



Analysis of the Pricing of SSE 50 ETF Options Based on Monte Carlo Simulation

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Abstract. Reasonable pricing is the key to option trading and stable market operation. As the first listed option in China, SSE 50ETF option's pricing accuracy is very important for investors' decision-making and market development. Taking SSE 50ETF's daily closing price from January 2016 to December 2025 as a sample, this paper describes the volatility characteristics by GARCH model and combines Monte Carlo simulation (100,000 simulations) to price SSE 50ETF European options. The results show that the logarithmic return of SSE 50ETF shows the characteristics of peak and thick tail, volatility concentration and ARCH effect, and GARCH (1,1) model has excellent fitting effect, and the annualized volatility is 18.85%. Monte Carlo simulation pricing conforms to the law of option price. After optimization by dual variance reduction technology, the average absolute error is 0.10283, and the overall root mean square error is reduced to 0.0090, which is within the acceptable range of the industry. This study verifies the effectiveness of GARCH-Monte Carlo combination method, provides quantitative support for investor arbitrage, market maker quotation and market price discovery, and helps the stable development of stock index option market.

Keywords: GARCH Model, Monte Carlo Simulation, Option Pricing.

1 Introduction

The SSE 50 ETF option was listed in February 2015. As the first listed option in China, its significance within the country is self-evident. Options, as an important financial derivative, have functions such as price discovery, hedging, and speculation. The implied volatility and strike price distribution in the option contract directly reflect the market's expectations for the future fluctuations of the underlying assets.

As a critical financial derivative traded in markets, the price of an option constitutes an indispensable component of market economic pricing. A rationally determined option price contributes to enhanced stability in the options market. Therefore, the proper valuation pricing of options is the primary issue in option trading. This paper provides a basis for market investors to make optimal investment strategy decisions and manage risks through the study of option pricing and offers scientific quantitative tools for the stable operation and innovative development of China's stock index option market.

In the field of derivative pricing, in 1973, American scholars Myron Scholes and Fischer Black derived the well-known B-S option pricing model [1]. In the 1980s, Engle proposed the ARCH model. The ARCH model addresses the issue of heteroscedasticity by using a linear combination of the lagged squared residuals to depict the conditional heteroskedasticity and then uses this for modeling [2]. Bollerslev further expanded this concept into the generalized ARCH (GARCH) model. By introducing the autoregressive term of conditional variance, it improved the fitting effect on the clustering characteristics of financial time series volatility [3]. Bollerslev further expanded this concept into the generalized ARCH (GARCH) model. By introducing the autoregressive term of conditional variance, it improved the fitting effect on the clustering characteristics of financial time series volatility [3]. In 2012, Cheng Songlin et al. proposed the Monte Carlo stochastic simulation method for option pricing, which could better solve the pricing problems of derivative securities compared to other numerical methods. Meanwhile, the randomly selected numbers play a significant role in determining the accuracy of the Monte Carlo stochastic simulation used for option pricing [4].

Based on the inspiration from the previous achievements, this paper proposes a research method for predicting the logarithmic return volatility of the SSE 50 ETF based on time series. Taking the data from the last 10 years as the sample, an empirical study was conducted, a GARCH model was established, and the changing patterns and trends of the model were predicted. Subsequently, the Monte Carlo simulation method was employed to simulate the options price, and the pricing results were analyzed.

2 Research Method

This paper selects the European options of the SSE 50 ETF as the research object of this study. During the initial stage of the listing of the SSE 50 ETF options, abnormal fluctuations may occur in the market. To avoid this situation, the study selected the daily closing prices of the Shanghai 50 ETF from January 4, 2016, to December 4, 2025, with a total of 2,411 samples. The data was sourced from the Wind database. In this paper, the closing prices of the SSE 50 ETF obtained were processed using the logarithmic difference method, and their logarithmic returns were obtained.

"Autoregressive Conditional Heteroskedasticity Model" is the full name of the GARCH model. Its ability to accurately predict the volatility in financial time series data is the advantage of the GARCH model, and it is particularly suitable for data containing volatility clusters [5]. A volatility cluster refers to a situation where the volatility in the financial market remains consistently high or low for an extended period. The GARCH model is capable of effectively capturing this characteristic. Furthermore, the GARCH model also exhibits excellent statistical characteristics. This model is based on actual observational data and uses the maximum likelihood estimation method to estimate and predict its parameters. Theoretically speaking, GARCH type models can make full use of a large amount of historical data to fit the model and thus have relatively high prediction accuracy.

