The dynamic properties of mine waste rock

Feihong Shang[1*]

[1]Central Research Institute of Building and Construction Co.,Ltd, MCC Group, Beijing 100088, China
2145256814@qq.com

Abstract. This paper systematically discusses the dynamic properties of mine waste materials, with a focus on the application of the equivalent viscoelastic model in the seismic analysis of tailings dams built from waste materials. The article first introduces the Hardin-Drnevich hyperbolic model and its suitability for use with coarse-grained materials. It then analyzes the methods for determining the maximum dynamic shear modulus (G_{max}), including the Seed formula and the Hardin formula. Based on this, the paper examines the relationship between the decrease in dynamic shear modulus and the increase in damping ratio in waste materials. It also compares the differences between waste materials and hydraulic rockfill materials in terms of particle size distribution, stress history, density, and dynamic properties. The study shows that the dynamic properties of waste materials are significantly influenced by factors such as confining pressure, fine particle content, and porosity. However, the lack of experimental data limits the ability to develop more precise seismic design methods. Finally, the paper calls for increased systematic research on the dynamic properties of waste materials, the establishment of a parameter database, and efforts to standardize their engineering applications.

Keywords: Waste rock, Equivalent viscoelastic model, Dynamic shear modulus, Damping ratio, Rockfill.

1 Introduction

In recent years, waste-rock centerline tailings dams, in which mine overburden waste rock is substituted for conventionally classified tailings as the principal construction material, have drawn growing interest for high tailings storage facilities in seismically active areas. This dam type merges waste rock recycling with tailings impoundment construction, simultaneously consuming solid mining waste and markedly improving the seismic resistance of the dam. Dynamic response and stability analyses of such structures under earthquake loading rely heavily on an accurate characterization of the dynamic properties of the mine waste rock.

In the field of hydraulic rockfill dams, the equivalent viscoelastic model has gained widespread acceptance—covering dynamic testing of rockfill for concrete-faced rockfill dams, model parameter calibration, and finite element seismic response analysis—thus forming a well-established technical pathway. This model was

© The Author(s) 2026
H. Wu et al. (eds.), *Proceedings of the 2026 3rd International Conference on Civil Engineering Structures and Concrete Materials (CESCM 2026)*, Advances in Engineering Research 312,
https://doi.org/10.2991/978-94-6239-865-8_5

formally developed by Hardin and Drnevich [1]. However, directly transferring it to the dynamic analysis of waste rock in centerline tailings dams poses several key challenges. Although both waste rock and hydraulic rockfill are coarse-grained materials, they differ systematically in particle size distribution, geological origin, fines content, and compaction state, and these differences inevitably manifest in their dynamic behavior.

At present, experimental investigations of the dynamic properties of waste rock remain significantly less advanced than those for rockfill. Systematic experimental determination of equivalent viscoelastic model parameters for waste rock is far from being as comprehensive as that for rockfill, and engineering practice often defaults to empirical analogies, limiting the level of refinement achievable in seismic design. Building on an exposition of the equivalent viscoelastic model principles, this chapter consolidates the dynamic behavior patterns of waste rock, thereby providing a basis for selecting parameter values in the subsequent dynamic response analysis.

2 Equivalent Viscoelastic Model

2.1 Hyperbolic Model

The core concept of the equivalent viscoelastic model is to equivalently decompose the nonlinear hysteresis behavior of soil under cyclic loads into an elastic component (energy storage) and a viscous component (energy dissipation), described by two parameters: the equivalent dynamic shear modulus G and the equivalent damping ratio λ. This model belongs to a linearized iterative approach: for a given dynamic shear strain amplitude, the nonlinear stress-strain hysteresis loop （See Figure 1） is equivalently represented as an ellipse with identical maximum shear stress and maximum shear strain, thereby determining the equivalent stiffness and equivalent damping corresponding to that strain level.

