Cross-Market Asset Allocation and Risk Management under Dynamic Network Structures: Evidence from Monte Carlo Simulations

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Abstract. High-dimensional portfolio optimization ($N \approx T$) inherently suffers from ill-conditioned covariance estimates, causing unstable weights and excessive transaction costs. This paper proposes a dynamic risk management framework using the **Rolling Graphical LASSO (Glasso)** to recover the sparse topology of precision matrices. We evaluate this approach against Sample Covariance and Ledoit-Wolf Shrinkage via a rigorous Monte Carlo simulation of 105 assets, explicitly modeling clustered correlation structures and regime-dependent crisis dynamics. Empirical results demonstrate that Glasso acts as a "risk firewall" during contagion, reducing annualized volatility by approximately 11% (24.86% vs. 27.78%) and, crucially, cutting portfolio turnover by over 80% (0.284 vs. 1.478) compared to the Ledoit-Wolf benchmark. Although strict sparsity constraints result in a conservative Sharpe Ratio (0.193 vs. 0.239), Glasso provides superior cost-efficiency and stability by filtering spurious correlations, effectively balancing the bias-variance trade-off.

Keywords: High-Dimensional Covariance; Rolling Graphical LASSO; Portfolio Risk Management; Regime Switching; Sparse Topology.

1 Introduction

Modern portfolio management increasingly relies on allocating capital across high-dimensional asset pools, where the number of assets (N) approximates or exceeds the observation horizon (T). In this context, the classic mean-variance framework faces the "curse of dimensionality," where the Sample Covariance matrix becomes ill-conditioned, yielding unstable portfolio weights and poor out-of-sample performance.[1] While Linear Shrinkage methods stabilize estimation by shrinking towards a structured target, they often fail to exploit the sparsity inherent in financial networks, where most pairwise conditional dependencies are statistically insignificant.

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This paper addresses these limitations by integrating the Graphical LASSO (Glasso) into a rolling-window framework.[5] Unlike global shrinkage, Glasso applies an L_1 penalty to the precision matrix ($\Omega = \Sigma^{-1}$), explicitly recovering the latent sparse topology of asset networks. This topological recovery is critical during market stress, where "contagion" manifests as a breakdown of structural independence.[2]

We rigorously evaluate the Rolling Glasso against Sample Covariance and Ledoit-Wolf benchmarks using a high-dimensional Monte Carlo simulation ($N = 105$). Unlike empirical studies lacking a "ground truth," our simulation incorporates regime-switching and block-diagonal correlation structures, allowing precise measurement of estimation error (Frobenius Norm Loss) against the true covariance Σ_{true}. We contribute by demonstrating that Glasso not only reduces volatility by 11% but also lowers turnover by 81%, offering a decisive advantage in transaction cost-adjusted performance.

2 Methodology

This part builds on the Markowitz framework to minimize risk in high-dimensional settings.[7] This section presents the portfolio objective and compares three covariance estimation methods.

2.1 Global Minimum Variance Portfolio (GMVP)

We consider a universe of N assets with returns $R_t \in \mathbb{R}^N$. The GMVP objective is to determine the weight vector w that minimizes portfolio variance $w^T \sum w$ subject to the budget constraint $w^T 1 = 1$. The analytical solution, $w_{GMVP} = \frac{\Sigma^{-1} 1}{1^T \Sigma^{-1} 1}$, highlights that portfolio stability is fundamentally contingent upon the condition number of the estimated precision matrix $\sum^{-1}$.

2.2 Covariance Estimation Models

We compare three rolling-window estimators using a lookback window T.

Benchmarks: To mitigate estimation error, we employ the Linear Shrinkage estimator proposed.[4] This method acts as a robust benchmark by shrinking the sample covariance S towards a constant-correlation target F: $\widehat{\Sigma_{LW}} = \delta F + (1 - \delta)S$, where the shrinkage intensity $\delta \in [0,1]$ is analytically optimized to minimize the expected quadratic loss.