Monte Carlo simulation is a method that approximates the overall expectation of model and distributed set by using the sample mean of random samples.

3 Research Outcome

3.1 Descriptive Statistic

As can be seen from Table 1, the kurtosis value of 8.8247 in Table 1 is greater than 3, and the skewness is negative, indicating that the distribution of logarithmic returns is left-skewed and has a thicker tail. The Shapiro-Wilk test indicates that it does not conform to a normal distribution.

Table 1. Descriptive statistics of the logarithmic return of the SSE 50 ETF.

Statistical quantity	Numerical value	Statistical quantity	Numerical value
Mean value	0.0002	Standard deviation	0.0117
Sample size	2410	Skewness	-0.0836
Minimum value	-0.0776	Maximum value	0.0848
Kurtosis	8.8247	Jarque-Bera	Significant deviation from normal distribution

3.2 Stationary Test

Before conducting the modeling, first, test whether the sequence of daily logarithmic return rates is stationary. If it is not stationary, the series needs to be reprocessed until it becomes stationary before proceeding with the modeling. This paper uses the ADF test to assess the stationarity of the sequence. This test method is used to check whether there is a trend in the time series that changes over time, and it is often employed in the stationarity test of time series [6,7].

The ADF test results indicate that the data can reject 1% significance level of the null hypothesis. Therefore, it can be concluded that the logarithmic daily return series is stationary and meets the prerequisite conditions for conducting GARCH modeling.

3.3 Autocorrelation Test

The partial autocorrelation coefficient (PACF) and the autocorrelation coefficient (ACF) of the logarithmic return rate of the SSE 50 ETF with a lag of 20 periods were calculated.

As shown in Figure 1, the q value begins to increase sharply from the 6th order, indicating the presence of autocorrelation. All subsequent analyses assume the existence of autocorrelation.

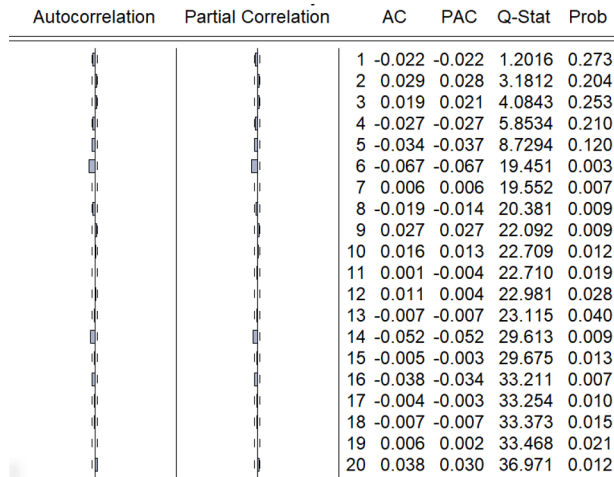


Fig. 1. Autocorrelation and partial autocorrelation diagrams.

3.4 ARCH Effect Test

In order to determine whether the GARCH model can be used to model and predict the volatility of the log return sequence of the options, this paper establish a mean equation for the log return sequence of the Shanghai 50 ETF and conduct an ARCH effect test on the residuals of the mean equation for the log return sequence of the SSE 50 ETF.

The P-value corresponding to Obs*R-squared under the ARCH test indicates that it rejects the null hypothesis, suggesting the presence of the ARCH effect, namely, heteroscedasticity. Thus, an ARCH model and a GARCH model can be established.

3.5 Estimation of GARCH Model

The volatility clustering characteristics of financial time series can usually be effectively characterized by the GARCH (1,1) model, and this model has simple parameters and strong interpretability, which meets the practical requirements of option pricing [8,9]. Therefore, the GARCH (1,1) model is used to model the volatility. The estimated parameter results of the model are as follows:

Table 2. GARCH (1,1) model

Variance Equation				
C	0.00000442	0.00000058	7.6251	0.0000
RESID (-1) ^2	0.0917	0.0049	18.6328	0.0000
GARCH (-1)	0.8769	0.0008	114.3934	0.0000

