



Dynamic Measurement of Risk Transmission Between Chinese and U.S. Stock Markets Under Uncertainty Theory: Based on URTI, URAI and GARCH Correction

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Abstract. Aiming at the core defects of traditional cross-border financial risk transmission indicators that rely on probability distribution assumptions, fails to characterize non-stochastic cognitive uncertainty and are difficult to capture time-varying volatility characteristics, this paper originally constructs the **Uncertain Risk Transmission Intensity Index (URTI)** and **Uncertain Risk Asymmetry Index (URAI)** based on the Uncertain Impulse Response Function (UIRF) under Liu Baoding's axiomatic system of Uncertainty Theory, and realizes the time-varying correction of the indicators through GARCH decomposition, forming an implementable dynamic monitoring tool for cross-border financial risks. The empirical study based on the intraday and overnight return data of Chinese and U.S. stock markets from 2016 to 2024 shows that there is a significant asymmetric one-way risk transmission mechanism between Chinese and U.S. stock markets, and the risk output from the U.S. stock market to the A-share market has long been dominant; the risk transmission intensity after the COVID-19 pandemic (2020-2024) is significantly higher than that before the pandemic; the time-varying Uncertain Risk Transmission Intensity Index (TV-URTI) corrected by GARCH has stronger ability to capture extreme event shocks than the static index, and can accurately identify the activation time and duration of risk transmission.

Keywords: Uncertainty Theory; Uncertain Impulse Response Function; Risk Transmission Intensity; Risk Transmission Asymmetry; GARCH Decomposition.

1 Introduction

With the acceleration of global financial integration, the scale of cross-border capital flows continues to expand, and the risk transmission effect between international stock markets is increasingly significant[4]. As the world's largest capital market, the volatility of the U.S. stock market has a profound impact on emerging markets, especially China's A-share market. Traditional risk transmission studies are mostly based on the probabilistic statistical framework, relying on strict distribution

assumptions and large-sample stability conditions[5][6]. When facing the fat-tailed distribution, structural mutation and cognitive uncertainty prevalent in financial markets, model specification errors and lagged risk identification often occur.

Uncertainty Theory was founded by Professor Liu Baoding in 2007[1]. By introducing uncertain measure to replace traditional probability measure, it provides a rigorous mathematical tool for characterizing non-stochastic uncertainty[7]. Based on the Uncertain Vector Autoregressive (UVAR) model[8], this paper constructs a complete quantitative indicator system for risk transmission under the uncertain framework, and introduces the GARCH model[2] for time-varying correction aiming at the limitation that static indicators cannot capture volatility clustering[9]. Taking Chinese and U.S. stock markets as research objects, this paper systematically tests the intensity, direction and time-varying characteristics of risk transmission in different periods, providing a new quantitative basis for preventing imported financial risks.[3]

2 Construction of Risk Transmission Indicators Under Uncertainty Framework

2.1 Uncertain Impulse Response Function (UIRF)

The Uncertain Impulse Response Function is the core tool for characterizing the shock transmission path between variables, which is extended from the classical impulse response theory[10][11]. For a k -order UVAR model:

$$Y_t = \Phi_0 + \sum_{q=1}^k \Phi_q Y_{t-q} + \varepsilon_t \quad (1)$$

Where $\varepsilon_t \sim \mathcal{N}_u(0, \Sigma)$ is a multivariate uncertain normal disturbance term[12]. The original errors are transformed into orthogonal structural shocks v_t through Cholesky decomposition, satisfying $\varepsilon_t = A v_t$ and $E_u[v_t v_t^T] = I$.

By transforming the UVAR model into an infinite-order Uncertain Moving Average (UMA(∞)) form, the **Uncertain Impulse Response Function** is defined as:

$$UIRF_{j,i}(h) = \Psi_h A e_j \quad (2)$$

Its economic meaning is: the response degree of the i -th variable at period $t+h$ after applying a 1-unit structural shock to the j -th variable at period t . The 95% uncertain confidence interval of UIRF can be constructed through Bootstrap resampling to judge the significance of shocks.

