Long-Memory Option Pricing: A Comparison Based on Fractional Brownian Motion and the Black-Scholes Model

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Abstract. Distribution properties of asset returns and assets market volatility are among the fundamental aspects that affect option pricing. Typically, these factors vary with the different market conditions thus creating biases in the conventional pricing models. The traditional Black-Scholes model is based on the assumption that the returns of the assets are characterized by the random walk, however, the empirical studies have shown that markets are often fractal. This paper analyses Shanghai Composite Index. Once the non-normality of return sequences has been established, it proves that the market is highly long memory ($H > 0.5$). Upon this, an option-based pricing model based on a fractional Brownian motion is built and the model is simulated against the traditional one. Findings reveal that the long memory fractional B-S model is much more effective when compared to the classical one in pricing long- run options, which best capture the actual volatility risk in the market. The given study enhances empirical evidence to the option pricing theory under fractal market theory, offering more practical theoretical backgrounds and methodological references to the option trading decisions of the long-term investors and the optimization procedure of the risk management of financial institutions.

Keywords: Long-Range Memory, Fractional Brownian Motion, Black-Scholes Model, Hurst Index.

1 Introduction

Since it is one of the key tools of financial derivatives market, options are an important tool in asset allocation and risk management. The Black-Scholes option pricing model gave a closed-form analytical answer to this question to price European options, which forms the foundation of the current financial mathematics [1]. The B-S model presumes that the underlying asset price is governed by the geometric Brownian motion simplifying the computation and extremely enhancing its initial popularity.

Nonetheless, the developments of financial markets have led to overwhelming empirical studies pointing out that the real-life asset price fluctuations are not entirely in the idealistic assumptions proposed by B-S model. Financial time series usually possess features of long memory and self-similarity, that is, the past changes of prices

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have long term effect on future trends instead of assuming the random walk in the B-S model [2]. Bayraktar et al. also affirmed the dominance of fractal dimensions and long-memory property, in financial markets, by looking at wavelet analysis of the S and P 500 index [3].

The conventional B-S model tends to cause large option prices in long-memorizing markets as the model underestimates tail risks. In order to overcome this shortcoming, fractional Brownian motion became a popular way of improving. Despite the theoretical problems of early models in terms of arbitrage opportunities, Hu and Øksendal used the fractional ItO approach to prove this observation with reference to the theory of the fractional ItO integrals [4]. On the basis of this, Necula formulated the pricing formula of the European options in the framework of the fractional Brownian motion. [5]. Assumptions were loosened in turn by the researchers.

As further studies of market microstructure have been undertaken, there have been numerous extensions of the concept of fractional Brownian motion models that aim to provide a solution to more involving market frictions and dynamics. Hu et al. used the combination of adaptative, differential evolution algorithms to suggest a model, based on fractional stochastic volatility, European option pricing, which has better model calibration [6]. Ma et al. investigated the practical problem of inadequate market liquidity by studying the price use of the hybrid models of fractional Brownian motion in illiquid markets [7]. Using a discrete-time model, Guo and colleagues explored the European option pricing within sub-fractional Brownian motion models and supplemented the theoretical system of discrete models [8]. Adding the sub-fractional models to the analysis of European options, to which Ren and Feng applied Vasicek pricing methodology, further widened the conceptual coverage of the sub-fractional models. [9]. On the issue of risk hedging applications, Dufera conducted simulated experiments to validate the usefulness of fractional Brownian motion in the option pricing and dynamic delta hedging with respect to both simulated and real options and proved that the model can be efficient in the real trading front [10].

Since the Chinese markets are highly fractal and the standard Black-Scholes models fail to perform on them, this paper seeks to empirically determine the Hurst exponent utilizing the actual market data to establish the effectiveness of the models of the fractional Brownian motion and compare the performance of the models based on their pricing effectiveness. The paper will authoritatively obtain market data to substantiate the need to introduce the fractional Brownian motion model and establish an empirical basis on which pricing deviations between the B-S model and the fractional Brownian motion model will be compared.

2 Data Analysis Data Description

To empirically test the applicability of the traditional Black-Scholes model assumptions in real markets, this study selected the Shanghai Stock Exchange Composite Index (SSE Composite Index, Code: 000001.SS) as the research subject. Being one of the fundamental indicators that display the general tendency of the Chinese stock market,

SSE Composite Index is effective in presenting the nature of volatility the emerging markets.

The data was obtained in the Yahoo Finance database that provides information about daily prices at the end of the day from January 2021 up to December 2025. There were 1, 211 valid observations after the processes of removing missing and anomalous data. It is a time that the study has seen various patterns of volatility of the markets at different stages and thus high levels of representativeness. The sample period has several market characteristics such as uptrends, downtrends, and sideways and adequate time to capture possible long-memory issues in financial time series. The rate of 10-year government bonds in China is used to get the risk-free rate, at which the yield is approximate.

