



Application of Uniform Design in Slope Stability Sensitivity Analysis

Gaofeng Chen^{1,2,*}, Xiqi Liu^{1,2}, Chenyuan Zhang³

¹Pearl River Water Resources Research Institute, Guangzhou 510611, China;

²Key Laboratory of Water Security Guarantee in Guangdong-Hong Kong-Macao Greater Bay Area of Ministry of Water Resources, Foshan 528244, China;

³School of Civil Engineering, University of South China, Hengyang 421001, China.

*863225578@qq.com

Abstract. To enhance slope stability sensitivity analysis, this study applies uniform design methodology, which significantly reduces required tests by ensuring even distribution of test points within the experimental range. Using a homogeneous soil slope as a case study, sensitivity analysis was conducted on five parameters: unit weight, internal friction angle, cohesion, slope angle, and seismic acceleration. Results indicate that cohesion and slope angle are the most sensitive factors in typical homogeneous soil slopes or loose rock-soil slopes. In regions with seismic fortification intensity below 7 degrees, soil unit weight and seismic acceleration are considered non-sensitive factors.

Keywords: Uniform design, slope stability, sensitivity analysis

1 Introduction

The construction of infrastructure such as highways, railways, and water conservancy projects has created numerous slopes. The increasing number of slope accidents has caused significant losses to the nation and its people. Slope failure results from the combined effects of multiple factors, with each factor's impact on slope stability determining the sliding mechanism and failure mode. Conducting sensitivity analysis on these influencing factors is crucial for slope management, monitoring, and prediction.

Slope stability sensitivity analysis has become a hot topic in current research, with numerous experts and scholars conducting in-depth studies. These methods can be broadly categorized into single-factor analysis^[1-5] and multi-factor analysis^[6-10]. The single-factor approach essentially involves selecting a specific parameter value, varying one factor while keeping others constant, and comparing how the baseline value changes. The sensitivity of each factor is then visually represented on the baseline-value versus factor-change curve. While this method provides a clear visualization of how factors affect the baseline, it relies on certain assumptions that may not align with real-world conditions. Multi-factor analysis methods include orthogonal design^[6-8], GIS^[9], and RBF artificial neural networks based on orthogonal design^[10]. These methods share

the common advantage of utilizing probabilistic statistical principles and computer technology to reduce experimental trials and computational workload. However, when dealing with multiple factors and levels, the required trial count and computational load remain substantial, posing challenges for experimental implementation and numerical analysis.

This paper introduces the uniform design test method to analyze the sensitivity of the factors affecting the stability of the slope, reducing the number of tests and the amount of calculation, making the test points more evenly distributed in the test range, and achieving a better effect.

2 Introduction of Uniform Design[11]and Its Application in Slope Sensitivity Analysis

Uniform design is a new method of multi-factor and multi-level experimental design, which is proposed by mathematician Fang Kaitai and Wang Yuan. It is similar to orthogonal design, and there are ready-made uniform tables to facilitate its application.

2.1 Composition of Uniform Table and Application Table

The construction methods of uniform table include good lattice method, Latin method, orthogonal table expansion method, folding method, cutting method, RBIBD method, flipping method, numerical optimization method, etc. The most commonly used good lattice method for constructing uniform design table is as follows:

(1) Given the number of trials n , find a number h smaller than n such that the greatest common divisor (GCD) of n and h is 1. The positive integer pairs that satisfy these conditions form a vector $h=(h_1, h_2 \cdots h_m)$.

(2) The j -th column of the uniform design table is generated by the following method

$$u_{ij} = (i \times h_j) [\text{mod } n] \quad (1)$$

In equation (1), $[\text{mod } n]$ denotes the congruence operation. If $i \times h_j$ exceeds n , subtract an appropriate multiple of n to bring the remainder into the range $[1, n]$. u_{ij} can be generated recursively.

$$u_{1j} = h_j \quad (2)$$

$$u_{i+1,j} = \begin{cases} u_{ij} + h_j & \text{if } u_{ij} + h_j \leq n \\ u_{ij} + h_j - n & \text{if } u_{ij} + h_j > n \end{cases} \quad (i = 1, \cdots, n-1) \quad (3)$$

The uniform design table constructed by the above steps is denoted as $Un(nm)$, and the vector h is referred to as the generating vector of the table. For a given n , the number m of h can be obtained from the Euler function $E(n)$ (i.e., the function that counts the number of integers less than or equal to n that are coprime with n).

