Research on the Time-Frequency Correlation between Green Bonds, Energy Markets and Metal Futures Markets

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Abstract. This paper employs continuous wavelet transform and wavelet coherence to investigate the time-frequency dynamic correlation between green bonds and crude oil, coal, natural gas, copper, nickel, zinc futures during 2017-2024. Results show energy futures are dominated by high-frequency volatility, while metal futures present medium-frequency periodic volatility. The correlation between green bonds and energy futures is policy-driven, and that with non-ferrous metals is featured by industrial linkage. These findings reveal multi-scale cross-market linkages and provide references for risk hedging and green finance supervision.

Keywords: Green Bonds; Energy Market; Non-ferrous Metal Futures; Wavelet Analysis; Wavelet Coherence

1 Introduction

Under global low-carbon transition and China's dual carbon goals, green bonds have developed rapidly. Traditional energy and non-ferrous metals are closely tied to energy transition, so research on their correlation and lead-lag relationships with green bonds is of great significance. Existing studies mostly adopt linear methods, lacking time-frequency analysis and simultaneous comparison between energy and non-ferrous metal markets. Using wavelet analysis and 2017–2024 daily data, this paper fills the research gap, provides multi-scale correlation evidence, and expands the research scope of green finance.

2 Literature Review

Existing literature confirms time-varying risk spillovers between green bonds and energy or commodity markets, with wavelet tools supporting time-frequency analysis. However, most studies focus on green bond-energy spillovers, ignore non-ferrous metals, and rarely compare the two markets simultaneously. This paper addresses these deficiencies.

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2.1 Green Bonds and the Energy Market

Most studies focus on risk spillover. Liu[1] found green bonds as net risk receivers with positive time-varying dependence on energy markets via wavelet analysis and DY/BK models. Wan[2] confirmed two-way volatility spillover from crude oil via TVP-VAR-DY model. Zhang et al.[3] noted short-term-dominated spillover intensified by major events via TVP-VAR frequency domain model. Wang[4] showed enhanced spillover in extreme conditions with energy as net transmitter via quantile spillover model. Lei[5] revealed time-frequency heterogeneous spillover affected by geopolitics via QVAR-BK model. Wu[6] found strong medium-long-term correlation and time-frequency heterogeneous lead-lag relationships via wavelet coherence analysis.

2.2 Green Bonds and the Metal Market

Relevant studies focus on the linkage between green bonds and commodity markets. Lei[5] and Wu[6] identify significant time-varying spillovers and strengthened cross-market linkages under extreme market conditions. However, few studies examine the time-frequency correlation between green bonds and non-ferrous metal futures, and none compare the differential linkages across energy and metal markets simultaneously.

This paper uses continuous wavelet transform and wavelet coherence to explore green bonds' time-frequency correlation with traditional energy and non-ferrous metal futures. Despite existing wavelet analysis applications, studies fail to compare green bonds with both asset types to fill non-ferrous metal research gaps and use multi-scale analysis to reveal dynamic correlation and lead-lag relationships.

3 Research Methodology

3.1 Wavelet Analysis Framework

A wavelet is generated from a mother wavelet and satisfies the admissibility condition for good time–frequency localization. This study uses the Morlet wavelet, which is widely adopted in financial time–frequency analysis:

$$\psi(t)=\pi^{-\frac{1}{4}}e^{iw_0t}e^{\frac{-t^2}{2}} \tag{1}$$

We set w_0=6 to balance time and frequency localization.

The continuous wavelet transform of series $W_x(s)$ is defined as:

$$W_x(s)=\int_{-\infty}^{\infty}x(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t}{s}\right), \tag{2}$$

where * denotes the complex conjugate, s is the scale parameter, and t is the time position.

The wavelet coherence is used to quantify the time–frequency correlation between two series :

$$R_n^2(s)=\frac{|(s^{-1}W_n^{XY}(s))|^2}{(s^{-1}|W_n^X(s)|^2)(s^{-1}|W_n^Y(s)|^2)} \quad (3)$$

In the formula, s is a smoothing operator for time and scale. $R^2(s)$ has similar characteristics to the correlation coefficient: the closer the value is to 0, the weaker the dependence between the two sequences; the closer the value is to 1, the stronger the dependence. Since the theoretical distribution of the wavelet coherence coefficient is not clear, its statistical significance needs to be determined by Monte Carlo simulation .

