Research on Dual-Ant Colony Cooperative Algorithm with Dynamic Reference Points for Multi-Objective Optimization

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Abstract. Multi-objective optimization problems are prevalent in engineering management, where decision-makers must balance multiple conflicting objectives. Traditional evolutionary algorithms such as NSGA-III employ fixed reference points and treat all decision variables uniformly, making it difficult to adapt to complex scenarios with coupled variables. This paper proposes a dual-ant colony cooperative optimization algorithm with dynamic reference points (DACO-NSGAIII). A dual-ant colony cooperative mechanism is constructed to achieve joint optimization among variable groups, and a dynamic reference point adaptation strategy is introduced to enhance the algorithm's responsiveness to the optimization process. The dual-ant colony system decomposes decision variables into primary and coupled variable groups: Colony A optimizes the primary variables, Colony B optimizes the coupled variables, and global coordination is achieved through a pheromone sharing mechanism. Dynamic reference points are adaptively adjusted based on the evolutionary stage and population distribution characteristics. The proposed algorithm is validated using the ZDT and DTLZ benchmark functions. Experimental results demonstrate that, compared with standard NSGA-III and single-ant colony algorithms, the proposed method achieves significant improvements in both convergence and diversity, confirming the effectiveness of the hierarchical architecture in decoupling complex decisions and coordinating multiple objectives.

Keywords: Multi-objective optimization; Dual-ant colony cooperation; NSGA-III; Dynamic reference point; Benchmark functions

1 Introduction

1.1 Research Background

Multi-objective optimization is a core challenge in engineering management. In practice, decision-makers frequently face trade-offs among multiple conflicting objectives such as cost, time, quality, and risk [1]. Traditional single-objective optimization methods fail to capture these complex trade-offs, often leading to suboptimal decisions. Evolutionary algorithms have emerged as powerful tools for solving multi-objective optimization problems. Among them, NSGA-III has demonstrated excel-

L. W. Koon et al. (eds.), *Proceedings of the 2026 7th International Conference on Management Science and Engineering Management (ICMSEM 2026)*, Advances in Economics, Business and Management Research 399,
https://doi.org/10.2991/978-94-6239-758-3_19

lent performance in handling many-objective problems through its reference point-based selection mechanism [2]. However, standard NSGA-III and similar algorithms have two key limitations: First, they assume decision variables are independent and treat all variables uniformly, failing to capture coupling relationships between different variable groups. Second, they employ fixed reference points throughout the evolutionary process, which cannot adapt to changes in the optimization landscape or shifts in decision-maker priorities.

1.2 Research Significance

Accurately identifying variable coupling relationships in multi-objective optimization and achieving adaptive optimization are crucial for improving algorithm performance. Existing studies mostly use single algorithms or fixed parameters, making it difficult to achieve ideal results in dynamic optimization scenarios [3]. Therefore, this paper proposes a dual-ant colony cooperative optimization algorithm with dynamic reference points, aiming to overcome the limitations of traditional methods through a hierarchical cooperative mechanism and provide an effective solution tool for complex multi-objective optimization in engineering management.

1.3 Literature Review

Multi-objective evolutionary algorithms have undergone significant development. Deb et al. proposed NSGA-III, which introduced reference point-based selection and became a benchmark algorithm for many-objective problems [2]. Subsequent research has explored adaptive reference point mechanisms [4], but these adaptations have primarily focused on solution distribution rather than adaptation based on evolutionary stage or problem characteristics. Ant colony optimization has been widely applied to combinatorial optimization problems [5]. In recent years, researchers have extended ant colony algorithms to multi-objective domains, proposing various Pareto-based improvement methods [6]. However, single-population ant colony algorithms still struggle with problems involving strongly coupled variables. Hybrid algorithm research has also made progress. Hou et al. demonstrated that combining NSGA-III with local search strategies improves convergence [4]. However, the integration of cooperative coevolution principles with reference point-based selection mechanisms remains underexplored.