2.2 Uncertain Risk Transmission Intensity Index (URTI)

Based on the cumulative shock effect of UIRF, and drawing on the construction idea of Diebold-Yilmaz spillover index[13], the **Uncertain Risk Transmission Intensity Index (URTI)** is constructed to quantify the dominance degree of a certain direction of risk transmission between two markets:

$$S_{A \rightarrow B} = \sum_{h=1}^H |UIRF_{A \rightarrow B}(h)| \cdot \frac{\sigma_A^{(u)}}{\sigma_B^{(u)}} \quad (3)$$

$$URTI = \frac{S_{A \rightarrow B}}{S_{A \rightarrow B} + S_{B \rightarrow A}} \quad (4)$$

where $\sigma_A^{(u)}$ and $\sigma_B^{(u)}$ are the uncertain standard deviations of residuals of market A and market B respectively, and H is the maximum lag period of impulse response.

2.2.1 Statistical Properties.

1. Value range $URTI \in [0,1]$;
2. Symmetry: If $S_{A \rightarrow B} = S_{B \rightarrow A}$, then $URTI = 0.5$, indicating balanced bidirectional transmission intensity;
3. Monotonicity: $URTI$ increases with $S_{A \rightarrow B}$ and decreases with $S_{B \rightarrow A}$.

Interpretation Rule: Set $URTI > 0.8$ as the strong risk transmission stage, indicating that the risk transmission from market A to market B is in a high-intensity activation state.

2.3 Uncertain Risk Asymmetry Index (URAI)

To further characterize the directional difference of risk transmission, and referring to the directional spillover measurement method of Diebold & Yilmaz[14], the **Uncertain Risk Asymmetry Index (URAI)** is constructed as follows:

$$URAI = \frac{S_{US \rightarrow CN} - S_{CN \rightarrow US}}{S_{US \rightarrow CN} + S_{CN \rightarrow US} + \epsilon} \quad (5)$$

where $\epsilon > 0$ is a stability correction term to avoid the extreme case of zero denominator.

2.3.1 Statistical Properties.

1. Value range: $URAI \in (-1,1)$;
2. Directionality: $URAI > 0$ indicates that the United States is a net risk exporter; $URAI < 0$ indicates that China is a net risk exporter;
3. Boundedness:

$$\lim_{S_{CN \rightarrow US} \rightarrow 0} URAI \rightarrow 1, \quad \text{meaning that when China has no risk spillover to the U.S.,}$$

$$\lim_{S_{US \rightarrow CN} \rightarrow 0} URAI \rightarrow -1 \quad \text{meaning that when the U.S. has no risk}$$

spillover to China, URAI approaches -1;

4. Monotonicity:

URAI strictly increases with $S_{US \rightarrow CN}$; URAI strictly decreases with $S_{CN \rightarrow US}$

3 Time-varying Correction of URTI Based on GARCH Decomposition

3.1 Limitations of Static URTI

Static URTI adopts fixed uncertain standard deviations within the full sample or rolling window, implying the assumption that residual volatility remains constant during the sample period. However, financial time series generally have the characteristic of volatility clustering[9]. Static indicators will underestimate the true risk transmission intensity under extreme event shocks and generate false strong transmission signals during market stability periods.

3.2 GARCH(1,1) Model and Time-varying Standard Deviation

For the residual sequence $\hat{\varepsilon}_{i,t}$ of the UVAR model, a standard GARCH(1,1) model proposed by Bollerslev[2] is constructed to capture its volatility clustering:

$$\sigma_{i,t}^2 = \omega + \alpha \varepsilon_{i,t-1}^2 + \beta \sigma_{i,t-1}^2 \quad (6)$$

where $\omega > 0$ is the constant term, $\alpha \geq 0$ is the ARCH term coefficient (characterizing the short-term impact of new information shocks), $\beta \geq 0$ is the GARCH term coefficient (characterizing the long-term volatility persistence), and the stationarity condition $\alpha + \beta < 1$ is satisfied.