3.2 Phase

The main function of the wavelet coherence phase difference is to capture the positive and negative correlation and leading-lagging relationship between two sequences in the time-frequency space.

$$\varphi_{XY}(s)=\tan^{-1}\left(\frac{I(s^{-1}W_n^{XY}(s))}{R(s^{-1}W_n^{XY}(s))}\right) \quad (4)$$

In the formula, $\Im$ and $\Re$ correspond to the imaginary and real parts of the smoothed power spectrum, respectively. The phase relationship can be intuitively represented by arrows: an arrow pointing to the right (left) means the sequences are in-phase (anti-phase), i.e., positively (negatively) correlated; an arrow pointing down (up) indicates that the second (first) sequence leads the first (second) sequence by 90°.

3.3 Data

This paper selects daily data of crude oil, coal, natural gas, nickel, copper, zinc and green bond indices from 2017 to 2024 to analyze their cross-time-scale dependence and causality. All data are from Wind Database, and price index yields are calculated by first difference of natural logarithm of index prices.

3.4 Descriptive Statistics

Descriptive statistics of core variables are shown in Table 1. Metal and energy futures have high price dispersion and volatility risk, while green bonds are less volatile. Green bonds have skewness close to 0 with flat distribution; most metal and energy futures have high skewness and are prone to extreme high prices. Jarque-Bera test shows GB, GAS, COAL, CU, NI and ZN reject the normal distribution hypothesis at $\alpha=0.1$ (P=0.0000), while OIL accepts it (P=0.118833>0.1), consistent with financial data's non-normality and leptokurtosis.

Table 1. Descriptive Statistics of Core Variables

	Green Bond	Natural Gas	Coal	Crude Oil	Copper	Nickel	Zinc
Mean	118.5013	4330.703	887.4704	59436.09	130685.8	70.77401	22043.41
Median	118.3900	4100.000	801.0000	57550.00	125210.0	72.40000	22050.00

Maximum	137.6000	9600.000	1991.000	87670.00	267700.0	129.4700	28650.00
Minimum	98.94000	2350.000	661.0000	37200.00	72080.00	23.00000	14420.00
Std.Dev	11.42697	1362.226	198.0711	11302.29	37076.90	17.76372	2592.824
Skewness	-0.149745	0.794437	1.594416	0.154068	0.836516	0.065940	-0.325811
Kurtosis	1.844726	2.999759	7.215823	1.584710	3.170075	3.197961	2.844128
Jarque-Bera	107.2419	190.0755	2103.786	157.9613	212.9218	4.260064	33.79907
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.118833	0.000000
Sum	214131.9	7825580	1603659	1.07E+08	2.36E+08	127888.6	39832445
Sum Sq.Dev.	235819.5	3.35E+09	70853302	2.31E+11	2.48E+12	569882.9	1.21E+10

4 Analysis of Empirical Results

4.1 Continuous Wavelet Analysis

Figure 1 presents the continuous wavelet analysis results: horizontal axis for time, vertical axis for period, solid curve for the cone of influence, and warmer colors for higher volatility. Most sequences respond to shocks at the 512-day 2-year scale.

Figure 1(a) green bonds: Significant short-medium term volatility in 2018, possibly affected by financial deleveraging and Fed rate hikes; strong medium-long term volatility from 2018 to 2022, possibly driven by dual carbon policies, pandemic and energy crisis.

Figure 1(b) crude oil: Extremely strong short-medium term volatility around 2020-2021, possibly caused by pandemic, OPEC+ production cuts and geopolitical risks.

Figure 1(c) natural gas: Discontinuous high volatility in high-frequency intervals, possibly affected by coal-to-gas policy, limited supply strategy and extreme cold wave.