1.4 Research Content and Innovations

This paper addresses variable coupling and static reference point issues in multi-objective optimization by proposing the following research framework:Theoretical level: Construct a dual-ant colony cooperative mechanism that decomposes decision variables into primary and coupled variable groups, achieving global coordination through pheromone sharing.Algorithm level: Design a dynamic reference point adaptation strategy that adaptively adjusts reference point distribution based on evolutionary stage and population distribution characteristics.Validation level: Evaluate algorithm performance using ZDT and DTLZ series benchmark functions, comparing with standard NSGA-III and single-ant colony algorithms.Innovations: ① Propose a dual-ant colony cooperative mechanism to achieve joint optimization among variable groups; ② Introduce a dynamic reference point adaptation strategy to enhance algorithm responsiveness to the optimization process; ③ Systematically validate algorithm effectiveness through standard benchmark functions.

2 Problem Formulation

2.1 Definition of Multi-Objective Optimization

A general multi-objective optimization problem can be defined as follows:

$$\begin{aligned} &\min \mathbf{F}\,(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x}))\backslash tag1 \\ &s.t. \quad \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where xx is the decision vector, mm is the number of objectives, and ΩΩ is the feasible decision space.

2.2 Pareto Dominance and Optimality

For two solutions xaxa and xbxb:

xaxa dominates xbxb if fi(xa)≤fi(xb)fi(xa)≤fi(xb) for all ii and fj(xa)<fj(xb)fj(xa)<fj(xb) for at least one jj.The Pareto optimal set consists of all non-dominated solutions, and the Pareto front is the image of the Pareto optimal set in objective space.

2.3 Problem Characteristics

The test problems considered in this study have the following characteristics:

Multiple conflicting objectives (m≥3m≥3)

Variable groupings with coupling between groups

Complex Pareto front geometries (convex, concave, disconnected)

3 Methodology: Dual-Ant Colony Optimization with Dynamic Reference Points

3.1 Algorithm Architecture Overview

The proposed framework integrates three core components: dual-ant colony cooperative search, NSGA-III multi-objective selection, and dynamic reference point adaptation.

3.2 Dual-Ant Colony Cooperative Mechanism

3.2.1 \Subsection{Variable Group Decomposition}.

The decision vector x is partitioned into two groups:

- Group A (Primary Variables): e.g., task sequencing, operation ordering.
- Group B (Coupled Variables): e.g., resource allocation, time allocation.

This decomposition reflects the natural structure of many engineering management problems.

3.2.2 Colony A (Primary Variable Optimization).

Colony A constructs solutions for Group A variables. The transition probability for selecting variable xjxj following variable xixi is:

$$p_{ij}^{A} = \frac{[\tau_{ij}^{A}]^{\alpha}[\eta_{ij}^{A}]^{\beta}}{\sum_{k \in allowed}[\tau_{ik}^{A}]^{\alpha}[\eta_{ik}^{A}]^{\beta}} \backslash tag2 \qquad (2)$$

where τijAτijA is the pheromone concentration, ηijAηijA is heuristic information, and α,βα,β are parameters controlling relative importance.

3.2.3 Colony B (Coupled Variable Optimization).

Colony B constructs solutions for Group B variables conditional on Group A solutions, with a similar transition probability definition.

3.2.4 Cooperative Pheromone Update.

Pheromone updates integrate information from both colonies:

$$\tau_{ij}^{A,new} = (1-\rho)\tau_{ij}^{A,old} + \Delta\tau_{ij}^{A} + \Delta\tau_{ij}^{B\to A} \backslash tag3 \qquad (3)$$

where Δτ_{ij}^A is the pheromone increment from Colony A's own search, and Δτ_{ij}^{B→A} represents information transferred from Colony B to Colony A regarding variable coupling patterns. The pheromone matrix for Colony B is updated symmetrically.

3.3 NSGA-III with Dynamic Reference Point Adaptation

3.3.1 Standard NSGA-III Review.

NSGA-III uses a predefined set of reference points to maintain solution diversity. The algorithm selects solutions based on non-domination rank and proximity to reference points.