- (1) When n is a prime number (i.e., n has no divisors other than 1), $E(n) = n-1$;
- (2) When n is a prime power, i.e., $n=p^l$ (where p is a prime number and l is a positive integer), $E(n) = n(1-1/p)$;
- (3) If n does not fall into the two aforementioned cases, then n can be expressed as the product of powers of distinct primes, where $p_1, p_2, \dots, p_m$ are distinct primes and $l_1, l_2, \dots, l_m$ are integers.

$$E(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_m}\right) \quad (4)$$

In the three scenarios mentioned above, the optimal choice is when n is a prime number, as it allows for up to $n-1$ columns, enabling the arrangement of $n-1$ factors for experimentation. However, when n is a non-prime number, the structure of the table may not support columns, and the number of possible columns could be insufficient. To address this, a method was proposed to construct $Un(nm)$ by removing the last row of $U^*n(nm)$. Since $U^*n(nm)$ exhibits better uniformity than $Un(nm)$, it is generally preferred for use.

Each uniform design table corresponds to a usage table. The usage table ensures that the selected s columns from $Un(nm)$ or $U^*n(nm)$ exhibit excellent uniformity, which is measured by the discrepancy. Let $x_1, x_2, \dots, x_n$ be n points in C_m , and any vector $x = (x_1, x_2, \dots, x_m) \in C_m$. Denote $v(x) = x_1, x_2, \dots, x_m$ as the volume of the rectangle $[0, x]$, and n_x as the number of points in $x_1, x_2, \dots, x_m$ that fall within $[0, x]$.

$$D(x_1, x_2, \dots, x_n) = \sup_{x \in C^m} \left| \frac{n_x}{n} - v(x) \right| \quad (5)$$

The deviation of the set $\{x_1, x_2, \dots, x_n\}$ in C_m .

2.2 Comparison between Uniform Design and Orthogonal Design

Uniform design and orthogonal design are fundamentally methods for selecting representative points within an experimental range. Orthogonal design selects representative points based on orthogonality criteria, ensuring these points reflect relationships between factors and test indicators within the experimental range. The representative points selected by orthogonal design exhibit two characteristics: uniform distribution and comparability. To ensure the "comparability" feature, orthogonal design requires at least q^2 trials (where q is the number of factor levels). Uniform design focuses solely on the even distribution of test points within the experimental range. Compared to orthogonal design, it has unique layout methods, primarily manifested in the following four aspects: (1) Each factor level is tested once and only once; (2) Test points for any two factors can be marked on a grid plane, with exactly one test point per row and column; (3) The uniformity of experimental schemes formed by any two columns in uniform design tables is generally not equivalent; (4) When factor levels increase, the number of trials increases proportionally with the number of levels.

2.3 Application of Uniform Design in Landslide Sensitivity Analysis

The sensitivity analysis of landslide stability influencing factors using uniform design includes the following steps: (1) Defining test parameters. Slope stability coefficient is typically used as the test parameter in slope sensitivity analysis. (2) Selecting test factors. Based on engineering practices and prior research, factors such as geotechnical properties, seismic forces, groundwater variations, and human engineering activities are selected for testing. (3) Determining factor levels. The value ranges of each factor are defined according to test conditions and practical experience, followed by setting appropriate levels within these ranges. (4) Designing uniform tables and arranging tests. (5) Establishing computational models for combined analysis. (6) Analyzing test results. Regression analysis is employed to interpret test results and identify sensitive factors.

3 Engineering Examples

For a homogeneous soil slope with a height of 20m, the soil has a unit weight of 25kN/m^3 , an internal friction angle of 17° , cohesion of 42kPa , and an earthquake coefficient of $0.05g$ (refer to [12] for data and figures). As shown in Figure 1, a computational model was established using the specified dimensions, with parameters assumed to fluctuate within their baseline ranges as detailed in Table 1. To facilitate statistical analysis of experimental data, the study employed a two-factor design with 10 levels per factor, employing a uniform design method to evaluate the sensitivity of unit weight, internal friction angle, cohesion, slope angle, and earthquake coefficient to slope stability.

