



Application of Copula Model in Risk Analysis of Financial Asset Portfolios

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Abstract. This study analyzes the daily returns of three core assets in the Chinese financial market. Data are obtained from the CSMAR database, the gold spot price series, the CSI 300 Index and the ChiNext Index. The sample covers the period from 14 March 2022 to 11 March 2026. Marginal distributions are modeled via the GARCH (1,1)-t model. Four copula functions, namely the Gaussian copula, t-copula, Clayton copula and Gumbel copula, are adopted to characterize the interdependence structures among assets. The copula-GARCH framework is further utilized to calculate the VaR and ES risk metrics for the investment portfolio. Empirical results based on 948 daily return observations indicate that the t-copula yields the optimal fit with an AIC value of -1175.88. The tail correlation coefficient between the CSI 300 Index and the ChiNext Index reaches 0.4344. This value indicates strong co-movement between the two assets under extreme market conditions. The tail dependence coefficient between gold and stock indices is still at a very low level, which shows the advantage of effective risk diversification.

Keywords: Copula Model, Risk Analysis, Financial Asset Portfolios.

1 Introduction

Correlations among assets in financial markets exhibit a continuous upward trend. Portfolio risk management thus becomes a core issue for investors and financial institutions. Conventional risk assessment methods rely on linear correlation metrics and the Gaussian distribution assumption. These methods thus present critical limitations in capturing heterogeneous dependence structures [1]. Copula models constitute a core paradigm shift in the field of financial econometrics. It is a very flexible tool for the modelling of the nonlinear dependencies, and the tail risks that the usual methods cannot catch enough [2].

The application of copula theory in financial risk management has received considerable attention in recent years. There is a lot of existing literature that shows that the application of the copula function in the financial sector has confirmed the superiority of the copula in relation to the linear correlation. The copula model on integration with extreme value theory can improve the accuracy of portfolio risk forecasting during the cycle of the market [3]. The use of dynamic copula models

shows significant of the shifts in the mechanism of portfolio risk during the post pandemic period [4]. The application of the copula method in the financial sector is moving quickly [5]. Improved the estimation methods for the Archimedes Copula give better predictive performance [6]. Vine Copula theory gives a new tool for the multi-asset portfolio modelling [7]. The copula-GARCH model can be applied to the international equity markets, and it is effective [8].

Despite these advances, research gaps remain. The unique characteristics of the multipolar era have not yet been fully incorporated into portfolio risk models, and the dynamics of the three-way tail dependence between gold, the CSI 300 Index, and the ChiNext Index during extreme market events remain to be explored. This study addresses these gaps by applying Copula models to analyse portfolio risk in the Chinese market from 2022 to 2026. At the theoretical level, it extends the application of Copulas to the post-pandemic multipolar market environment. At the practical level, it provides actionable guidance for portfolio diversification strategies within a dynamic economic landscape.

2 Theoretical Framework and Methodology

Optimizing portfolio risk and return is the foundation of modern portfolio theory. Portfolio risk is not the weighted average of the risks of individual assets, but the risk is determined by the covariance structure among the constituent assets [9]. There are several dimensions of the financial portfolio risk: market risk, credit risk, liquidity risk, and operational risk. This paper focuses on market risk as its core research subject. Only market risk is taken into consideration in this study. Three key determinants of market risk are identified and examined. The first determinant is the volatility of individual assets, which measures the magnitude of price fluctuations. The second determinant is the correlation structure, which indicates the direction of co-movement among assets. The third determinant is portfolio weights, which represent the asset allocation decisions. The correlations range from the perfect positive correlation to the perfect independence, and then to the perfect negative correlation. The empirical research shows that the financial correlations usually have non-linear characteristics and asymmetric dependencies, especially under extreme market conditions. The issue of the tail dependence, which the traditional correlation metrics are not able to capture, is becoming a big problem that needs to be solved as soon as possible.