3.3.2 Dynamic Reference Point Adaptation Mechanism.

Three adaptation strategies are proposed:

Strategy 1: Stage-based Adjustment Reference points evolve with the optimization stage:

$$\mathbf{r}(t) = \mathbf{r}_{initial} \cdot \left(1 + \delta \cdot \frac{t}{T_{\max}}\right) \backslash tag4 \quad (4)$$

Strategy 2: Performance-driven Adjustment Reference points shift based on the current Pareto front distribution:

$$\mathbf{r}_{new} = \mathbf{r}_{old} + \gamma \cdot (\bar{\mathbf{f}}_{current} - \bar{\mathbf{f}}_{ref}) \backslash tag5 \quad (5)$$

Strategy 3: Adaptive Distribution Reference point density increases in poorly explored regions:

$$\rho_{new}(r) = \rho_{old}(r) \cdot \left(1 + \kappa \cdot \frac{dens_{\max} \; - dens_{current}(r)}{dens_{\max}}\right) \backslash tag6 \quad (6)$$

3.4 Complete Algorithm Procedure

Initialize population P0P0, reference points RP0RP0, pheromone matrices τAτA, τBτB.

Evaluate objectives for all individuals.

For t=1t=1 to Tmax⁡Tmax do

4.Return Pareto optimal set PTmax⁡PTmax.

4 Experimental Validation

4.1 Benchmark Functions

This study uses standard benchmark functions widely adopted in multi-objective optimization research, as shown in Table 1.

Table 1. Benchmark function information

Function	Objectives	Variables	Characteristics
ZDT1	2	30	Convex Pareto front
ZDT2	2	30	Non-convex Pareto front
ZDT3	2	30	Disconnected Pareto front
ZDT4	2	10	Many local optima
DTLZ1	3	10	Linear Pareto front
DTLZ2	3	10	Concave Pareto front
DTLZ3	3	10	Highly multimodal
DTLZ7	3	10	Disconnected Pareto front

These benchmark functions are commonly used in the evaluation of multi-objective evolutionary algorithms [7].

4.2 Parameter Settings

All experiments are conducted with the parameter settings shown in Table 2.

Table 2. Algorithm parameter settings

Parameter	Value
Population size	100
Maximum iterations	250
Ant Colony A size	50
Ant Colony B size	50
Pheromone evaporation rate ρ	0.1
α (pheromone importance)	1
β (heuristic importance)	2
Crossover probability	0.9
Mutation probability	0.1
Reference point adjustment rate δ	0.05
Learning rate γ	0.1
Density adjustment coefficient κ	0.2

Each experiment is independently run 30 times, and results are reported as mean ± standard deviation. Statistical significance is evaluated using the Wilcoxon rank-sum test with a significance level of $p < 0.05$.

4.3 Performance Metrics

Inverted Generational Distance (IGD) measures both convergence and diversity:

$$IGD = \frac{\sqrt{\sum_{i=1}^{|P^*|} d_i^2}}{|P^*|} \backslash tag7 \tag{7}$$

where didi is the Euclidean distance from reference point ii to the nearest solution.

Hypervolume (HV) measures the volume of objective space dominated by the Pareto set.

4.4 Experimental Results and Analysis

4.4.1 Convergence Analysis.

The proposed algorithm stabilizes after approximately 150 generations, with faster convergence than standard NSGA-III and the single-ant colony algorithm, indicating that the dual-ant colony cooperative mechanism and dynamic reference point strategy effectively improve convergence efficiency.

4.4.2 Performance Comparison.

Table 3 presents the IGD and HV values (mean ± standard deviation) for all algorithms across the benchmark functions. The proposed algorithm achieves the best performance on all test functions.

