Improved TLS-ESPRIT Algorithm Research for Uniform Line Array

Yuanlei Si

Jiangsu Jianzhu Institute

Xuzhou, China

E-mail:siyuanlei@163.com

Abstract It has proved that the improved TLS-ESPRIT algorithm is more accurate than the TLS-ESPRIT algorithm in DOA estimation such as azimuth and elevation estimation at different SNR through experiments. The improved algorithm can show its superiority especially under the condition of the low SNR, its variance and root-mean-square are lower than that of the original algorithm. However, which is more superior of two algorithms when the number of sub array element is actually the same. This paper uses MATLAB to demonstrate the performance of the improved algorithm with M array and the original algorithm with M+1 array to show the performance of the both at the different SNR cases. And it also points out the deficiency of the improved algorithm and its theoretical basis is presented through analysis.

Keywords:DOA estimation; root-mean-square; sub array; TLS-ESPRIT

1. Introduction

DOA estimation is the rapid rise of the edge technology involved in interdisciplinary professions in recent years, which has a very broad application prospects in radar, sonar, communication, seismic signal processing and so on. The improved TLS-ESPRIT algorithm based on uniform linear array adopts backward-shifting technique to construct the corresponding rotation matrix, makes full use of each array element to construct the two sub array and expands the effective aperture of the array, which can improve the DOA estimation precision. Therefore, in theory, that, the original algorithm of M array and the improved algorithm of M-1 array should be of the same performance. And the 2-d improved TLS-ESPRIT algorithm means that it is extended from planar to three-dimensional spaces. It has proved that the improved TLS-ESPRIT algorithm is more accurate than the TLS-ESPRIT algorithm in azimuth and elevation estimation at different SNR through experiment. The improved algorithm can show its superiority especially

under the condition of small SNR and its variance and root-mean-square are lower than the original algorithm. However, which is more superior of two algorithms when the number of sub array element is the same. This paper makes further study and analysis on the improved TLS-ESPRIT algorithm.

2. one-dimensional uniform linear array model

The improved TLS-ESPRIT algorithm based on uniform linear array adopts the backward-shifting technique, which splits one M array into two M sub array instead of the front M-1 ones and the back M-1 ones. Each pair of array in the two M sub array is of the same shift vector. That is difference between the two M sub array by a rotation matrix reflected on the mathematical expression[1].
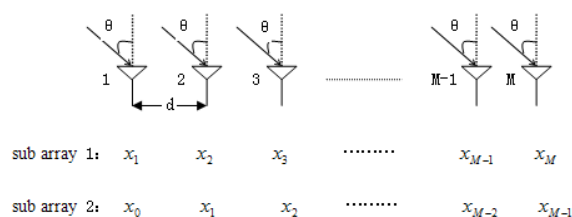

Figure 1 Uniform linear array model of DOA estimation

The TLS-ESPRIT algorithm is to define the translation vector y(k)=x(k+1) of the observation vector x(n), it uses the forward-shifting technique to divide the M array into the front M-1 sub array and the back M-1 ones, each sub array has M-1 elements. At the same time,datas received by the two sub arrays are as followed：

$$X_{sub1}(k)=[x_1(k) \ x_2(k) \ x_3(k) \ \dots \ X_{M-1}(k)]^T \quad (1)$$

$$X_{sub2}(k)=[x_2(k) \ x_3(k) \ x_4(k) \ \dots \ X_M(k)]^T \quad (2)$$

However, the improved TLS-ESPRIT algorithm is to define the translation vector y(k)=x(k-1) of the observation vector x(n), it uses the backward-shifting technique to divide the M array into the two M sub arrays and each sub array has M elements. At the same time,datas received by the two sub arrays are as

followed:

$$Y_{sub1}(k)=[x_1(k) \; x_2(k) \; x_3(k) \; \ldots \; X_M(k) \;]^T \qquad (3)$$

$$Y_{sub2}(k)=[x_0(k) \; x_1(k) \; x_2(k) \; \ldots \; X_{M-1}(k)]^T \qquad (4)$$

Among them $x_0(k)$ physical meaning: if it exists an array element x_0 to the left, the x_1 sample of the number k snap on the number zero array element is $x_0(k)$. that is to say, the observed value of the latter sub array on the time k is equal to the one of the front sub array overall shifting a array spaced .