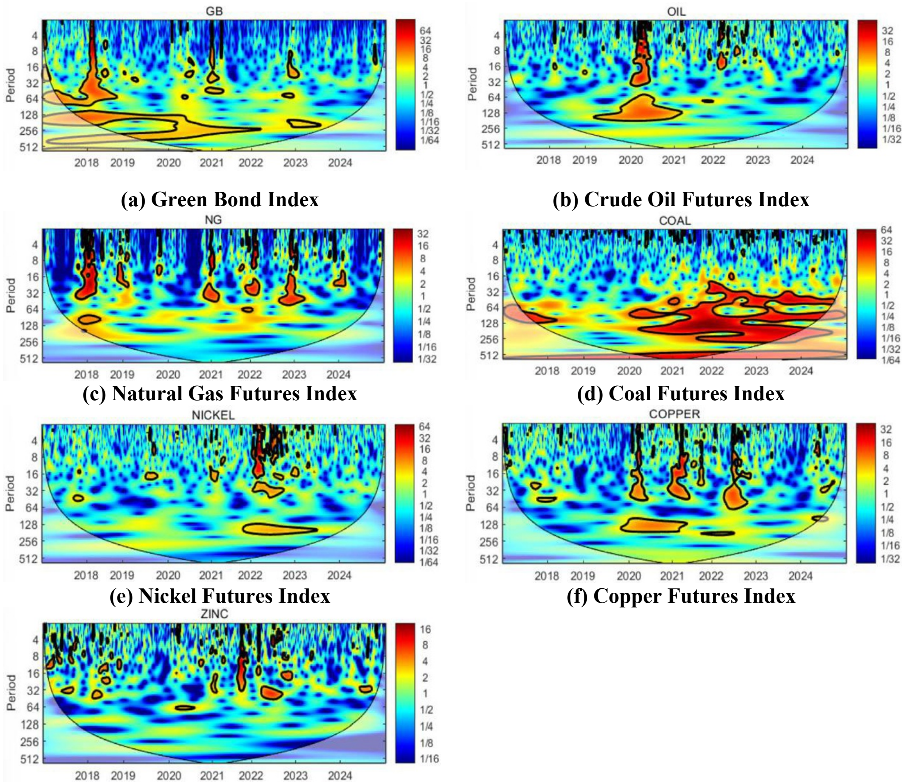
Figure 1(d) coal: Sustained strong medium-long term volatility from 2020 to 2024, possibly driven by dual carbon policy constraints, Russia-Ukraine conflict and import restrictions.

Figure 1(e) nickel: Strong high-frequency volatility from 2022 to 2023, possibly caused by low inventories, export sanctions and LME nickel short squeeze event.

Figure 1(f) copper: Discontinuous strong medium-term volatility from 2020 to 2023, possibly related to Fed policy adjustments including quantitative easing and interest rate hikes.

Figure 1(g) zinc: High high-frequency volatility around 2022, possibly affected by energy crisis and rising smelting costs from Russia-Ukraine conflict.

Overall, energy futures including crude oil and natural gas are dominated by high-frequency volatility; metal futures including copper and nickel show medium-frequency periodic volatility, possibly due to supply-demand differences. Coal has medium-low frequency high volatility, possibly reflecting energy transition supply-demand contradictions. Both energy and metal futures showed strong volatility from 2020 to 2023, possibly affected by macro policies, geopolitics and pandemic.



(a) **Green Bond Index** (b) **Crude Oil Futures Index**

(c) **Natural Gas Futures Index** (d) **Coal Futures Index**

(e) **Nickel Futures Index** (f) **Copper Futures Index**

(g) **Zinc Futures Index**

Fig. 1. Continuous Wavelet Analysis of Green Bond and Futures Indices

4.2 Wavelet Coherence Analysis

Wavelet coherence analysis reveals the time-frequency correlation between green bonds and various futures indices: vertical axis for period 4-512, horizontal axis for time 2017-2024, warm colors for high correlation, cold colors for low correlation, and arrows for phase difference and leading-lag relationship.