Core Model: Rolling Graphical LASSO (GlassoCV): Our proposed approach estimates the sparse precision matrix $\Omega = \sum^{-1}$ by assuming a multivariate Gaussian distribution and solving the L_1-penalized log-likelihood maximization problem:

$$\widehat{\Omega} = \arg\min_{\Omega \succ 0} \left(-\log\det\Omega + \mathrm{tr}(S\Omega) + \lambda \textstyle\sum_{i \neq j} |\Omega_{ij}| \right) \tag{1}$$

where S is the sample covariance matrix and $\lambda \geq 0$ is the sparsity penalty parameter. Crucially, we employ a recursive Rolling Cross-Validation procedure.[3] To ensure reproducibility and prevent look-ahead bias, we utilize a fixed lookback window of $L = 500$ days. At each rebalancing time t, the penalty parameter λ is selected via 3-fold cross-validation performed strictly within the historical window $[t - L, t - 1]$.

2.3 Out-of-Sample Performance Evaluation

Performance is evaluated via Annualized Volatility (σ_p), Sharpe Ratio (SR), and Portfolio Turnover (TO). Turnover is explicitly defined to account for asset drift: $TO = \frac{1}{M}\sum_{t=1}^{M} \parallel w_{t+1} - w_{t+} \parallel$, assuming a monthly rebalancing frequency (21 days) and a proportional transaction cost of 10 basis points

3 Simulation Design

A high-dimensional Monte Carlo simulation was constructed and repeated 100 times to ensure statistical robustness.[8] The reported performance metrics are the ensemble average across these independent simulation runs. This allows precise error quantification against a known ground truth—unachievable with empirical data alone.

To rigorously evaluate estimator robustness against structural breaks, we synthesize a high-dimensional universe of $N = 105$ assets distributed across three market segments ($N_A = 75, N_B = 15, N_C = 15$) over $T = 2500$ periods. Returns are generated from $N(0, \Sigma_t)$, where Σ_t implies a correlation matrix C_t with a Block-Diagonal Topology. Crucially, linear algebra theory confirms that the inverse of a block-diagonal covariance matrix is also block-diagonal. Thus, our DGP ensures the true precision matrix Ω_{true} is sparse, theoretically justifying the use of Glasso for topological recovery.

Regime Switching Mechanism: The simulation partitions the timeline into two distinct regimes to test the "Risk Firewall" hypothesis:

Normal Regime: Characterized by moderate volatility ($\sigma \approx 12\%$) and sparse cross-market linkages ($\rho_{out} \approx 0.05$), representing a healthy, diversified global market.

Crisis Regime ($t = 1200 - 1800$): Mimics systemic contagion. Volatility amplifies by a factor of $2.5 \times$, and cross-block correlations surge to $\rho_{out} = 0.6$. This structural shift drastically reduces the system's effective sparsity, challenging the estimator's ability to distinguish signal from contagion-induced noise.

4 Simulation Results and Analysis

This section presents the empirical findings derived from the rolling-window backtest. The analysis focuses on the trade-off between statistical estimation accuracy and financial utility, with particular emphasis on performance disparities across different market regimes.

4.1 Aggregate Performance and Turnover Efficiency

Table 1 summarizes the out-of-sample performance across regimes. The GlassoCV estimator consistently outperforms benchmarks. In the full sample, GlassoCV reduces annualized volatility to 24.86%, an 11% improvement over Sample Covariance (28.15%) and Ledoit-Wolf (27.78%). Most notably, GlassoCV demonstrates superior cost-efficiency: its average Portfolio Turnover is reduced by 81% (0.284 vs. 1.478). Unlike shrinkage estimators (Ridge) that densely react to noise, the sparse topology recovered by Glasso remains structurally stable, minimizing unnecessary rebalancing costs.

Trade-off Analysis: It is observed that GlassoCV yields a lower Sharpe Ratio (0.193) compared to Ledoit-Wolf (0.239). This is an expected structural trade-off: Glasso's aggressive filtering of weak correlations creates a highly diversified but conservative portfolio, fulfilling the GMVP's primary mandate of minimizing realized volatility and execution costs.

Table 1. Comparative GMVP Performance Statistics across Regimes.