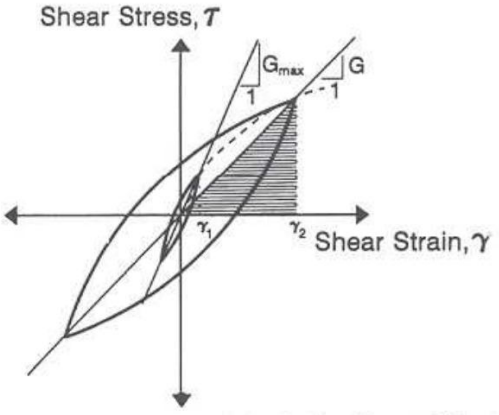


Figure 1. Nonlinear stress-strain hysteresis loop.

The mechanical equations for the equivalent viscoelastic model typically employ the Hardin-Drnevich hyperbolic model, with the G/G_{max}-γ curve and λ-γ curve serving as its core components. Subsequent researchers have continuously refined this model, enabling the hyperbolic approach to more flexibly fit various soil types. Through large-scale resonance column tests on gravel soils, Hardin and Kalinski demonstrated that the modified hyperbolic model with two parameters (a and b) better captures the G/G_{max} attenuation characteristics of gravel soils [2].

$$\frac{G}{G\max} = \frac{1}{1+\frac{\gamma}{\gamma_r}\left[1+a\exp^{(-b\gamma/\gamma_r)}\right]} \tag{1}$$

$$\frac{\lambda}{\lambda_{\max}} = \frac{\frac{\gamma}{\gamma_r}}{1+\frac{\gamma}{\gamma_r}} \tag{2}$$

Where G_{max} represents the maximum dynamic shear modulus; γ_r denotes the reference shear strain, physically representing the shear strain level when $G/G_{max} = 0.5$; and λ_{max} indicates the maximum damping ratio.

However, waste rock differs from standard gravels due to its high fines content and irregular particle shapes. These characteristics may not be adequately captured by existing models. Therefore, modifying the Hardin-Drnevich equations by incorporating additional coefficients is recommended to better represent energy dissipation mechanisms like particle fragmentation and intergranular friction.

2.2 Maximum dynamic shear modulus G_{max}

The maximum dynamic shear modulus G_{max} (also known as the small-strain shear modulus or initial shear modulus) is a key parameter characterizing soil stiffness under extremely low strain conditions ($\gamma < 10^{-5}$). Within this strain range, contact between soil particles undergoes primarily elastic deformation, while particle sliding and rearrangement have not yet occurred; consequently, the soil exhibits linear elastic behavior, with G remaining constant at G_{max}. The most fundamental method for determining G_{max} involves field or laboratory wave velocity measurements. Its physical basis stems from elastic wave propagation theory: in an isotropic elastic medium, the relationship between the shear wave propagation velocity Vs and the shear modulus G is given by:

$$G_{\max} = \rho \bullet V_s^2 \tag{3}$$

where G_{max} represents the maximum dynamic shear modulus; ρ denotes the density; and Vs indicates the shear wave velocity.

This relationship forms the physical basis for G_{max} measurement—whether using field methods such as the trans-hole method, downhole method, or MASW, or indoor

tests like the Bender Element or resonance column test — all involve determining shear wave velocity (Vs) and calculating G_{max} through density conversion, as shown in Table 1. Based on field measurements of the relationship between shear wave velocity and burial depth, Park and Kishida analyzed data from 28 fill dams via regression analysis and found that the shear wave velocity (Vs) of rockfill dam shells ranges from 150 to 540 m/s at depths of 0–40 m [3]. Iwashita reported shear wave velocities of 240–580 m/s for rockfill materials over a broader depth range[4].

Table 1. The burial depth of the shear wave velocity for rockfill materials.

Shear wave velocity (m/s)	Depth range (m)	Method	Material type	G_{max} (MPa)
200-500	0-35	In-situ	Rockfill	88-550
200-400	0-12	In-situ	Rockfill	88-352
240-580	0-50	In-situ	Rockfill	127-738
150-540	0-40	In-situ	Shell zone of rockfill	50-615

Building upon the wave velocity method for determining G_{max}, researchers have established empirical relationships between G_{max} and key soil physical parameters (such as confining pressure, porosity, and overconsolidation ratio) through extensive experimentation, enabling estimates when actual wave velocity data are unavailable. The two most representative formulas are the Seed formula and the Hardin formula.