Table 2 presents the GARCH (1,1) model's parameter estimation results. The model passed the significance test for the parameters at the 5% significance level. The sum of the parameters is close to 1, indicating that the model is effective in the long term. The volatility equation of the model is:

$$\sigma_t^2=0.00000442+0.8769\sigma_{t-1}^2+0.0917\varepsilon_{t-1}^2 \quad (1)$$

According to the long-term variance formula, it can be known that:

$$V_t=\frac{\omega}{(1-\alpha-\beta)} \quad (2)$$

Here, $\alpha=0.0917$, $\beta=0.8769$, $\omega=0.00000442$. Therefore, the SSE 50 ETF's long-term variance of the logarithmic return is 0.000141. Assuming there are 252 trading days each year, the annualized volatility of the SSE 50 ETF is 18.85%.

Conduct an ARCH effect test on the residual sequence of the model. The p-value indicates the heteroscedasticity has been eliminated.

From the above text, the GARCH (1,1) model's volatility equation can be obtained as follows:

$$\sigma_t^2=0.0000042-06+0.8769\sigma_{t-1}^2+0.0917\varepsilon_{t-1}^2 \quad (3)$$

From this, this paper can obtain the specific volatility sequence from January 5, 2016, to December 4, 2025. The line graph of this sequence is shown in Figure 2.

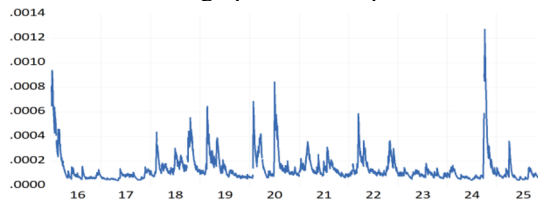


Fig. 2. Volatility sequence chart

The meaning of its volatility sequence is 0.0001, the minimum value is 0.0000429, the maximum value is 0.0013, the standard deviation is 0.0001. The volatility of the SSE 50 ETF exhibits a distinct clustering characteristic, which is consistent with the fitting assumptions of the GARCH model, laying the foundation for subsequent pricing.

3.6 Monte Carlo Simulation Model Construction and Result Analysis

This paper selects the December contract of the 2025 SSE 50 ETF option as the research object for empirical analysis. The first trading date of this contract is April 24, 2025, and the expiration date is December 24, 2025. The feature of European options is that they can only be exercised on the exercise date. The parameters of the Monte Carlo simulation are as follows:

Strike Price: The strike price range for the December option contract is 2.5 - 3.6, with a total of 17 different exercise prices. Due to space limitations and for ease of analysis, this article only selects the call options with exercise prices ranging from 2.5 to 3.1 and the put options with exercise prices ranging from 3.1 to 3.6 for research. **The price of the underlying asset S0:** On December 4, 2025, the closing price of the SSE 50 ETF was 3.116. **Risk-free interest rate:** In China, market-making system is adopted for option trading. To reflect the actual trading situation in the country, the one-year

SHIBOR rate (Shanghai Interbank Offered Rate) of the same year is used in this paper to estimate the risk-free interest rate. The one-year Shibor rate on December 4, 2025, was 1.65%. Stock volatility standard deviation σ : The annualized volatility of the CSI 50 ETF is 18.85%. Option expiration date T: The actual trading days of the option within one year are approximately 252 days. By annualizing the remaining trading days before the contract expiration, the option validity period can be obtained. As of today (December 4, 2025), the calculation method for the remaining validity period is to conduct an annualization calculation based on the remaining 14 trading days. $T=14/252=0.0556$. Simulation times Nu: The number of simulations for the selected option is 100,000. The simulation was conducted 100,000 times. As it was found through the convergence test, the standard deviation of the pricing results was less than 0.001, when the number of simulations exceeded 50,000, indicating a stable trend.