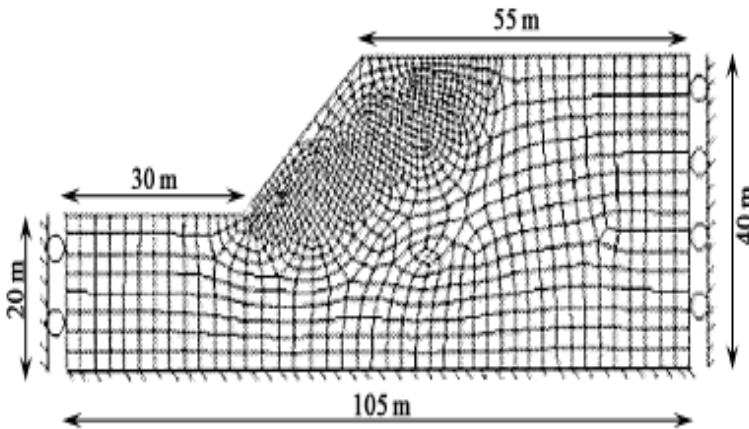


Fig. 1. Computational section

Select columns 1, 3, 4, 5, and 7 from the U*10(108) uniform design table. The experimental design combinations and stability coefficients calculated using the simplified Bishop method are shown in Table 4. The pseudo-static method is employed to introduce seismic effects, with the calculation formulas presented in Equations (6) to (7)^[13]. The parameters are summarized in Table 2 and Table 3:

$$K = \frac{\sum_{i=1}^n [c_i b_i + (W_i - u_i b_i) \tan \phi_i] \sec \beta_i}{(1 + \tan \beta_i \tan \phi_i / K) \left[\sum_{i=1}^n W_i \sin \beta_i + \sum_{i=1}^n Q_{Hi} \cos \beta_i \right]} \quad (6)$$

$$Q_{Hi} = K_H C_Z \alpha_i W_i \quad (7)$$

Table 1. Factor Value Range

volume-weight γ	internal friction angle φ	Cohesion c	slope angle β	seismic coefficient a
/ (kN/m ³)	/ (°)	/ (kPa)	/ (°)	/ (cm/s ²)
18~27	15~33	27~54	31~49	10~100

Table 2. Uniform Design Table U*10(108)

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	7	7	10
2	2	4	6	8	10	3	9	9
3	3	6	9	1	4	10	5	8
4	4	8	1	5	9	6	3	7
5	5	10	4	9	3	2	1	6
6	6	1	7	2	8	9	10	5
7	7	3	10	6	2	5	8	4
8	8	5	2	10	7	1	6	3
9	9	7	5	3	1	8	4	2
10	10	9	8	7	6	4	2	1

Table 3. Usage Table of Uniform Design Table U*10(108)

S	Column Number	D
2	1 6	0.1125
3	1 5 6	0.1681
4	1 3 4 5	0.2236
5	1 3 4 5 7	0.2414
6	1 2 3 5 6 8	0.2994

Table 4. Results of Uniform Design Calculation

test number	1(γ)	2(ϕ)	3(c)	4(β)	5(a)	stability coefficient K
1	18	19	36	39	90	1.596
2	19	25	48	49	70	1.845
3	20	31	27	37	50	1.534
test number	1(γ)	2(ϕ)	3(c)	4(β)	5(a)	stability coefficient K
4	21	15	39	47	30	1.294
5	22	21	51	35	10	2.581
6	23	27	30	45	100	1.253
7	24	33	42	33	80	2.325
8	25	17	54	43	60	2.094
9	26	23	33	31	40	1.728
10	27	29	45	41	20	1.942

The regression analysis was performed using the backward method, and the regression equation was established with a significance level of $\alpha=0.05$:

$$y=b_0 + b_1 \times \gamma + b_2 \times \phi + b_3 \times c + b_4 \times \beta + b_5 \times a$$