3.3 Framework Compatibility of Probability and Uncertainty

Theorem: If the UVAR residual $\varepsilon_{i,t}$ follows a time-varying uncertain normal distribution $N(0, \sigma_t)$, then the time-varying conditional standard deviation $\sigma_{i,t}$ estimated by the GARCH model can be used as the time-varying scale parameter of the uncertain normal distribution without violating any axioms of Uncertainty Theory[15].

Proof:

1. The definition of uncertain normal distribution does not restrict the scale parameter to be a constant, only requiring $\sigma_t > 0$ to be an $\mathcal{F}_{t-1}$ -measurable deterministic sequence[12];
2. $\hat{\varepsilon}_{i,t}$ estimated by GARCH is a deterministic value calculated based on historical information, not a random variable. It only changes the dispersion degree of the uncertain distribution and does not change the core attributes of uncertain variables;
3. The two have consistent economic connotations, both used to characterize the volatility level of variables.

3.4 Time-Varying URTI with GARCH Correction

Based on time-varying uncertain standard deviations, the **Time-varying Uncertain Risk Transmission Intensity Index (TV-URTI)** is reconstructed:

$$S_{A \rightarrow B, t} = \sum_{h=1}^H |UIRF_{A \rightarrow B}(h)| \cdot \frac{\sigma_{A, t}}{\sigma_{B, t}} \quad (7)$$

$$TV-URTI_t = \frac{S_{A \rightarrow B, t}}{S_{A \rightarrow B, t} + S_{B \rightarrow A, t}} \quad (8)$$

Risk Level Classification:

Strong risk transmission: $TV-URTI_t > 0.8$ (corresponding to the 90th percentile of the full sample);

Moderate risk transmission: $0.5 < TV-URTI_t \leq 0.8$;

Balanced transmission: $TV-URTI_t \leq 0.5$.

4 Empirical Analysis

4.1 Sample Selection and Data Processing

The daily data of China's Shanghai Composite Index (000001) and the U.S. Dow Jones Industrial Average (DJIA) from January 1, 2016 to December 31, 2024 are selected. Samples with mismatch-ed trading days are excluded, and a total of 2054 valid observations are obtained. The sample is divided into two sub-periods with the outbreak of COVID-19 as the cutoff point[3]:

1. Period T1 (Pre-pandemic): 2016.01.01-2019.12.31 (884 observations);
2. Period T2 (Post-pandemic): 2020.01.01-2024.12.31 (1170 observations).

Calculate the **intraday return** and **overnight return** of each index:

$$r_{d, t} = (\ln p_{i, t}^c - \ln p_{i, t}^o) \times 100 \quad r_{n, t} = (\ln p_{i, t}^o - \ln p_{i, t-1}^c) \times 100$$

Using the natural design of the trading time difference between Chinese and U.S. stock markets to isolate bidirectional endogeneity interference, this paper focuses on analyzing the risk transmission from U.S. stock market intraday returns to A-share market overnight returns. The model lag order is determined by the fixed origin cross-validation method proposed by Liu & Yang[16], and the optimal lag order is 2.

4.2 Empirical Results of Static URTI and URAI

4.2.1 Full Sample Period.

As shown in Fig. 1 and Fig. 2: There are 1466 observations with $URTI > 0.8$, accounting for 71.37%, indicating that the risk output from the U.S. stock market to

the A-share market is dominant in most of the time; URAI: The United States was a risk exporter for 1723 days, accounting for 83.89%, while China's reverse spillover only accounted for 16.11% and showed a pulse-like characteristic, which is consistent with the findings of Auer et al[16].