Copula theory gives a mathematically rigorous, but not depending on the marginal distributions, way to model the multivariate dependence structures. Sklar's theorem attains a separation between the marginal behaviour and the dependence structures, which gives much flexibility for the financial application. In this study, four representative Copula models are compared: the Gaussian Copula is based on a multivariate normal distribution, the parameter is the correlation coefficient, it has no tail dependence, and is suitable for the case that the assets are linearly related but do not have extreme co-movement. t-Copula is based on the multivariate t-distribution. The parameters are the correlation coefficient and the degrees of freedom. It has

symmetric tail dependence and is suitable for most financial assets [10]. Clayton Copula is a kind of Archimedean Copula. It has lower-tail dependence and is suitable for the case of synchronous decline to be modelled. Gumbel Copula is a kind of extreme-value Copula. The parameter is $\theta \geq 1$, and it has upper-tail dependence, which is suitable for the case of synchronous rise to be modelled. The study uses the method of two-stage inference to construct the copula models. For each asset return series, in this paper, the study uses the GARCH (1,1)-t model to catch the clustering characteristics of time-varying volatility. After the GARCH estimation, the study transforms the standardised residuals into uniform variables through a probability integral transform and uses the uniform variables as input data for the copula parameter estimation. The study uses the method of maximum likelihood estimation to estimate the copula parameters. The combined diversification strategy gives a way of action.

3 Data Processing and Model Construction

3.1 Data and Descriptive Statistics

The spot gold price is based on the Au99.99 contract quoted by the Shanghai Gold Exchange. Daily logarithmic returns are calculated from the closing prices, with the three series aligned on common trading days. First, data cleaning has been done, that is, the detection of missing values in all the time series data. The results show that there are no missing values in the data collected, so the study has all the sample period covered. Second, the outliers have been detected according to the 3sigma principle. For the outlier analysis, the extreme observations are about 0.32% in all the asset classes. After that, the Winsorisation method, that is, the truncation in the 1st and 99 percentiles, has been applied to correct these extreme values, to diminish their effect, but keeping the data.

According to the descriptive statistics, first, the average daily return on gold is 0.111%, the annualised return is 28.04%, which is much larger than 3.01% for the annualised return of the CSI 300 and 6.79% for the annualised return of the ChiNext Index. Second, the ChiNext Index has the highest volatility, the daily standard deviation is 1.70%, the annualised volatility is 26.95%, the daily standard deviation of the CSI 300 is 1.06%, and the annualised volatility is 16.90%, the daily standard deviation of gold is 1.05%, and the annualised volatility is 16.69%. The most interesting thing in the correlation analysis is that the correlation of the CSI 300 Index and the ChiNext Index is 0.8680, which indicates they have a very high degree of integration in the China mainboard market and its growth-oriented market. And the gold only has weak positive correlation with two stock indices, the correlation coefficient is 0.0959 and 0.0292, respectively, which may indicate that it may have diversification benefit during this period. This kind of characteristic is very important in the risk hedging practice. When the stock market faces a systemic shock, the price of gold may have counter-cyclic volatility, and the low correlation can well reduce the overall risk of the portfolio.

3.2 GARCH Model Estimation

Following data pre-processing, a GARCH (1,1)-t model is fitted to each return series to obtain the standardized residuals. The study fits the GARCH (1,1)-t model, because the residuals follow a student's t-distribution with degrees of freedom, the heavy-tailed nature of the returns can be well captured. Table 1 presents the estimation results of the GARCH (1,1)-t model.

Table 1. GARCH (1,1)-t model estimation results.

Parameter	Gold	CSI 300	ChiNext
$\mu (\times 10^{-3})$	80.72	-18.08	-53.70
$\omega (\times 10^{-6})$	44,541.99	98,846.42	351,392.11
α (ARCH term)	0.1882	0.1014	0.1658
β (GARCH term)	0.7866	0.8040	0.7117
$\alpha + \beta$ (persistence)	0.9747	0.9053	0.8776
v (degrees of freedom)	3.8682	3.9478	3.5787

All models exhibit high persistence, with $\alpha + \beta$ ranging between 0.88 and 0.97, indicating that volatility shocks decay slowly over time. The ChiNext Index has the lowest degree of freedom ($v = 3.5787$), suggesting that its returns have the thickest tails and the highest probability of extreme volatility events. Gold exhibits the highest volatility persistence, with $\alpha + \beta = 0.9747$, indicating that volatility shocks in the gold market decay the most slowly.