Regression coefficient bi: $b_0 = 1.75$, $b_1 = -2.50 \times 10^{-2}$, $b_2 = 2.14 \times 10^{-2}$, $b_3 = 4.62 \times 10^{-2}$, $b_4 = -4.34 \times 10^{-2}$, $b_5 = -3.18 \times 10^{-4}$, $R^2 = 9.36$;

Significance test of regression equation, as shown in Table 5:

Table 5. Variable Analysis Table

source of variation	quadratic sum	variance	mean square	mean square ratio
recurrence	$U=1.65$	$K=5$	$U/K=0.329$	$F=52.20$
surplus	$Q=2.52 \times 10^{-2}$	$N-1-K=4$	$Q/(N-1-K)=6.31 \times 10^{-3}$	
sum	$L=1.67$	$N-1=9$		

The sample size was $N=10$, the significance level was $\alpha=0.05$, the test value was $F_t=52.20$, the critical value of $F(0.05,5,4)=6.265$, $F_t > F(0.05,5,4)$, the regression equation was significant.

F-test results: $F(1)=5.992$, $F(2)=17.50$, $F(3)=184.3$, $F(4)=72.27$, $F(5)=9.707 \times 10^{-2}$. The regression contributions of the equation terms, in descending order, are c 、 σ 、 β 、 ϕ 、 γ and a , with term a having the smallest contribution. The significance test for term a yields $F(0.05,1,4)=7.709$, and since $F(5) \leq F(0.05,1,4)$, term a is statistically insignificant and should be excluded.

Remove insignificant equation terms and create a new regression equation:

$$y=b_0 + b_1 \times \gamma + b_2 \times \phi + b_3 \times c + b_4 \times \beta$$

regression coefficient bi: $b_0 = 1.73$, $b_1 = -2.42 \times 10^{-2}$, $b_2 = 2.10 \times 10^{-2}$, $b_3 = 4.65 \times 10^{-2}$, $b_4 = -4.38 \times 10^{-2}$, $R^2 = 9.54$;

Significance test of regression equation, as shown in Table 6:

Table 6. Variable Analysis Table

source of variation	quadratic sum	variance	mean square	mean square ratio
recurrence	$U = 1.65$	$K = 4$	$U/K = 0.412$	$F = 71.61$
surplus	$Q = 2.59 \times 10^{-2}$	$N - 1 - K = 5$	$Q/(N - 1 - K) = 5.17 \times 10^{-3}$	
sum	$L = 1.67$	$N - 1 = 9$		

The sample size was $N=10$, the significance level was $\alpha=0.05$, the test value was $F_t=79.61$, the critical value of $F(0.05,4,5)=5.192$, $F_t > F(0.05,4,5)$, the regression equation was significant.

F-test results: $F(1)=7.312$, $F(2)=21.95$, $F(3)=242.6$, $F(4)=95.81$. The contribution of equation terms to regression, in descending order, is: c 、 β 、 ϕ 、 γ . Among these, term γ has the smallest contribution. Its significance test yields $F(0.05,1,5)=6.608$, with $F(1) > F(0.05,1,5)$, indicating this term is statistically significant, as shown in Table 7.

Residual analysis:

Table 7. Residual Analysis Table

NO	The observed value of K	Regression value of K	Observed value-regression value	(Regression value-observed) / observed $\times 100(\%)$
1	1.60	1.66	-6.00×10^{-2}	3.75
2	1.85	1.88	-3.00×10^{-2}	1.62
3	1.53	1.53	0.00	0.00
4	1.29	1.29	0.00	0.00
5	2.58	2.48	0.10	-3.88
6	1.25	1.16	9.00×10^{-2}	-7.20
7	2.33	2.35	-2.00×10^{-2}	0.86
8	2.09	2.11	-2.00×10^{-2}	0.96
9	1.73	1.76	-3.00×10^{-2}	1.73
10	1.94	1.98	-4.00×10^{-2}	2.06

The results of regression analysis show that the seismic action a is not significant factor in the area of seismic fortification intensity below 7 degrees, while the bulk density γ , internal friction angle ϕ , cohesion c and slope angle β are significant factors (significance level $\alpha=0.05$), and the order of significance is as follows: cohesion c , slope angle β , internal friction angle ϕ , bulk density γ .