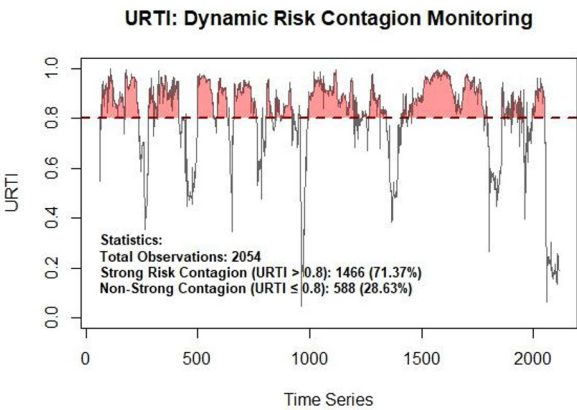


Fig. 1. Full-sample URTI

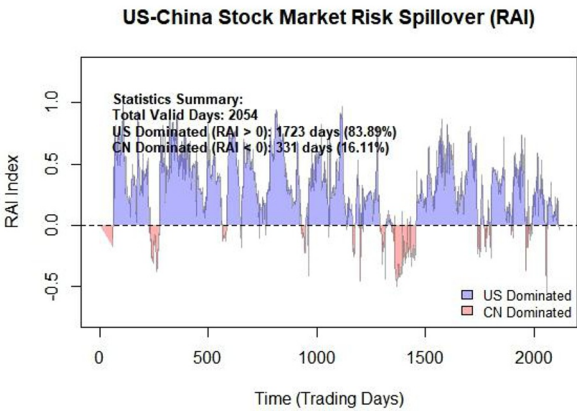


Fig. 2. Full-sampleURAI

4.2.2 Sub-period Comparison.

As shown in Figs. 3, 4, 5 and 6.

Table 1. Proportion of strong risk transmission in different periods

Period	Proportion of URTI>0.8	Proportion of URAI>0
T1 (16-19)	72.06%	90.05%
T2 (20-24)	72.70%	78.74%