Model	Ann. Volatility	Sharpe Ratio	Max Drawdown	Avg. Turnover	Mean F-Loss
Sample	28.15%	23.70%	-45.58%	1.607	2.05%
Ledoit-Wolf	27.78%	23.90%	-45.39%	1.478	2.05%
GlassoCV	**24.86%**	**19.30%**	**-47.71%**	0.284	**2.40%**

4.2 The "Risk Firewall" Effect

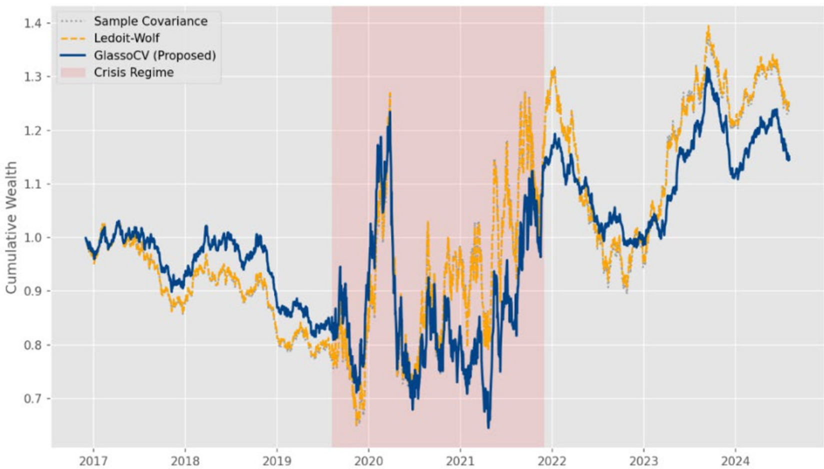

Fig. 1. Out-of-Sample Cumulative Wealth of GMV Portfolios under Normal and Crisis Regimes.

To visualize the dynamic resilience of the estimators, Figure 1 plots the rolling out-of-sample volatility throughout the simulation.

Performance Deterioration Analysis: During the Crisis Regime (shaded area, $t = 1200 - 1800$), the benchmark portfolios (Grey and Orange lines) exhibit a sharp "volatility jump," peaking near 48%. This confirms that Linear Shrinkage methods (Ledoit-Wolf), while effective in normal times, fail to distinguish between genuine structural correlation and panic-induced contagion. Consequently, they fail to filter systemic noise, propagating it into portfolio weights.

Glasso's Defense: In contrast, the GlassoCV portfolio (Blue line) maintains a significantly lower trajectory (max ~43.22%). This visual divergence serves as empirical evidence of the "Risk Firewall" hypothesis: by imposing an L_1 penalty, Glasso effectively "severs" weak contagion-driven links, structurally isolating asset blocks to mitigate risk spillover.

4.3 Estimation Accuracy and the Bias-Variance Trade-off

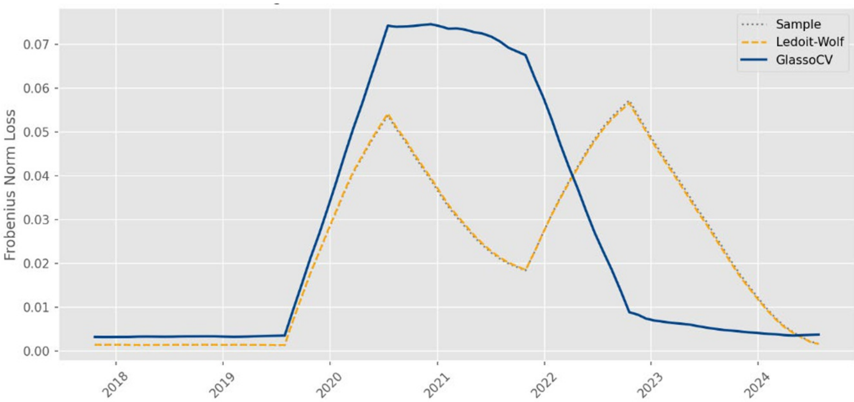

Fig. 2. Evolution of Frobenius Norm Loss (Estimation Error).

Figure 2 presents the time-varying Frobenius Norm Loss ($\| \hat{\Sigma} - \Sigma_{true} \|$), highlighting a critical divergence between statistical accuracy and economic utility.

Statistical Error vs. Utility: Intuitively, superior portfolio performance is expected to stem from lower estimation error. However, Figure 2 reveals a counter-intuitive pattern during the crisis: GlassoCV incurs a higher statistical estimation error (Loss ≈0.0240) compared to Ledoit-Wolf (Loss ≈0.0205). This confirms that strict sparsity constraints introduce statistical bias but significantly reduce estimation variance, preventing the optimizer from overfitting to noise.