3. two parallel uniform linear arrays model

The two parallel uniform linear arrays model is shown in Fig 2. Supposed array elements are perpendicular to the plane XOY and the interval between array elements is d both in X axis and Y axis.
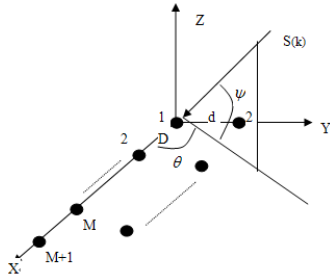

Figure 2 Two parallel uniform linear arrays model

As it is shown in figure 2 above, we can divide the array into three sub arrays. The array element of 1—M on the X axis is defined sub array one, The array element of 2—M+1 on the X axis is defined sub array two, and The array of M array elements Parallel to the X axis is named sub array three.

output vector of the array

$$X_1(t)=[x_{11}(t), \; x_{21}(t), \ldots \; x_{M1}(t)]^T \qquad (5)$$

noise vector of measurement

$$N_1(t)=[n_{11}(t), \; n_{21}(t), \ldots \; n_{M1}(t)]^T \qquad (6)$$

S(t) means $P \times 1$ dimension column vector

$$S(t)=[s_1(t), \; s_2(t), \; \ldots \; s_p(t)]^T \qquad (7)$$

likewise, the output signal column vector form of the sub array 2

$$X_2 （t） =[x_{21}(t), \; x_{31}(t), \ldots \; x_{(M+1)1}(t)]^T \qquad (8)$$

$$F=diag(f_1, f_2, \ldots \; f_p)_{p \times p} \qquad (9)$$

$$f_k=exp(-j2\Pi(d/\lambda)cos(\theta_k)cos(\psi_k)) \qquad (10)$$

$$N_2(t)=[n_{21}(t), \; n_{31}(t), \ldots \; n_{(M+1)1}(t)]^T \qquad (11)$$

Similarly, the output signal column vector form of the sub array 3

$$X_3 （t） =[x_{12}(t), \; x_{22}(t), \ldots \; x_{M2}(t)]^T \qquad (12)$$

$$G=diag(g_1, g_2, \ldots \; g_p)_{p \times p} \qquad (13)$$

$$g_k=exp(-j2\Pi(d/\lambda)sin(\theta_k)cos(\psi_k)) \qquad (14)$$

$$N_3(t)=[n_{12}(t), \; n_{22}(t), \ldots \; n_{M2}(t)]^T \qquad (15)$$

f_k and g_k of the same location in vector F and G contains the DOA information of the same signal.

The main idea of the 2-d improved TLS-ESPRIT algorithm is to adopt M array elements on the X axis and parallel to the X axis, which can reduce an array element. Both two parallel arrays employ backward-shifting technique. Thus:

$$A^{/}(\Theta)=[a^{/}(\Theta_1), a^{/}(\Theta_2), \ldots a^{/}(\Theta_p)] \qquad (16)$$

Compared to the original algorithm, it reduces a row. The corresponding output signal column vector of the sub array will have been changed.

The output signal column vector form of the sub array 1 is transformed as followed.

$$X^{/}_1(t)= A^{/}(\Theta)S(t)+N^{/}_1(t) \qquad (17)$$

Among them:

$$X^{/}_1(t)= [x_{11}(t), \; x_{21}(t), \ldots \; x_{M1}(t)]^T \qquad (18)$$

$$N^{/}_1(t)= [n_{11}(t), \; n_{21}(t), \ldots \; n_{M1}(t)]^T \qquad (19)$$

The output signal column vector form of the sub array 2 is transformed as followed.

$$X^{/}_2(t)= A^{/}(\Theta)F^{/}S(t)+N^{/}_2(t) \qquad (20)$$

Among them:

$$X^{/}_2 （t） =[x^{*}_{21}(t), \; x_{11}(t), \ldots \; x_{(M-1)1}(t)]^T \quad (21)$$

$$F^{/}=diag(f^{/}_1, f^{/}_2, \ldots \; f^{/}_p)_{p \times p} \qquad (22)$$

$$f^{/}_k=exp(-j2\Pi(d/\lambda)cos(\theta_k)cos(\psi_k)) \qquad (23)$$

$$N^{/}_2(t)=[n^{*}_{21}(t), \; n_{11}(t), \ldots \; n_{(M-1)1}(t)]^T \qquad (24)$$

