The Multi-Objective Decision-Making Based on DEA and Analytic Hierarchy Process

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Abstract—**As a result of the diversity of people's demand, the multi-objective of the social and economic activities begins to appear, in order to meet these diverse demands. However, these objectives are not compatible. This article mainly adopts the combination of the data envelopment analysis (DEA) and analytic hierarchy process (AHP) to coordinate the conflict.**

Keywords- multi-objective decision-making; data envelopment analysis; analytic hierarchy process;

I. INTRODUCTION

In multi-objective decision-making problems, a decision maker often needs to select or rank alternatives that are associated with non-commensurate and conflicting objects, which can be represented as follows:

$$D = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad (1)$$

and $\quad W = \begin{bmatrix} w_{m1} & w_{m2} & \dots & w_n \end{bmatrix} \quad (2)$

where A_1, A_1, ..., A_m are the alternatives to be evaluated, C_1, C_2, ..., C_n are performance measures against, which performance of alternatives are measured, a_{ij} is the performance rating of i th alternative against j th criterion, and w_j is the weight of j th criterion[1-3].

This paper formulates a multi-objective decision-making problem as a multiple objective data envelopment analysis model where inputs correspond to cost objects and outputs correspond to benefit objects.

II. PRESENTATION OF MULTI-OBJECTIVE DECISION

We are quite familiar with single objective decision problems and other optimization fields such as inventory management, and project management. However, many decision situations call for decision making when more than one objective needs to be considered (called multi-objective decision making). Such an approach is typical to investment problems where commercial banks need to balance return and risk because of legal and ethical obligations demand.

Other examples where multi-objective optimization fit the nature of the problem in question are listed next:

①The design of a mechanical part needs to meet several goals, such as sensitivity maximization, rigidity maximization, and cost-minimization.

②The optimal location of a new airport should be a function of minimizing cost to the federal government, capacity increase of airport facilities, system safety improvement, reduction in noise levels, and so on.

③Building Electrical Power facility is to be determined based on increasing electrical power as much as possible, health considerations of the residents, economic conditions of the residents, the economy of the city and the state, local politics.

The difficulty of identifying an optimal solution for a multi-objective decision problem lies in the possible conflicts that may exist between the optimal solutions for the separate objectives. Even the definition of "optimality" in this context is not straight-forward. The best solution for a certain objective might be the worst for another. The following graphical presentation illustrates the presence of possible conflict among objectives.
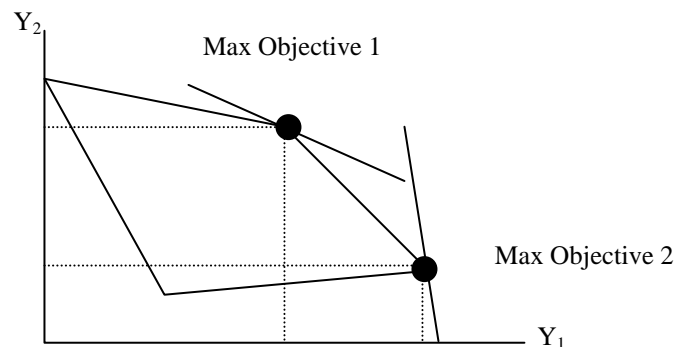

Figure 1. Multi-objective Decision Diagram

Where is the optimal solution to such a problem? Objective 1 and objective 2 are maximized at two different extreme point of the feasible region.

Resolving these conflicts is the essence of finding an "optimal" solution on multi-objective ground. Thus, a solution that finds a good compromise acceptable to the decision maker with respect to all objectives is sought. It is with this understanding that we proceed to study a few approaches to locating a satisfactory (thus multi-objective optimal) solutions.

From figure 1 we can see efficient and consistent decision making is a hard challenge. The decision maker needs to account for multiple, often conflicting objectives, resulting in a very large number of options to consider. By using such as Pareto optimization we can reduce the number of alternatives to an optimal set known as the Pareto Frontier. Provided with this set, we can understand the conflicts between the objectives, explore the available tradeoffs and see the value of a selected solution. To meet this end, we can also use smart visualization and analytical recommendation mechanism that altogether enable an easy and intuitive exploration of the Frontier. This approach increases the transparency of the selection process when multiple parties need to reach a consensus, and enables better evaluation of alternatives.

The multidimensionality of the objective world results in the multiplicity of the people's demands. As a result, the multiple criteria (multiple objectives and multiple purposes) of the social and economic activities begin to appear, in order to meet these diverse demands.