The classical formula proposed by Seed and Idriss expresses the maximum dynamic shear modulus as:

$$G = 1000 \bullet K_2 \bullet \left(\sigma_m^{'}\right)^{1/2} \tag{4}$$

where G represents dynamic shear modulus, measured in psf; K_2 denotes the modulus coefficient; and σ_m' ndicates the average effective principal stress, also measured in psf.

Seed demonstrated through large-scale cyclic triaxial tests that the $K_{2,\,max}$ value for gravelly soils ranges from 80 to 180, which is 1.35 to 2.5 times higher than that for sandy soils (30–75).

Through systematic resonance column tests, Hardin and his collaborators developed the G_{max} expression that accounts for the effects of void ratio and over consolidation ratio. This formula was first introduced by Hardin and Black in their study on the vibration modulus of normally consolidated clay, and later refined by Hardin to apply to various soil types. The calculation formula is as follows:

$$G_{\max} = 1230 \bullet \frac{(2.973 - e)^2}{1 + e} \bullet (OCR)^K \bullet \left(\overline{\sigma}_0\right)^{1/2} \tag{5}$$

where e denotes the porosity ratio; this equation characterizes the effect of density on G_{max} through the porosity ratio function F(e). OCR represents the overconsolidation ratio, while K is an index associated with the plasticity index PI.

When PI = 0 and K = 0, the maximum compressive strength (G_{max}) of non-plastic soils (such as clean sand or gravel) is unaffected by OCR. As PI increases, the K value rises, and OCR's enhancing effect on G_{max} becomes more pronounced. Hardin noted that this formula applies to soils with a void ratio e within the range of 2.0; for e > 2, the formula may yield underestimated predictions. The constant coefficient 1230 in the formula was derived from regression analysis of experimental data at the time, and its value has been adjusted in subsequent studies depending on soil type and testing method. Hardin and Blandford further developed an elastic modulus expression within a three-dimensional constitutive framework, introducing the stiffness coefficient S_{ij} to differentiate between various soil types and stress states, thereby enabling more flexible application of the Hardin formula across diverse soil conditions.

Both formulas share the same fundamental logic: they both assert that G_{max} is proportional to the square root of the average effective stress and increases with increasing density. The primary distinction lies in their approaches: Seed's formula is concise, using a single parameter K_2 to comprehensively account for multiple influencing factors, making it more widely applicable in engineering; Hardin's formula has a more rigorous theoretical foundation, enabling it to capture the independent effects of parameters such as void ratio, overconsolidation ratio, and plasticity index, with subsequent researchers further expanding its parametric physical implications.

In summary, the primary factors influencing G_{max} include:

(1) confining pressure. Confining pressure not only affects the absolute value of Gmax but also influences the shape of the G/Gmax–γ curve—increasing confining pressure shifts the curve upward[5]. Zhu further confirmed this trend through experiments on 12 types of gravel under confining pressures ranging from 227 to 4000 kPa [6].

(2) Porosity. Due to the characteristics of mining blasting and transportation accumulation, waste rock typically exhibits high and uneven initial porosity. Tests on Ok Tedi waste rock revealed that the initial loose porosity range of samples was 0.38–0.46, with average porosities before compression and shearing reaching 0.25 (unsaturated) and 0.20 (saturation), respectively[7].

(3) Density. The higher the density of a sample, the greater the G_{max} value and the lower the damping ratio; loose waste rock exhibits faster stiffness decay and a more significant increase in damping ratio under cyclic loading.

(4) Particle size distribution and fine particle content. Fine particles can reduce G_{max} by weakening direct contact between coarse particle skeletons, while simultaneously increasing the damping ratio due to enhanced energy dissipation from frictional sliding between fine particles.