And here "*" represents conjugation.

the output signal column vector form of the sub array 3 is transformed as followed.

$$X^{/}_3(t)= A^{/}(\Theta)G^{/}S(t)+N^{/}_3(t) \qquad (25)$$

there into:

$$X^{/}_3 （t） =[x_{12}(t), \; x_{22}(t), \ldots \; x_{M2}(t)]^T \quad (26)$$

$$G^{/}=diag(g^{/}_1, g^{/}_2, \ldots \; g^{/}_p)_{p \times p} \qquad (27)$$

$$g^{/}_k=exp(-j2\Pi(d/\lambda)sin(\theta_k)cos(\psi_k)) \qquad (28)$$

$$N^{/}_3(t)=[n_{12}(t), \; n_{22}(t), \ldots \; n_{M2}(t)]^T \qquad (29)$$

therefore, the 2-d DOA estimation can be obtained as$(\theta^{/}_k, \psi^{/}_k)$.

4．the simulation experiment and results

The simulation condition one：For further study on the performance of the original algorithm and the improved algorithm, the uniform linear array of 8 elements or 9 elements are used respectively. The distance between two array elements half of the wavelength, that is d=0.5λ,4 incoherent source direction （P=4）：$0^0,20^0,25^0$ and 40^0. Sampling 100 times，at different SNR cases, the TLS-ESPRIT algorithm uses 9 array elements, but the improved TLS-ESPRIT algorithm uses 8 array elements. The simulation result is shown in figure 3.
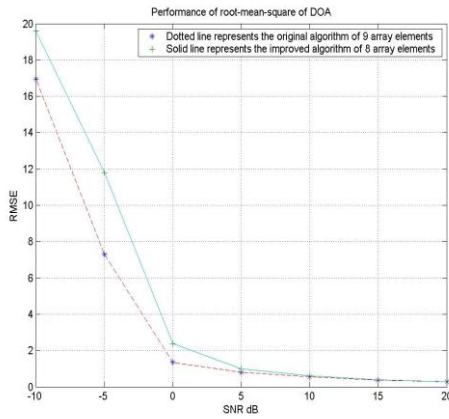

Figure 3 Performance of root-mean-square of DOA estimation

The purpose of this simulation experiment is to do some comparison. Both performance and simulation result should be the same under the same conditions of two kinds of algorithms in the same number of array elements, however, obviously the performance of the TLS-ESPRIT algorithm is superior to that of the improved one by the root mean square error from experiments. Through analysis and study carefully, we found the cause of the difference. In the improved TLS-ESPRIT algorithm, we construct the second sub array, where there is the data $x_0(k)=x_2^*(k)$，$x_2^*(k)$ includes the noise $n_2^*(k)$，but in fact $n_2^*(k)=n_2(k)$，that is to say，in the second sub array, the noise of the first array element is the same with that of the third one , the noise power increases after autocorrelation which results in the decrease of estimation precision. With the improvement of signal-to-noise ratio, this influence is waning, when SNR is large, the performance of the two algorithms is approached to the same. The simulation results have verified it.

The simulation condition two: The 2-d TLS-ESPRIT algorithm and the improved TLS-ESPRIT algorithm

will be discussed in the following. In the original algorithm, we adopt the uniform linear array of 9 elements on the X axis, the parallel array to the X axis uses the uniform linear array of 9 elements (named 9 + 8 structure), The improved algorithm adopts the uniform linear array of 8 elements on the X axis , the parallel array to the X axis uses the uniform linear array of 8 elements o(named 8 + 8 structure). The distance between two array elements half of the wavelength, that is d=0.5λ,2 incoherent source direction（P=2）：$(15^0,30^0)$ and$(25^0,45^0)$. Sampling 100 times，at different SNR cases, Both algorithms are compared. The simulation result is shown in figure 4,5 and 6.
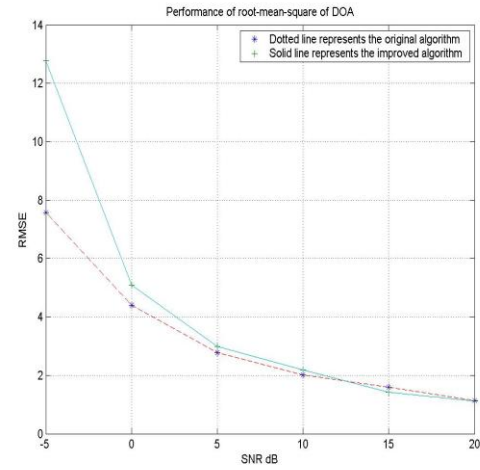