To remove any non-stationarity exactly in the price data, and because the paper aims at carrying out statistical analysis, the target of the study is the log returns of the asset. Relevant indicators are as follows: The underlying asset price will be defined as the close price of Shanghai Composite Index at time t. Second, the log returns are additive and thus consistent with financial modeling. These log returns are represented as:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

Historical volatility is a measure of the strength of changes in price of assets. The Hurst index is an indicator to measure the memory behavior of a time series. Although H=0.5 equals a conventional random walk process, values of 0.5 H 1 are persistent in the time series. The distribution properties of the returns are described using descriptive statistics in this paper. Also, J-B test is used to know whether the sequence of returns is normally distributed. The findings with a significant rejection by the J-B statistic to the null hypothesis imply that the market has fat-tail nature thereby invalidating the assumptions of the standard B-S model. In order to resolve the long-memory properties within the SSE Composite Index, the Hurst index is estimated with the relabeled range analysis. Linear regression is carried out on data that are plotted in the double-log scale, which is based on empirical formulae.

3 Results and Discussion

3.1 Descriptive Statistics and the Test of Normality

The paper tracks the fluctuations of SSE Composite Index price in the sample. Instead of having the nature of a stationary random walk, the index prices were highly trendy and volatile. At the same time, the market also featured strong so-called volatility-clustering: high volatility was found to be followed by high volatility, and low volatility was followed by low volatility.

This non-linear dynamic property implies that there are complicated autocorrelation structures of the market, which the simple geometric Brownian motion assumption would otherwise not can accommodate. As a test of the major assumption of the B-S model that logarithmic returns are normally distributed, 8 this paper performs the statistical analysis of the logarithmic returns of the SSE Composite Index.

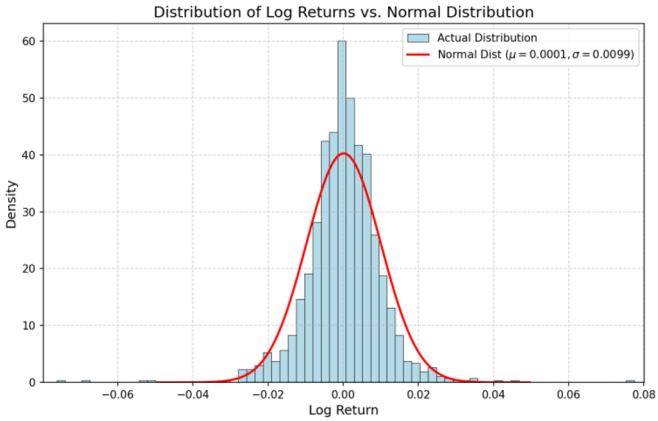


Fig. 1. Comparison of standard normal distribution and Log-Return distribution.

The actual return of distribution is visually compared to the standard normal distribution as shown in Figure 1. The blue histogram is the frequency distribution of actual sample returns, and the red curve is the probability density function of the standard normal distribution that is being fitted depending on the sample standard deviation and the sample mean.

The true sequence of returns cannot be properly represented by a normal distribution, and to which it displays strong leptokurtosis and fat tails. Whereas in the central area of the distribution, the actual returns density is much greater than the normal distribution maximum, stable trading days in the market are the main, and those fluctuation cases are much more common than stipulated by the normal distribution. Actual data of occurrences at the two ends of the distribution have greater likelihood of occurrence indicating that traditional models under owes probability of the occurrence of the black swan events or extreme market conditions.

The sample skewness value (0.3750) demonstrates that the skewness of a given sample is a right skewed distribution with a somewhat unequal tendency to move at extreme values that is more pronounced, as compared to what is otherwise easy to think is equal in terms of probability on both sides, that is emphasized in the normal distribution. The gross kurtosis stands at 9.3654 which is far above the normal value of 0 indicating that the returns are concentrated about the mean and the probability of extreme values is more pronounced. The J-B test outcome indicates a statistic of 4454.0876 and a p-value which is much less than 0.01. The null hypothesis that the returns follow a Gaussian distribution is rejected. The B-S model where normal distribution is used does not embody the extreme volatility, and option tail risks are grossly underestimated, introducing some price bias. Thus, it becomes logical and desirable to introduce a model of a fractional Brownian motion.

3.2 Empirical Long-Memory Properties Test of the Market

The Hurst exponent of Shanghai Composite Index was obtained as the relabeled range method to measure nonlinear behavior of the market. Various time spans n was calculated and then the corresponding $(R/S)_n$ values were used in the linear regression analysis using a two-logarithmic coordinate system.