Table 3. Option pricing results from Monte Carlo simulation as of December 4th

Option contract code	Option type	Strike price	Simulated price	Antithetic variates	Actual price
10009217	Call option	2.5	0.6182	0.6178	0.616
10009218	Call option	2.55	0.5683	0.5682	0.5672
10009219	Call option	2.6	0.5185	0.5181	0.5174
10009220	Call option	2.65	0.4682	0.4681	0.4675
10009221	Call option	2.7	0.4188	0.4182	0.4176
10009222	Call option	2.75	0.3689	0.3683	0.3679
10009223	Call option	2.8	0.3195	0.3188	0.318
10009224	Call option	2.85	0.2695	0.2695	0.267
10009225	Call option	2.9	0.2221	0.2217	0.2195
10009311	Call option	2.95	0.1754	0.1753	0.1702
10009317	Call option	3	0.1330	0.1324	0.1225
10009497	Call option	3.1	0.0648	0.0649	0.0413
10009498	Put option	3.1	0.0463	0.0461	0.0239
10009624	Put option	3.2	0.1053	0.1054	0.09
10009728	Put option	3.3	0.1876	0.1875	0.184
10009790	Put option	3.4	0.2823	0.2828	0.284
10009814	Put option	3.5	0.3806	0.3811	0.384
10010172	Put option	3.6	0.4811	0.4810	0.484

As can be seen from Table 3, the exercise price of a call option moves in the opposite direction to the option price. It conforms to the changing pattern between the intrinsic

value of the option and the exercise price, indicating that the simulation pricing of call options is reasonable. The exercise price of a put option moves in the same direction as the option price. It conforms to the changing pattern between the intrinsic value of the option and the exercise price, indicating that the simulation pricing of the put option is reasonable. Furthermore, from the results of the Monte Carlo simulation for option pricing, it can be seen that the Monte Carlo simulation has relatively large errors for option contracts with smaller prices [10].

3.7 Analysis of Pricing Result Errors

In order to comprehensively analyze the results of the pricing accuracy based on the GARCH volatility estimation model, the pricing effect is calculated by conducting error analysis on the simulated prices obtained from each model. Specifically, this is done through the calculation methods of root mean square error and average absolute error, to measure the errors between the option prices simulated by each model and the corresponding actual option prices [11].

Table 4. Pricing error

Option type/evaluation indicator	Root Mean Square Error	Mean Absolute Error
Call option	0.00772	0.06049
Call option (after Antithetic variates)	0.00763	0.05950
Put option	0.01134	0.19127
Put option (after Antithetic variates)	0.01126	0.18950
Overall	0.00909	0.10409
Overall (after Antithetic variates)	0.00900	0.10283

As can be seen from Table 4, the error in pricing call options is smaller than that in pricing put options. This is because the number of put options selected in this study is relatively small, and the prices of some of these options are relatively low. It can be seen that the paired variance reduction technique effectively reduces the pricing error, making the Monte Carlo simulation of option pricing closer to the actual value. This paper also found that there are general errors in the option pricing using Monte Carlo simulation. This is because the errors in Monte Carlo simulation stem from the inherent characteristics of random sampling. Although they cannot be completely eliminated, by increasing the number of simulations (100,000 times), the errors have been controlled at a relatively low level, which does not affect the practicality of the pricing results.

4 Conclusion

Since the development of option derivatives, option pricing has always been one of the most crucial aspects in option trading. Therefore, the importance of using a model to

price options is self-evident. A simulation method for the SSE 50 ETF options. During the process of modeling volatility, the long-term variance of the GARCH model is solved and incorporated into the Monte Carlo simulation for option pricing. In the simulation pricing of the SSE 50 ETF, the risk-neutral pricing principle and geometric Brownian motion are used to suggest the Monte Carlo simulation method. The pricing errors in this study are all less than 5%, which fall within the acceptable range in the industry. They can provide references for investors' trading decisions and market makers' quotations and indirectly promote the improvement of market price discovery efficiency.

The conclusions drawn in this paper are as follows: Firstly, the log return of the SSE 50 ETF exhibits a phenomenon of sharp peaks, heavy tails and volatility clustering, and this sequence also has an ARCH effect. Therefore, a GARCH volatility estimation model can be established for it. Second, the exercise price of a call option moves in the opposite direction to the option price. The exercise price of a put option moves in the same direction as the option price.

Meanwhile, the model of Monte Carlo simulating option pricing also has limitations. For instance, the simulation results are highly dependent on the accuracy of the data, and incorrect model parameters can lead to erroneous prices obtained through precise calculations. And there will be random errors that occur. The simulation results themselves have statistical errors, which need to be controlled by conducting a large number of mini-iterations and variance reduction techniques. In further research, the fractional Brownian motion can be combined to better describe the sample distribution situation, and the study can also focus on how to use Monte Carlo simulation for option pricing to manage risks.

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