visualization Facilitation, Logical Correction, and Independent Construction. These categories form the "AI-Supported Thinking Breakthrough Model" (Figure 4). The model reveals that the core mechanism of this technological intervention is the construction of a "trial-error-feedback-correction" closed loop, effectively transforming implicit mathematical thinking into explicit understanding.

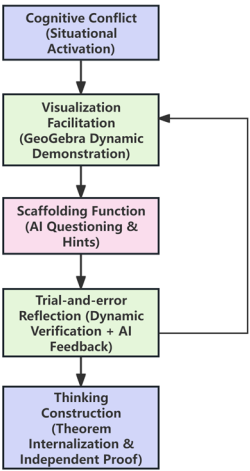


Fig. 4. Model of Thinking Breakthrough Mechanism Based on Grounded Theory

Typical Case Analysis.

Figures 5 and 6 illustrate this via a struggling student, Wang Qiang. Misbelieving that "two equal sides imply congruence," Wang used GeoGebra's dynamic dragging to visualize the shape's non-uniqueness, triggering cognitive conflict. Rather than providing direct answers, DeepSeek's Socratic scaffolding ("Does the figure still have degrees of freedom...?") guided him to translate this intuition into logical reasoning. Wang's independent derivation of the SSS theorem confirms that integrating AI with

dynamic geometry effectively bridges intuitive experience and rigorous argumentation.

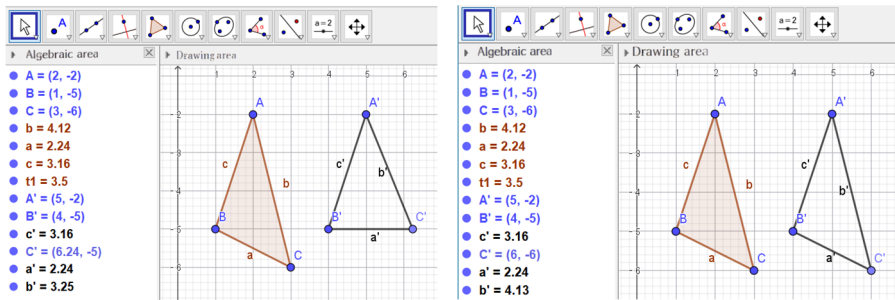


Fig. 5. Student using GeoGebra for dynamic verification of triangle conditions (trial-and-error process).

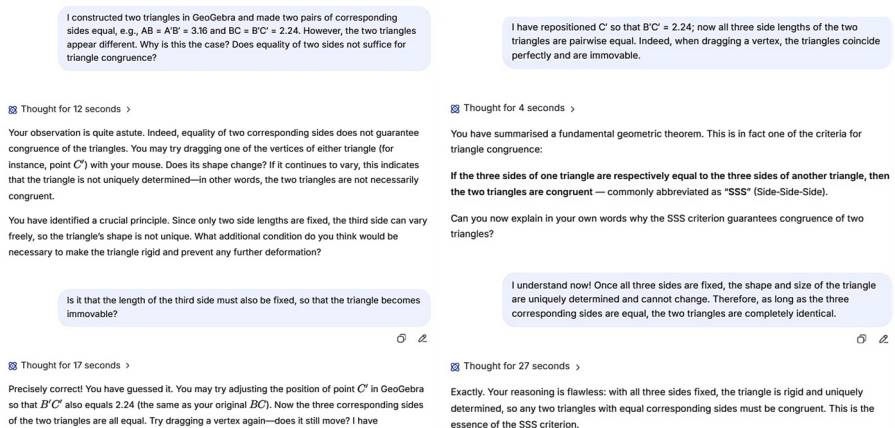


Fig. 6. Example of a dialogue interface showing AI providing Socratic questioning

4 Conclusion

To address the abstract nature of traditional mathematics instruction and the lack of personalized support, this study empirically validated a "dual-engine" teaching model integrating generative AI and GeoGebra in middle school geometry. Mixed-methods results demonstrate that this model significantly improved academic performance and logical reasoning. AI-driven Socratic scaffolding successfully externalized implicit mathematical thinking, bridging intuitive perception and rigorous argumentation. Furthermore, process-oriented evaluation reduced cognitive load and boosted classroom engagement. This study offers a practical, innovative pathway for mathematics education reform in the intelligent era. Future research should utilize larger samples and longitudinal designs across diverse educational stages to confirm the model's long-term generalizability and value-added effects.

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