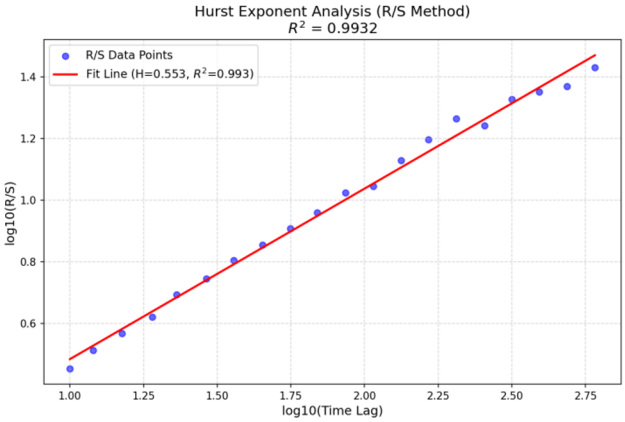


Fig. 2. Two-log regression plot of R/S analysis

Figure 2 indicates that the relationship between $ln(R/S)$ and $ln(n)$ show very linear relationship with the regression goodness of fit of 0.9932 and this implies that the result of the estimation is very reliable. The regression line slope is the Hurst exponent, H=0.5529.

It was analyzed that SSE Composite index returns are not distributed normally, and the Hurst exponent of the analyzed data was significantly larger than 0.5. This continuity proves the fact that the SSE Composite Index is not trading under regular brownian motion, but fractions brownian motion. Thus, the addition of the parameter of the Hurst exponent to option pricing can theoretically more sufficiently reflect the actual structure of market volatility.

The fractal market hypothesis provides that a market of H=0.5 is characterized by a random walk. When 0.5 This is persistence and it proves the fact that SSE Composite Index is not controlled by standard Brownian motion but fractional Brownian motion. Thus, using the Hurst exponent parameter as an element of the option pricing theoretically allows more precisely expressing the actual structure of volatility in the market.

3.3 Simulation and Comparison of the Pricing models of the options

There are two price models; (1) In the traditional black-Scholes model, the price of the underlying is assumed to be a geometric Brownian motion of fixed volatility. (2) Fractal Brownian motion model: According to the derivation of Necula, the conventional volatility shall be modified to a time-dependent process under the consideration of

market long memory. This model has more theoretical merits of employing nonlinearity of the market [6].

In order to measure the particular effects of the long-range memory in the option pricing, they were simulated numerically using the B-S model and the fBS model, with parameters determined previously.

Conventional B-S model presupposes volatility diffusion with the rate of $\sqrt{T}$ (i.e.,H=0.5). The volatility term structure changes under the fractional Brownian motion to T^H. This means that at H> 0.5, long-term volatility becomes accumulated more rapidly than as assumed in the traditional assumption. The simulation parameters will be established as below: To begin with, the underlying asset price (S_0) is the closing price of the end of the sampling period (December 31, 2025). In parallel with it, the risk-free rate of 1.83% (r) is mentioned; The standard deviation is transformed into the annualized volatility, which is 15.73 percent (σ): The Hurst exponent with the help of in the result of the above R/S analysis is $H \approx 0.5529$.

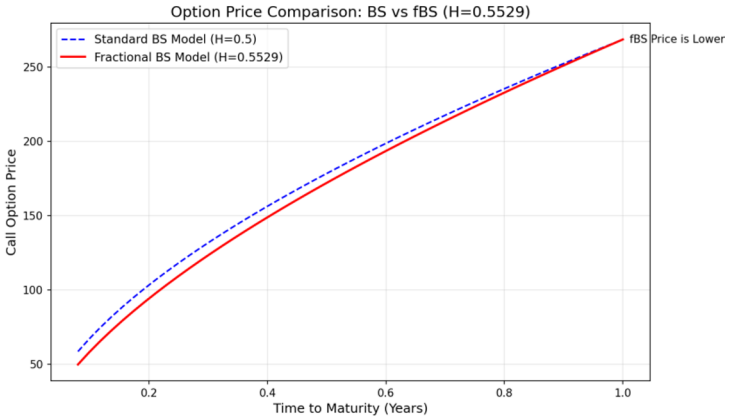


Fig. 3. Comparison of the option prices between various maturities (BS vs. fBS)

As illustrated in Figure 3, in the one-year time window all prices of the option based on the fBS model (red) are at all times lower than the traditional Black-Scholes model (blue). This means that in the short-term outlook comprising of less than one year, trend memory markets experience reduced uncertainty in the price movements and less noise as opposed to totally chaotic random walk markets.

Therefore, the fBS model willer realize slower accrual of actual risk in this period, thus the price of options is cheaper as compared to the standard model. The classical B-S model is also prone to overpricing the time value of options with short-term (T < 1) options. The fBS model removes this pricing bubble by rectifying the term structure of volatility due to disregard of the long-term memory of the market, which would give a more accurate pricing reference point.

The at-the-money option prices versus the remaining time to expiration shown in Figure 4 demonstrate the dependence on the remaining time to expiration where the long-term memory effects are the strongest.