3.3 Copula Model Selection and Estimation

After the GARCH estimation is carried out, the standardized residuals are taken out and are given in the form of uniform variables by a probability integral transform. For each asset, an empirical cumulative distribution function is applied to transform the residuals into uniform variables in the interval [0,1]. With PIT, all the marginal distributions are given in the form of a uniform distribution, which is a necessary condition for the Copula modelling. The results of the model comparison with the log-likelihood, AIC, and BIC criteria show that the t-Copula has the better fit for the AIC and the BIC. The AIC of the t-Copula is -1175.88, which is much better than the other three models, as shown in Table 2.

Table 2. Comparison of the four copula models for the CSI 300–ChiNext pair.

Model	Parameter estimates	Log-likelihood	AIC	BIC
Gaussian	$\rho = 0.8399$	578.71	-1155.42	-1150.57
t-Copula	$\rho = 0.8399, v = 6.80$	589.94	-1175.88	-1166.17
Clayton	$\theta = 2.1881$	448.57	-895.14	-890.29

Gumbel	$\theta = 2.6281$	564.52	-1127.04	-1122.19
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Table 3. T-Copula tail dependence coefficient estimation results.

Asset pair	Copula ρ	d.o.f. v	Lower tail λ	Upper tail λ
Gold–CSI 300	0.0675	14.03	0.0029	0.0029
Gold–ChiNext	0.0061	21.27	0.0002	0.0002
CSI 300–ChiNext	0.8399	6.80	0.4344	0.4344

As shown in Table 3, there is a tail correlation between the CSI 300 Index and the ChiNext Index, with the tail correlation coefficient as large as 0.4344. That is to say, when one index experiences an extreme decline, the conditional probability that another index also experiences an extreme decline is approximately 43.44%, which means the potential return from the diversification strategy is not much under the condition of an extreme market. This result provides an important implication for portfolio construction. Investors who want diversification in the Chinese stock market may need to worry about the allocation strategy besides the CSI 300–ChiNext index.

On the contrary, however, the tail correlation coefficients between the gold and the stock indices are much lower (0.0029 for gold and the CSI 300 Index, and 0.0002 for gold and the ChiNext Index). It shows that even in the tail, i.e., the cases of the extreme of the market, the gold is still able to achieve risk diversification well. In particular, the tail correlation coefficient between the gold and the ChiNext Index is near zero, which means that the diversification benefit of the gold can also take effect for the growth sector of the Chinese stock market. It is good empirical evidence that gold is a safe-haven asset in the Chinese investment portfolio.

3.4 Risk Calculation

The study calculates the Value-at-Risk and Expected Shortfall using the historical simulation method based on the estimated t-Copula model. VaR is defined as the α -quantile of the portfolio return distribution. ES is defined as the expected loss conditional on the loss exceeding the VaR threshold. As shown in Table 4, the highest risk has the ChiNext Index, with a Value at Risk of -4.02% at the 99% confidence level. It means that there is a probability of 1% that in a single day it has a loss bigger than 4%. The risk is reduced by diversification, thanks to the fact that the equally weighted portfolio has a VaR of -1.25% at the 95% confidence level, which is between the VaR values of the individual assets at the same confidence level, clearly showing the risk reduction of the portfolio diversification. Although it has the advantage of being more risk-diversified than high-risk equity assets, it has higher risk than low-risk gold assets.

Table 4. VaR and ES estimation results for the three assets based on the t-Copula-GARCH model.

Asset	VaR ₉₀ (%)	VaR ₉₅ (%)	VaR ₉₉ (%)	ES ₉₅ (%)	ES ₉₉ (%)
Gold	-0.78	-1.18	-2.96	-2.41	-4.77

CSI 300	-1.05	-1.46	-2.50	-2.24	-3.84
ChiNext	-1.56	-2.10	-4.02	-3.40	-5.72
Equal-weighted Portfolio	-0.93	-1.25	-2.28	-1.99	-3.45

A comparative analysis of the four copula models shows that there are some differences in the way the four copula models capture the asset correlation structures. The Gaussian copula is simple to parameterise, but does not capture tail correlation, and it underestimates the risk under extreme market conditions. The Clayton captures the lower-tail correlation but does not take the upper-tail correlation. The Gumbel Copula captures the upper-tail dependence but does not capture the lower-tail dependence. The t-Copula can capture both symmetric upper- and lower-tail dependence, so it is the copula that can be the most suitable for modelling the interdependence structure between the CSI 300 and the ChiNext.