3 The G/G_{max}-γ curve and the λ-γ curve for waste rock

By normalizing the G-γ and λ-γ curves under different confining pressures using G_{max} and λ_{max} respectively, the normalized backbone curves of the material can be obtained. Extensive experiments have demonstrated that for a given material, the normalized curves under different confining pressures largely overlap within a narrow band, facilitating reduced testing requirements in engineering applications. Research on the G/G_{max}-γ and λ-γ curves of waste rock materials remains relatively limited; studies have therefore drawn upon analyses of similar gravel-like materials.To overcome this shortage, more extensive laboratory testing is needed under varying confining pressures, particle size distributions, and saturation conditions. Such systematic experimentation would allow the construction of a reliable parameter database for the maximum shear modulus (G_{max}) and damping ratio of waste rock – something that is currently missing. Moreover, adopting standardized testing protocols would facilitate cross-study comparisons and enable the development of empirical relationships for seismic design.

Existing dynamic tests on waste rock materials demonstrate that the dynamic shear modulus G decreases with increasing dynamic shear strain γ under cyclic loading, exhibiting a typical nonlinear behavior. Within the narrow strain range ($\gamma < 10^{-5}$), G approaches its maximum value G_{max}; as shear strain increases further, the ratio G/G_{max} declines progressively, indicating gradual reduction in material stiffness. For the damping ratio curve, the D-γ curve shifts downward with rising confining pressure—meaning higher confining pressures result in lower damping ratios at identical shear strains. The equivalent damping ratio λ of waste rock exhibits an initial rapid increase followed by a slower decline as γ rises. In the range $\gamma < 10^{-4}$, λ primarily reflects small-strain damping effects; when γ increases to 10^{-4}–10^{-2}, energy dissipation mechanisms involving intergranular frictional slip and interlocking dislocation gradually become active, leading to a sharp rise in λ; after $\gamma > 10^{-2}$, the growth rate slows significantly.

Seed established the G/G_{max}–γ relationship curve for gravelly soils through cyclic triaxial tests, demonstrating that the normalized shear modulus attenuation curve of gravel exhibits a slightly gentler slope compared to sand, while its damping ratio curve range closely resembles that of sand[8]. In 1998, Rollins synthesized approximately 980 data points from 15 studies to derive mean curves and ±1 standard deviation ranges for both G/G_{max}–γ and D–γ relationships of gravelly soils [9], later refining these models in 2020 using broader experimental data from 18 studies [10]. The core improvement lies in recognizing that gravel's dynamic behavior depends not only on shear strain but is significantly influenced by confining pressure and nonuniformity coefficients: increased confining pressure shifts the dynamic modulus attenuation curve upward and to the right (resulting in greater linearity and stiffness), whereas elevated Cu values move it downward and to the left (enhancing nonlinearity and susceptibility to shear modulus attenuation at low strains). Building upon Stokoe [11] hyperbolic formulation, Rollins defined the reference shear strain γ_{ref} as a function of confining pressure and Cu. The generalized formula for the improved dynamic modulus attenuation curve is as follows:

$$\frac{G}{G_{\max}} = \frac{1}{1+\left[\frac{\gamma}{0.0046(Cu)^{-0.197}(\sigma_0')^{0.52}}\right]^{0.84}} \tag{6}$$

where reference shear strain $\gamma_{ref} = 0.0046(Cu)^{-0.197}(\sigma_0')^{0.52}$。

Regarding the damping ratio curve, Rollins '1998 model relied solely on an empirical fitting formula for shear strain and failed to account for copper's direct influence on the damping ratio, resulting in an unclear physical mechanism. To address this, Rollins proposes a new regression equation incorporating γ, Cu, and σ_0'.

$$D = 26.05\left(\frac{\gamma}{1+\gamma}\right)^{0.375} C_u^{0.08}\sigma_0'^{-0.07} \tag{7}$$

The approach demonstrates superior prediction accuracy and clearly elucidates the physical principles: the damping ratio increases with higher Cu content (due to increased particle contact points caused by complex gradation) and decreases with higher confining pressure (due to greater soil density). This finding represents a fundamental improvement over the 1998 model, indicating that the dynamic behavior of gravel should be understood as a composite function of strain, confining pressure, and gradation rather than solely dependent on strain. It is noteworthy that Rollins demonstrated through the analysis of gravelly soils that the fine particle content had minimal impact on the shapes of the G/G_{max}-γ and D-γ curves within the range of 0–9%. However, this conclusion was drawn for cases with relatively low fine particle content. Exercise caution when extrapolating these results to high fine particle concentrations, as deviations from the mean may occur.