Figure 2(a) green bonds and crude oil futures: strong short-medium term correlation 0-128 days, high long-term negative correlation 256-512 days 2019-2022, with green bonds leading, possibly driven by carbon neutrality policies and safe-haven demand.

Figure 2(b) green bonds and coal futures: frequent short-medium term high correlation 4-128 days, significant long-term negative correlation 128-256 days 2022-2024, with coal futures gradually leading, possibly related to China's dual carbon policy stages.

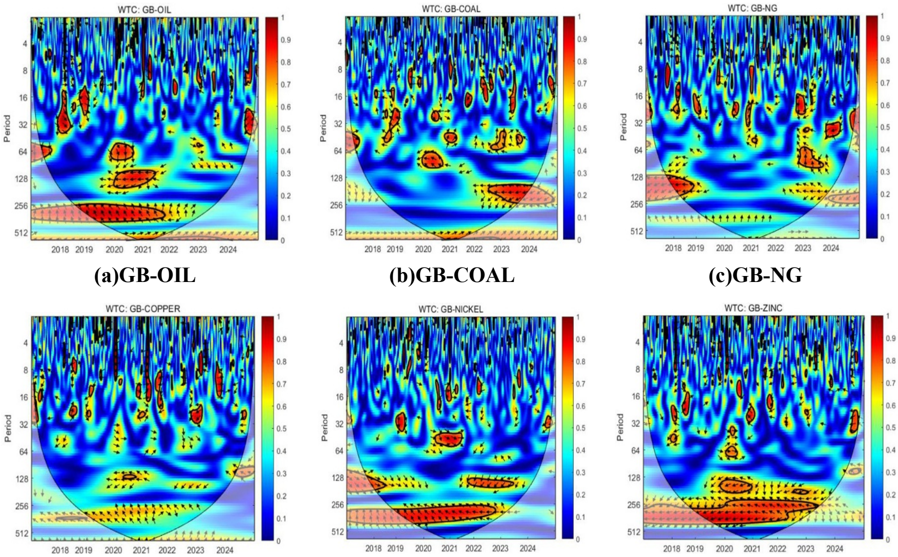
Figure 2(c) green bonds and natural gas futures: no obvious leading-lag relationship, overall negative correlation, possibly due to new energy and natural gas demand substitution.

Figure 2(d) green bonds and copper futures: overall weak correlation, local medium-term high correlation 32-64 days 2020-2021, unclear leading-lag relationship, possibly due to hedging effect of copper's dual demand.

Figure 2(e) green bonds and nickel futures: significant long-term negative correlation 256-512 days 2019-2023, with green bonds leading, possibly because green bonds respond to industry expectations earlier.

Figure 2(f) green bonds and zinc futures: strong long-term negative correlation 256-512 days 2019-2023, leading relationship shifting from green bonds to zinc futures after 2022, possibly driven by new energy project construction.

In summary, green bonds' coherence with energy futures is policy-driven and with metal futures is industry-linked. Coal has a stable leading effect on green bonds; the leading relationship between metal futures and green bonds reverses cyclically.



(a)GB-OIL **(b)GB-COAL** **(c)GB-NG**

(d)GB-COPPER **(e)GB-NICKEL** **(f)GB-ZINC**

Fig. 2. Wavelet Coherence Analysis between Green Bonds and Various Futures Indices

5 Research Conclusions and Policy Recommendations

5.1 Research Conclusions

Continuous wavelet transform shows metal futures have medium-frequency volatility, energy futures high-frequency volatility, and coal medium-low frequency volatility. Wavelet coherence indicates green bonds are long-term negatively correlated with traditional energy with green bonds leading, and show dynamic lead-lag relationships with non-ferrous metals.

5.2 Policy Recommendations

We should build multi-time scale risk early warning systems based on market volatility characteristics, leverage cross-market synergy to formulate policies, establish key non-ferrous metal strategic reserves, use green bond indicators for industry monitoring and develop green financial derivatives, and strengthen green bonds' capital allocation role as well as optimize their issuance and information disclosure mechanisms to promote the interaction between green finance and the real economy.

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