The results of model selection criteria also clearly show that the t-Copula is the best model. From the economic point of view, the symmetric tail dependence of the t-Copula is the same as the real performance of the A-share market: in the cases of the market rise, the investor sentiment is high and the capital comes in, then the multiple indices will rise at the same time. In the cases of the market fall, the panic spreads and the capital go out, then the multiple indices will fall at the same time.

4 Empirical Results Analysis

4.1 Correlation Analysis

Fig. 1 is the correlation heatmap of the interdependence structure among the three asset classes. The darker the area, the stronger the correlation. The results demonstrate that the correlation structure has hierarchical characters: the CSI 300 Index and the ChiNext Index have strong positive correlation, while the gold has a weaker correlation with the two major stock indices. It can be confirmed that the gold has diversification potential in a Chinese equity portfolio.

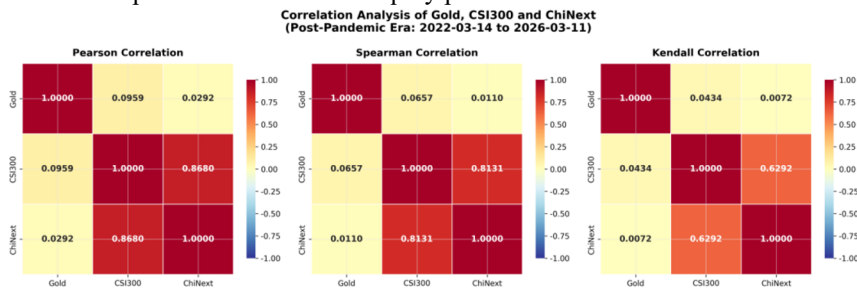


Fig. 1. Correlation heatmap of Gold, CSI 300, and ChiNext index (Source: author's own calculation based on CSMAR data.).

4.2 Portfolio Risk Assessment

The Value at Risk of the investment portfolio is calculated using a historical simulation method based on an estimated t-Copula model. At a 95% confidence level, the VaR of the equally weighted portfolio is -1.25%, implying that the probability of the portfolio suffering a single-day loss exceeding 1.25% is 5%. Meanwhile, the ES reaches -1.99%, indicating that when extreme losses occur, the average loss magnitude is significantly greater. As the confidence level is raised to 99%, VaR and ES expand to -2.28% and -3.45% respectively, with tail risk characteristics becoming increasingly pronounced.

It is worth noting that the inclusion of gold in the portfolio results in a reduction in risk metrics compared to a pure equity portfolio. To further investigate the non-linear impact of gold weighting on risk metrics, this paper constructs a dynamic weighting scan analysis. When the gold weighting falls within the 15%–25% range, risk metrics show the most pronounced decline. Once the 30% threshold is exceeded, the marginal risk-reduction effect weakens significantly. This finding provides Chinese investors with clear guidance on asset allocation—the optimal risk-hedging proportion for gold is not necessarily higher is better, but rather exhibits a distinct range effect.

Based on the mean-variance framework, three portfolio strategies were constructed: (1) the minimum variance portfolio. (2) the maximum Sharpe ratio portfolio. (3) the risk-parity portfolio. The risk-free rate is set to 2.5% for the Sharpe ratio calculation. The optimisation results are shown in Table 5. The portfolio with the highest Sharpe ratio recommends an allocation of 95.52% to gold assets, driven primarily by gold's outstanding performance during the sample period. This portfolio achieved an annualised return of 26.01% and a Sharpe ratio of 1.50, significantly outperforming the equally weighted portfolio. However, such a high allocation to gold also exposes the portfolio to the risk of a gold price correction. Investors should adjust their positions in light of their assessment of future market trends.

Table 5. Comparison of different portfolio strategies.