To account for the impact of fine particle content, a parameter called "fine content index" has been introduced, representing the weight percentage of fine particles.The γ_{ref} should be influenced by the fine particle content. The modified reference shear strain formula is as follows:

$$\gamma_{ref,mod} = \gamma_{ref,Rollins} \bullet \frac{1}{1+\alpha \bullet f_c} \tag{8}$$

Where f_c is the fine particle content index; α is material parameter, representing the sensitivity of modulus decay to fines content.

Zhu conducted experiments on 12 types of gravel within an average effective stress range of 227 – 4000 kPa; the results are shown in Figures 2 and Figures 3, demonstrating that higher confining pressures correspond to slower rates of modulus decay. The overall experimental data fall within the ranges recommended by Seed and Rollins[8].

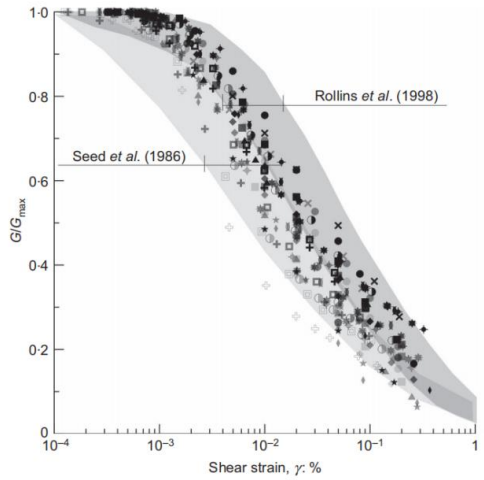


Figure 2. The G/G_{max}-γ relationship tested under different confining pressure conditions(For each data point, the darker the color indicates the greater confining pressure).

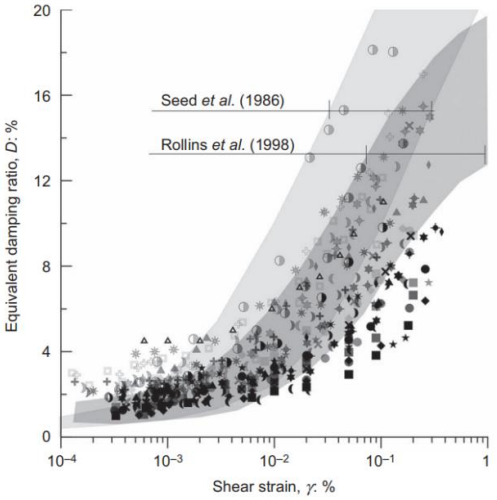


Figure 3. The D-γ relationship tested under different confining pressure conditions(For each data point, the darker the color indicates the greater confining pressure).

4 Conclusion

Current research on the dynamic properties of rockfill materials for hydraulic rockfill dams has accumulated extensive systematic experimental data. In contrast, dynamic test data for waste rock materials remain significantly scarce. Published literature contains limited reports on triaxial dynamic tests of waste rocks, with most

parameters derived from engineering analogies or existing rockfill test results rather than targeted calibration. Additionally, sampling representativeness of waste rocks poses challenges: mining waste exhibits wide particle size distributions, and no standardized approach exists for appropriate scale reduction in laboratory tests to balance particle size effects and material representativeness. Using an equivalent viscoelastic model framework, this chapter systematically investigates the dynamic behavior of waste rocks in linear tailings dams and identifies key conclusions:

(1) the dynamic shear modulus decreases with increasing shear strain; the equivalent damping ratio follows a nonlinear trend that initially increases rapidly before stabilizing with increasing strain – patterns that can be effectively described within the Hardin-Drnevich hyperbolic framework.

(2) G_{max} exhibits a positive correlation with effective confining pressure, conforming to the power-law relationship $G_{max} = C(\sigma_m')^n$. The equivalent damping of waste rock may be higher than that of stockpiled rock under identical conditions, primarily attributed to energy dissipation from particle edge fragmentation and fine-grained friction. the impact of elevated fine-grained content requires further validation.