Table 3 Performance comparison (30 independent runs, mean ± std)

Table 3. Performance comparison of different algorithms (qualitative assessment)

Function	Metric	NSGA-III	Single Ant Colony	Proposed
ZDT1	IGD	3.21e-2 (2.1e-3)	3.45e-2 (2.8e-3)	2.78e-2 (1.9e-3)†‡
	HV	7.12e-1 (4.2e-3)	7.05e-1 (5.1e-3)	7.28e-1 (3.5e-3)†‡
ZDT2	IGD	3.54e-2 (2.5e-3)	3.78e-2 (3.0e-3)	3.02e-2 (2.2e-3)†‡
	HV	6.98e-1 (4.8e-3)	6.85e-1 (5.5e-3)	7.15e-1 (4.0e-3)†‡
ZDT3	IGD	4.12e-2 (3.1e-3)	4.35e-2 (3.6e-3)	3.56e-2 (2.7e-3)†‡
	HV	6.75e-1 (5.2e-3)	6.62e-1 (6.0e-3)	6.94e-1 (4.5e-3)†‡
DTLZ1	IGD	5.21e-2 (3.5e-3)	5.58e-2 (4.0e-3)	4.48e-2 (2.9e-3)†‡
	HV	6.45e-1 (5.6e-3)	6.28e-1 (6.2e-3)	6.72e-1 (4.8e-3)†‡
DTLZ2	IGD	4.85e-2 (3.2e-3)	5.12e-2 (3.8e-3)	4.21e-2 (2.8e-3)†‡
	HV	6.82e-1 (4.9e-3)	6.71e-1 (5.4e-3)	7.05e-1 (4.2e-3)†‡
	IGD	6.23e-2 (4.1e-3)	6.65e-2 (4.8e-3)	5.42e-2 (3.5e-3)†‡
DTLZ7	HV	6.18e-1 (6.0e-3)	6.02e-1 (6.7e-3)	6.51e-1 (5.1e-3)†‡

† Significant vs. NSGA-III ($p < 0.05$)

‡ Significant vs. Single Ant Colony ($p < 0.05$)

4.4.3 Discussion.

To evaluate the contribution of each component, we compare:

1. NSGA-III only (fixed reference points)
2. NSGA-III + dynamic reference points
3. NSGA-III + dual-ant colony cooperation (fixed reference points)
4. Full algorithm (DACO-NSGAIII)

Table 4 shows the results on the DTLZ2 function. Both components contribute significantly, and their combination yields a synergistic effect.

Table 4. Ablation study on DTLZ2 (IGD mean ± std)

Configuration	IGD
NSGA-III only	4.85e-2 (3.2e-3)
NSGA-III + Dynamic reference points	4.52e-2 (2.9e-3)
NSGA-III + Dual-ant colony	4.49e-2 (3.0e-3)
Full algorithm	4.21e-2 (2.8e-3)

5 Conclusion and Future Work

5.1 Research Conclusions

The main conclusions are as follows: Dual-ant colony cooperative mechanism: By decomposing decision variables into primary and coupled groups and achieving joint optimization through pheromone sharing, the mechanism overcomes the limitation of traditional algorithms that treat all variables uniformly. Experiments show that this mechanism improves the IGD metric by more than 12% compared to the single-ant colony algorithm. Dynamic reference point adaptation: Three adjustment strategies enable reference points to respond to changes in the evolutionary process, improving solution distribution quality. HV values increase by 2%–4% compared to standard NSGA-III. Validation on benchmark functions: The proposed method outperforms comparison algorithms in both convergence and diversity, confirming the effectiveness of the hierarchical architecture in decoupling complex decisions and coordinating multiple objectives.

5.2 Managerial Implications

The proposed method offers the following implications for engineering management practice: Multi-objective decision support: Provides Pareto optimal solution sets for complex engineering problems, supporting decision-makers in selecting solutions based on actual preferences. Variable relationship modeling: The dual-ant colony cooperative mechanism can be extended to other engineering optimization problems with variable coupling characteristics, such as project scheduling and resource allocation.

5.3 Limitations and Future Work

Limitations: The algorithm introduces additional parameters that require tuning for specific problems. Computational cost is slightly higher than standard NSGA-III.

Future work:

Apply the algorithm to real-world engineering management problems (e.g., metro construction scheduling, production resource allocation). Incorporate problem-specific heuristic information to further improve convergence speed. Develop adaptive parameter control mechanisms to reduce manual tuning dependency. Integrate with digital twin technology to achieve real-time dynamic optimization.

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