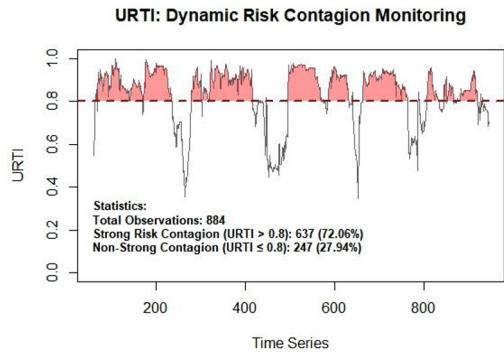


Fig. 3. URTI in period T1

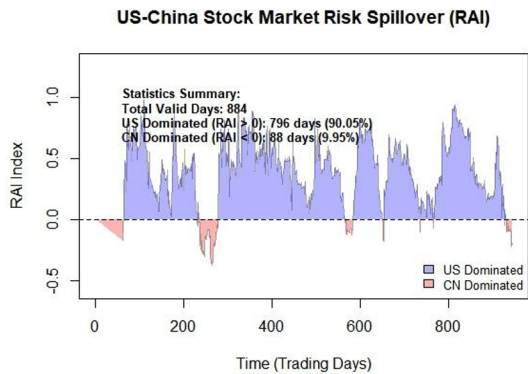


Fig. 4. URAI in period T1

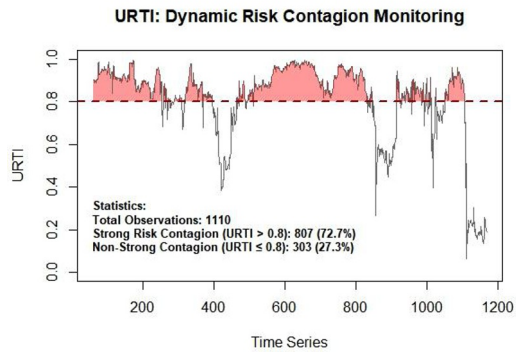


Fig. 5. URTI in period T2

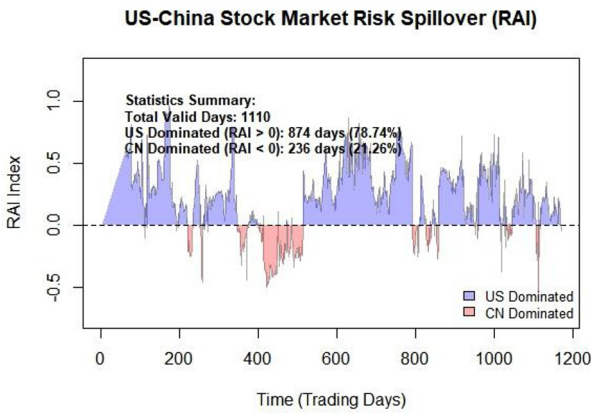


Fig. 6. URAI in period T2

As shown in Table 1, the proportion of high-risk values with $URTI > 0.8$ is 72.06% at time T1 and 72.70% at time T2. $URAI > 0$ The proportion of $URAI > 0$ reaches 90.05% at T1 and 78.74% at T2.

4.3 Empirical Results of TV-URTI

4.3.1 ARCH Effect Test.

The ARCH-LM test proposed by Engle[9] is performed on the UVAR residuals. The results show that both the LM statistic and F statistic of the residuals of Chinese and U.S. markets reject the null hypothesis at the 1% significance level, indicating significant volatility clustering and verifying the necessity of GARCH modeling.

4.3.2 GARCH(1,1) Parameter Estimation.

Table 2. Parameter Estimation

Parameter	Chinese Market (rn1 residual)	U.S. Market (rd3 residual)
ω	0.0031***	0.0272***
α	0.0330***	0.1732***
β	0.9535***	0.7689***
$\alpha + \beta$	0.9866	0.9421

Note: *** indicates significance at the 1% level.

As shown in Table 2 The results show that the U.S. market is more sensitive to the short-term response of new information shocks (larger α), while the Chinese market has stronger long-term volatility persistence (larger β), which is consistent with the characteristics of emerging markets.

4.3.3 Comparison Between TV-URTI and Static URTI.

The proportion of observations with >0.8 in the full sample TV-URTI is 74.13%, higher than 71.37% of static URTI;