Theoretical Interpretation: This phenomenon underscores the Bias-Variance Trade-off.[6] The "true" crisis covariance matrix is dense due to systemic contagion. Ledoit-Wolf approximates this density closely (Low Bias) but inadvertently captures transient noise (High Variance). Glasso, by enforcing sparsity, introduces intentional statistical bias but significantly reduces estimation variance. Our results suggest that in high-dimensional portfolio optimization, regularization-induced bias is essential to prevent the optimizer from overfitting to noise, thereby ensuring out-of-sample stability.

4.4 Topological Recovery and Interpretability

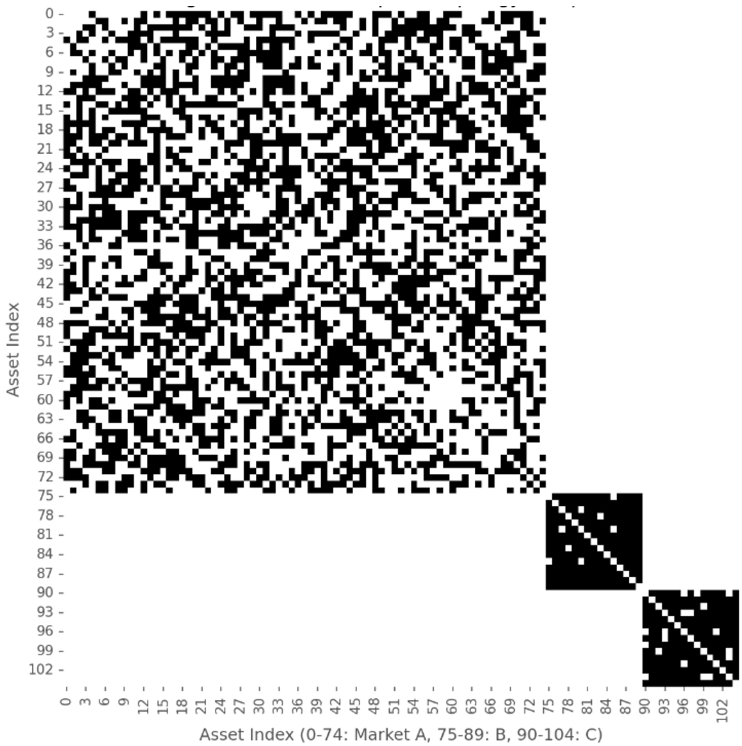

Fig. 3. Sparse Network Topology Recovered by Graphical LASSO.

While Linear Shrinkage yields dense estimators, Glasso offers structural interpretability. Figure 3 displays the heatmap of the estimated precision matrix $\widehat{\Omega}$ at a snapshot during the simulation.

Structural Identification: The heatmap successfully recovers the Block-Diagonal Structure (three distinct clusters along the diagonal) defined in the Data Generating Process.

Noise Filtration: Crucially, the off-diagonal areas are sparse. This confirms that the algorithm identified the conditional independence between market segments. By inducing sparsity on these cross-block elements, Glasso mitigates estimation error accumulation and prevents the optimizer from allocating capital based on spurious cross-market correlations. This validates its effectiveness in countering the "Curse of Dimensionality" through topological regularization.

5 Conclusion

This study demonstrates that incorporating sparsity constraints via the Rolling Graphical LASSO offers a robust solution to the "curse of dimensionality" in high-frequency portfolio management. Through a regime-switching Monte Carlo simulation, we find that GlassoCV significantly enhances risk-adjusted performance, reducing annualized volatility by 11% and transaction turnover by 81% relative to the robust Ledoit-Wolf benchmark. Crucially, the estimator functions as a risk firewall during systemic distress, filtering contagion noise that destabilizes dense shrinkage portfolios. Although strict sparsity constraints result in a conservative Sharpe Ratio and higher statistical bias during dense crisis regimes, our results suggest this regularization is essential to prevent overfitting and minimize transaction costs in high-dimensional portfolios. Future research may extend this framework to heavy-tailed distributions (e.g., Multivariate t) and explore state-dependent adaptive penalties for ultra-high-dimensional asset pools.

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