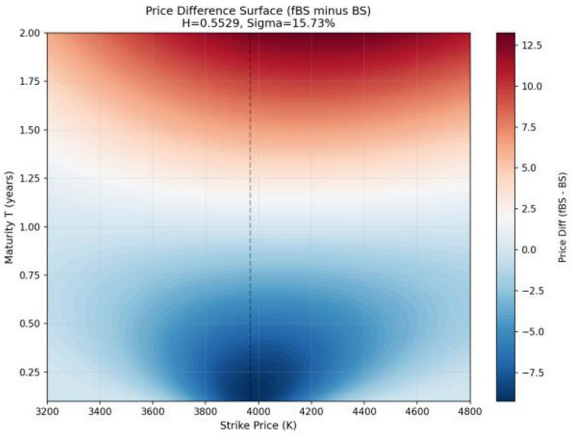


Fig. 4. Pricing disparity heatmap

With a short time horizon (T < 1 year), In general, the values obtained with the fBS model are lower than the values obtained with the conventional B-S framework. Despite the persistence in the market, between the time interval T < 1, the factor of volatility diffusion is numerically less than in the assumption of random walks. In the short run, fBS model is that the market noise effects are less important in trend and systemic risk is accrued slower than other traditional models, wallowing in a comparatively lower risk pricing.

The fBS model has a non-linear increase in prices over the conventional B-S approach as the maturity increases; in fact, the difference between these two approaches is significantly higher in the case of the fBS model, with these heights increasing with the length of maturity.

Since H> 0.5, the rate at which the volatility accumulates with time of a fractional Brownian motion accumulation increases at a greater rate than that of traditional models that accumulate time T. In the long run, tail risks are exaggerated and result in greater pricing by the fBS model and nonlinear growth in the implied volatility over time. Such an outcome is a strong indication that neglected long-memory properties in markets ultimately result in the traditional models grossly undervaluing the value and risk of long-term options.

Considering unknown long-memory (H>0.5): in standard models under the accurate assumption that the long-memory effect is ignored, they will slightly overvalue option value in the near term and grossly undervalue tail risk in the far-off period. The fBS model represents a significantly more realistic pricing aid of long-term investment and risk hedging by repricing the 50/50 long-run contracts and using the Hurst coefficient to rectify the term-structure of volatility, which represents more fully certain risks associated with trend-amplification.

4 Conclusion

The present paper discusses the long memory nature of financial time series with reference to the empirical information presented through the SSE Composite Index and how option prices can be obtained through the implementation of the long memory property. Both descriptive statistics and normality tests show that the returns on the Chinese stock markets have major characteristics of heavy tails that do not conform to the principle of normal distribution. At the same time, R/S analysis shows that there is a Hurst index of about 0.5529, which indicates the presence of significantly strong long-memory characteristics in the market where there is neither random walk nor background of price change. As option pricing simulation indicates, the long-memory parameter conclusively has a significant effect on the value of the long-term derivatives. The fractional B-S model gives better valuations of long-term options as compared to the traditional B-S model.

In conclusion, the results of mechanistic application of the classical B-S model to fractal markets can result in bias in pricing and the wrong estimation of risk exposures. Long-term risk management has been better served by offering better theoretical resources to investors through the introduction of Hurst-Index-adjusted price models which better reflect the changes of market over time which are nonlineal by nature.

The research possesses a number of limitations that need to be tightened in future: Firstly, the sample is limited to the Shanghai Composite Index, and not to different industries, small-cap indices, or cross-border market markets. This can restrict the generalizability of the findings to heterogeneous market settings as well as its ability to fully capture the long-memory behavior across asset classes. Second, the long-memory estimation is based purely on the re-scaled range analysis technique which can be biased in short time samples. The lack of cross-validation might undermine the strength of the estimates of Hurst exponents. Third, the process of option pricing simulation ignores market real-life friction, and the comparisons made are on model assumptions. Such a distortion of the actualities of financial markets weakens the practicality of the model.

To counter these shortcomings, future studies may increase as follows: First, a wider cover can be given to them, such as the ChiNext Index, STAR Market Index, industry classification indices and most international stock market indices. This would contrast the power of the long-range memory in the various market arrangements and features of any asset, which would still indicate the universality of the fractional B-S model. Second, maximize techniques in the measurement of the long-range memory by cross-validating memory, Hurst exponents, through a number of techniques, including modified R/S method, local Whittle estimation and wavelet transformation. Carry out sensitivity studies that determine the effect of various approaches on the pricing results and improve the strength of research findings. Third, this study increases the more realistic nature of the pricing model, by adding market friction factors to its fractional B-S model. This builds a longer model more capitalistic to actual trading conditions. The practical value of the guidance of the model is enhanced by empirical testing of the model using the actual real option transaction data.

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