4 Conclusion

This paper introduces the uniform design test method into the slope sensitivity analysis, and uses the uniform distribution of test points in the test range. Taking a homogeneous soil slope as an example, the sensitivity analysis of the slope is carried out by five factors, such as the unit weight, the internal friction angle, the cohesion, the slope angle and the seismic acceleration. The main conclusions are as follows:

(1) Uniform design provides an effective method for sensitivity analysis of factors affecting slope stability, particularly in multi-factor and multi-level scenarios, significantly reducing the required number of tests. (2) When applying uniform design to slope sensitivity analysis, determining the variation ranges of influencing factors is critical, as it directly impacts the accuracy and rationality of the conclusions. (3) In common homogeneous soil slopes or loose rock-soil slopes, cohesion (σ) and slope angle (β) are the most sensitive factors. In regions with seismic fortification intensity below 7 degrees, soil density (γ) and seismic acceleration (a) are considered non-sensitive factors.

This paper has obtained a general method of landslide stability sensitivity analysis, which is of great significance to the mechanism of slope disaster and its prevention and control, especially for the research and application of slope prevention and control in the Three Gorges Reservoir area.

Reference

1. Tian Yu, Zhao Fuwan, Sui Sugang, et al. Analysis of the Impact of Spatial Variability of Soil Parameters on Landslide Stability [J]. *Geological Hazards and Environmental Protection*, 2025,36(02):20-30.
2. Zhou Lanting, Chen Sisi, Sun Yongming. Research on a watershed reservoir water level prediction model based on deep learning for spatiotemporal information extraction [J]. *Hydropower Energy Science*, 2024,42(04):133-136+132.
3. Zheng MZ, Jian WB, Wu MM. Analysis of landslide susceptibility factors and countermeasures[J]. *Geotechnical Engineering*, 2004,7(12):29-31.
4. Chen Chong, Wang Wei, Lü Huayong. Stability analysis of slope reinforcement based on composite anti-slide pile model [J]. *Rock and Soil Mechanics*, 2019,40(08):3207-3217.
5. Zhao Wei, Gao Jingwei, Zhu Pengyu, et al. Sensitivity analysis of shear strength of soil-rock mixtures based on FLAC3D [J]. *Highway Engineering*, 2025,50(04):159-167.
6. Lei Lei, Wang Xiaodong, Wei Xiaolong, et al. Characteristics of Loess Water Infiltration under Non-accumulated Rainfall Conditions and Its Impact on Slope Stability [J]. *Journal of Water Resources and Hydraulic Engineering*, 2025,36(02):153-160+170.
7. Xi Fusong. Application of optimization algorithms in slope stability reliability analysis [J]. *Heilongjiang Transportation Science and Technology*, 2025,48(07):28-32.
8. Deng P, Wang JH, Yang GL, et al. Research on Stability Calculation Methods for Angoumen Landslide in Yuelè Town, Huachi County, Gansu Province [J]. *Gansu Geology*, 2025,34(04):63-69.
9. Yan Wenxiang, Guo Tingting, Yang Yanwu, et al. Regional landslide susceptibility analysis based on the WOE-PPMCC model [J]. *Geological Sciences*, 2024,59(06):1775-1785.

10. Xia Yuanyou, Xiong Haifeng. Artificial Neural Network Analysis of Slope Stability Influencing Factors Sensitivity[J]. Journal of Rock Mechanics and Engineering, 2004,23(16):2703-2707.
11. Fang Kaitai. Uniform Design and Uniform Design Table [M]. Beijing: Science and Technology Press, 1994.
12. Fu Wenguang. Reply to the Inquiry on the Impact of Load and Resistance Misalignment in the Stability Safety Factor Calculation Formula [J]. Journal of Geotechnical Engineering, 2021,43(11):2147-2148.
13. Yang Jinliang. Soil Mechanics [M]. Beijing: China Water & Power Press, 2006.

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