(3) Although both waste rock and hydraulic rockfill possess a coarse-grained skeletal framework, they differ substantially in particle size distribution, fines content, particle shape, and compaction density. To facilitate seismic design, quantitative comparisons — especially strain-dependent modulus decay rates — should be carried out following standardized testing protocols.

(4) A core obstacle in current research is the insufficient experimental dataset for the dynamic behavior of waste rock, especially under varied confining pressures, particle sizes, and saturation conditions. Unlike hydraulic rockfill, waste rock does not have well-established databases or recommended parameter ranges. Consequently, engineers often resort to analogies or borrow data from rockfill studies, which undermines the confidence of seismic analysis. Therefore, there is an urgent need for systematic laboratory testing and the development of a dedicated parameter database for waste rock.

Acknowledgments. The authors acknowledge that no specific funding was received for this work..

Disclosure of Interests. he authors have no competing interests to declare that are relevant to the content of this article.

References

1. Hardin, B. O., & Drnevich, V. P. (1972). Shear modulus and damping in soils: Design equations and curves. Journal of the Soil Mechanics and Foundations Division, ASCE, 98(7), 667-692. https://doi.org/10.1061/JSFEAQ.0001760
2. Hardin, B. O., & Kalinski, M. E. (2005). Estimating the shear modulus of gravelly soils. Journal of geotechnical and geoenvironmental engineering, 131(7), 867-875. https://doi.org/10.1061/(ASCE)1090-0241(2005)131:7(867)

3. Park, D., and Kishida, T. (2018). Shear wave velocity profiles of fill dams. Soil Dynamics and Earthquake Engineering, 104, 250-258. https://doi.org/10.1016/j.soildyn.2017.10.013.
4. Yasuda, N., Nakamura, A., & Takeda, O. (1995). Dynamic Deformation Characteristics of Rockfill Materials from Laboratory Test, In-Situ Test and Earthquake Motion Analysis.https://scholarsmine.mst.edu/icrageesd/03icrageesd/session02/9.
5. Li D, Du Jun, Wang Xiaodong, Zhang Rui, & Xiong Zhiming. (2026). Experimental Study on the Dynamic Properties of Rockfill Materials for Dam Construction. Hydropower Energy Science, 44(02),100–104+99. https://doi.org/10.20040/j.cnki.1000-7709.2026.20250726.
6. S. Zhu,G. Yang,Y. Wen & L. Ou.(2014).Dynamic shear modulus reduction and damping under high confining pressures for gravels.Géotechnique Letters,4(3),179-186. https://doi.org/10.1680/geolett.14.00030.
7. Wang, D., Nash, T., Sullivan, T., & Thurstun, D. (2023). Shear strength and modulus of mine waste rock–Large scale laboratory testing results. Australian Geomechanics Journal, 58(4), 61-82.DOI: 10.56295/AGJ5842
8. Seed, H. B., Wong, R. T., Idriss, I. M., & Tokimatsu, K. (1986). Moduli and damping factors for dynamic analyses of cohesionless soils. Journal of geotechnical engineering, 112(11), 1016-1032. https://doi.org/10.1061/(ASCE)0733-9410(1986)112:11(1016).
9. Rollins, K. M., Evans, M. D., Diehl, N. B., & III, W. D. D. (1998). Shear modulus and damping relationships for gravels. Journal of geotechnical and Geoenvironmental Engineering, 124(5), 396-405.https://doi.org/10.1061/(ASCE)1090-0241(1998)124:5(396).
10. Rollins, K. M., Singh, M., & Roy, J. (2020). Simplified equations for shear-modulus degradation and damping of gravels. Journal of Geotechnical and Geoenvironmental Engineering, 146(9), 04020076.https://doi.org/10.1061/(ASCE)GT.1943-5606.0002300.
11. Stokoe, K. H., Darendeli, M. B., Andrus, R. D., & Brown, L. T. (1999). Dynamic soil properties: laboratory, field and correlation studies. In Earthquake geotechnical engineering (pp. 811-845).

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (http://creativecommons.org/licenses/by-nc/4.0/), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