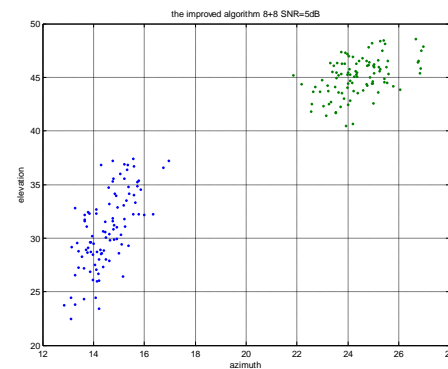**Figure 4 Performance of root-mean-square of the two dimensional DOA estimation**

Figure 5 Distribution of azimuth and elevation based on the improved algorithm
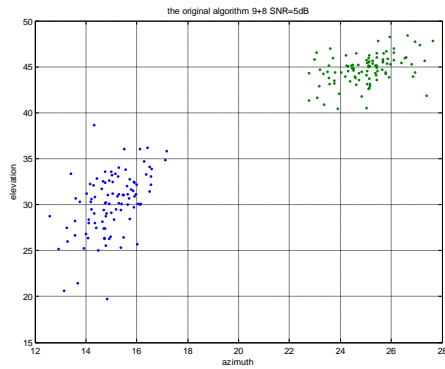
the original algorithm 9+8 SNR=5dB

Figure 6 Distribution of azimuth and elevation based on the original algorithm

Generally, the improved algorithm adopts 8 + 8 array structure, but the original algorithm uses 9 + 8 array structure as if the latter is apparently less one array element. Actually the two algorithms use the same number of array elements.

From the results of experiments, it can be detected that the performance of the original algorithm is superior to that of the improved one by the root mean square error when SNR is low .With the improvement of signal-to-noise ratio, the performance of the two algorithms is approached to the same.

5．Conclusion

For the use of one-dimensional uniform linear array with M , the improved algorithm, compared to the original algorithm can improve the precision of DOA estimation and enlarge the effective aperture of array, especially in the case of small SNR it even more reflects its advantages, but cannot reach the same precision with M+1 array.

Under the two-dimensional parallel uniform linear arrays，no matter observed from the estimation precision of azimuth and elevation , or root-mean-square, the original algorithm is obviously superior to the performance of the improved algorithm in the case of small SNR. However, when the SNR ratio is increased gradually, the performance of various aspects of the improved algorithm is basically close to the original algorithm, and when SNR is large enough, the root mean square error performance is compared with that of the original algorithm.

REFERENCES

[1] Yuanlei Si, Minghua Yang, Hongyu Huo, Yang Bai. "A Novel ESPRIT Algorithm Research for Uniform Line Array". Journal of Yunnan University（NaturalSciences），2006, 28(4):312-315.

[2] Yuanlei Si. "2-D Improved TLS-ESPRIT Algorithm Research for Uniform Line Array". Communications Technology, 2010,43 (4):25-30.

[3]G.Xu,S.Silverstenin, R Roy and T.Kailath. "Beanspace ESPRIT".IEEE Transactions on Signal Processing, vol.42, Feb 1994,349-356.

[4]Mahata K. "A subspace algorithm for wideband source localization without narrowband filtering". IEEE Transactions on Signal Processing, 2011, 59(7): 3470-3475.

[5]Bilik I. "Spatial compressive sensing for direction-of-arrival estimation of multiple sources using dynamic sensor arrays". IEEE Transactions on Aerospace and Electronic Systems, 2011,47(3):1754-1769.

[6]Gurbuz A C, Cevher V, McClellan J H. "Bearing estimation via spatial sparsity using compressive sensing". IEEE Transactions on Aerospace and Electronic Systems, 2012,48(2):1358-1369.