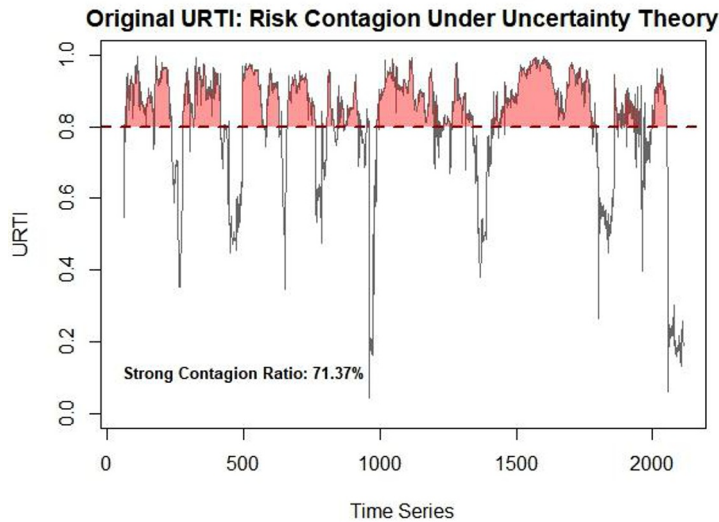


Fig. 7. Full-sample Original URTI

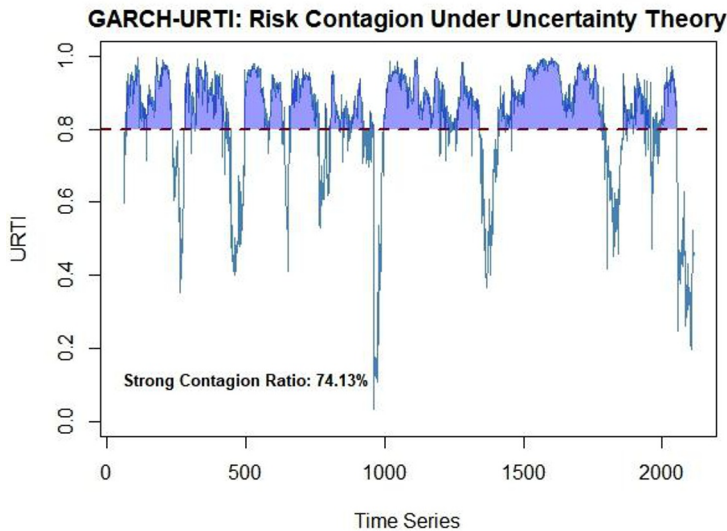


Fig. 8. Full-sample GARCH-URTI

As shown in Figs. 7, 8, 9 and 10 Window analysis shows that TV-URTI can accurately capture the phased risk transmission improvement that static URTI cannot identify, such as the U.S. tax reform debate period in October 2017 and the Federal Reserve policy adjustment period in March 2021.

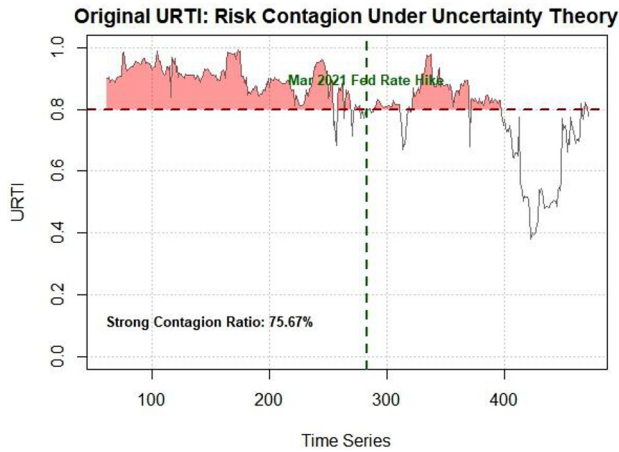


Fig. 9. 2020-2021 Original URTI

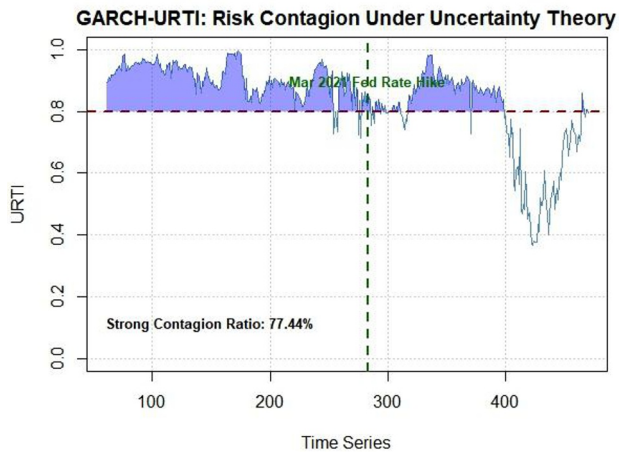


Fig. 10. 2020-2021 GARCH URTI

5 Conclusions and PolicyRecommendations

5.1 Research Conclusions

- 1. There is a significant asymmetric risk transmission mechanism between Chinese and U.S. stock markets. The one-way risk output from the U.S. stock market to the A-share market has long been dominant, and this transmission effect has been further enhanced after the COVID-19 pandemic;
- 2. The URTI and URAI indicators constructed in this paper can effectively characterize the cross-border risk transmission characteristics under uncertain environments, and the conclusions have passed multi-dimensional robustness tests;

3. The TV-URTI indicator corrected by GARCH has stronger risk identification ability than the static indicator, and can monitor the dynamic changes of risk transmission in real time.

5.2 Policy Recommendations

1. Establish a real-time cross-border financial risk monitoring system based on TV-URTI, and timely activate the risk early warning mechanism when the indicator exceeds the strong risk threshold of 0.8;
2. During extreme event shocks, strengthen macro-prudential management of cross-border capital flows to prevent imported risks caused by sharp fluctuations in the U.S. stock market;
3. Continuously promote the institutional construction of the A-share market, enhance the endogenous stability of the market, and reduce the sensitivity to external shocks.

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