Strategy	Gold weight	CSI 300 weight	ChiNext weight	Annual return (%)	Annual volatility (%)	Sharpe ratio	VaR95 (%)
Minimum variance	50.70%	49.30%	0.00%	15.07	12.18	1.03	-1.08
Maximum Sharpe ratio	95.52%	0.00%	4.48%	26.01	15.70	1.50	-1.12
Risk parity	45.52%	32.92%	21.56%	14.61	13.48	0.90	-1.18
Equal-weighted	33.33%	33.33%	33.33%	12.11	15.18	0.63	-1.25

5 Robustness Checks

To check whether the conclusions are robust, the study splits the sample period into two sub-periods for comparison: the period from March 2022 to March 2024 and the period from March 2024 to March 2026. Under the sub-period analysis, the study finds that the correlation coefficients of the gold and equity indices are still lower than 0.15 in both periods, and the characteristics of tail dependence are similar to those in the whole sample. The correlation coefficient of CSI 300 and ChiNext increases a little in the second sub-period, but the tail dependence is still significant. This means that the core conclusions are robust over time.

In addition to the t-Copula, the study also considers the Gaussian, Clayton, and Gumbel Copulas. The AIC and BIC model selection criteria show that the t-Copula has the best fitting performance for most of the asset pairs, especially for characterizing the symmetric tail dependence relationship between CSI 300 and ChiNext. The Gaussian Copula is the worst since it cannot catch the tail dependence, which also supports the choice of the t-Copula.

In this test, the study employs the Kupiec unconditional coverage test to test model prediction accuracy formally. As shown in Table 6, this test is to compare the theoretical violation rate with the actual violation rate: if the p-value is larger than 0.05, then the model passes the test. At the 90%, 95%, and 99% confidence levels, the Copula-VaR model of equal-weighted portfolio passes the backtesting test, which means the prediction of the model has good accuracy. The actual violation rates are close to the theoretical violation rates. So, it can be said that the t-Copula-GARCH framework applied to portfolio risk management in Chinese markets is valid.

Table 6. Kupiec backtesting results for Equal-weighted portfolio.

Confidence level	VaR (%)	Violations	Expected violations	Violation rate (%)	P-value	Result
90%	-0.93	95	95	10.02	0.9827	Pass
95%	-1.25	48	47	5.06	0.9289	Pass
99%	-2.28	10	9	1.05	0.8664	Pass

6 Conclusion

In this study, 948 sets of daily data from March 2022 to March 2026 are used to empirically study the dependence structures and portfolio risks of gold, the CSI 300 Index, and the ChiNext Index with the GARCH (1,1)-t model and four copula methods. By comparing these models, the t-Copula model is the best fit with the AIC = -1175.88, being much better than the other three models. The result shows that the returns of all the three asset classes have significant leptokurtic and heavy-tailed characteristics and a clustering of volatility. The GARCH (1,1)-t model can capture their conditional heteroscedasticity. The tail correlation coefficient between the CSI 300 and the ChiNext Index is as high as 0.4344, means that the two are very

interdependent with each other under the extreme market condition and there is little diversification effect. The tail dependence between the gold and stock indices is relatively low, providing good risk diversification effect. The strategy with the highest Sharpe ratio recommends allocating 95% to the gold assets, which can get the annualized return of 26.01% and the Sharpe ratio of 1.50.

The main contribution shows that the t-Copula is the best in characterizing asset interdependence structures. It systematically uncovers the dependency structures and risk spillover characteristics between the gold and stock indices. This paper addresses the gaps that the existing research does not pay enough attention to the post pandemic phase of the emerging market and does not have a comparison of multiple Copula models. The research confirms that the t-Copula-GARCH model is more robust in the portfolio risk measurement, which gives practitioners risk management methods, and gives the academic community a new empirical benchmark. The finding contributes to the asset allocation, the risk management system, and the market participants who would like to cope with the extreme volatility. This study only covers three asset classes, uses static Copulas, relies on in-sample backtesting, and does not incorporate machine learning. In the future, one can incorporate bonds and commodities, use the time-varying dynamic copulas, and combine deep learning to improve the predictive accuracy of the high-dimensional portfolios, so as to better adapt to the complex financial environment.

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