For instance, we often consider the following evaluative principles, such as "cost", "quality", "profit", etc. in the economic management work. And we will establish the goal of management according to these standards, such as "cost at the least", "best quality", "maximum profit" and so on. Of course, it will be the ideal state if the objective world can satisfy all the objectives.

However, in general, it is impossible to achieve this kind of ideal state. Or it is not compatible among those multiple objectives of the human social activities. In this sense we can say that human being's real decision-making activities are the efforts to solve the conflict.

This article mainly adopts the combination of the Data Envelopment Analysis (DEA) and Analytic Hierarchy Process (AHP) to coordinate the conflict.

A. The principle of Data Envelopment Analysis

Many problems in decision science involve a complex set of choices. The study of choices tends to be multivariate in nature, which is sometimes difficult to model in parametric form. Complex decision analysis, depending on analysis methodology, may require assumptions on distributions or functional forms of e.g. a production function. The study of choices could furthermore involve problems with a large number of inputs and outputs whereby some methodologies can be sensitive to the frequency distributions of the respective inputs.

Since the introduction of the DEA in 1978, it has been widely used in efficiency analysis of many business and industry application.

Data Envelopment Analysis (DEA) is a nonparametric mathematical programming approach which allows for comparisons of decision options with similar objectives, canonically termed decision making units (DMUs) in the following.

Data Envelopment Analysis can be applied to many complex decision problems, such as in evaluating the relative efficiency of various decision alternatives, optimize - maximize for a certain output decision variable, establish target values of given decision input variables, identify peer decision alternatives, etc. Its main benefit is its ability to handle the evaluation of alternatives with input and output vectors consisting of elements on different units[4-7].

B. The principle and steps of Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) is a simple method for human beings to make decisions of the more complicated and fuzzier problems. It is especially suitable for those problems, which are difficult to make a complete quantitative analysis. And it's a multi-criteria decision method of simple, vivid and practical characteristics and put forward by an American operations researcher T. L. Saaty in the early 70s.

During the process of the system analysis of social, economic and scientific management problems, the people are usually faced with a system, which is composed of many integrating, interrelating and interdependent factors. However, these factors are complex and often lack of quantitative data. Analytic hierarchy process (AHP) provides a new, simple and practical method of modeling for decision-making and sort of this kind of problem[8-10].

Modeling by the way of Analytic Hierarchy Process (AHP) generally can be divided into four steps.

① Establish a hierarchical structure model.

② Build all the judgment matrix of each level.

③ Order in single level and have a consistency check.

④ Order in whole levels and have a consistency check.

III. ILLUSTRATE THE MULTI-OBJECTIVE DECISION OF THE LIMITED SCHEME BY CASES

Multi-objective decision analysis has been presented for helping decision makers to make their final decisions in MCDM (Multiple Criteria Decision Making) problems. One of the main tasks in this research is how to incorporate value judgments of decision makers in decision support systems. If decision makers can make their decisions by seeing efficiencies (or inefficiencies) of alternatives, the idea of DEA (Data Envelopment Analysis) can be applied to MCDM problems. In this event, it is important to know what value judgment the domination structure of each DEA model reflects. Moreover, a model which can treat a wide range of value judgments of decision makers is required. A generalized DEA model is proposed and discussed for practical use in MCDM problems.

Someone intends to purchase a home. There are four places (schemes) to be chosen. The related information is just as follows:

TABLE I.   EXAMPLE OF MULTI-OBJECTIVE DECISION ANALYSIS

Scheme (Place)	Price y_1 (ten thousand yuan)	Distance from the working place y_3 (km)	Usable floor area y_4 (y_4)	Facilities y_4	Environment y_5
X_2	3.0	10	100	7	7
X_2	2.5	8	80	3	5
X_3	1.8	20	50	5	11
X_4	2.2	12	70	5	9

It is a decision that with five objectives. Among them, the usable floor area, facilities and environment are as the benefit objectives and can be regarded as the output. As for the output, the bigger, the better. The price and the distance can be regarded as the input. In contrary, the smaller, the better. Obviously, it's a multi-objective decision of the limited scheme.

We should make sure if the four schemes are effective with DEA in the first step. According to the above analysis, we regard the price and the distance from the working as the input and the usable floor area, facilities and environment as the output.

TABLE II.   THE DATA OF DATA ENVELOPMENT ANALYSIS

Scheme(Place)	Input		Output		
	y_1 (ten thousand yuan)	y_3 (km)	y_4 (y_4)	y_4	y_5
X_2	3.0	10	100	7	7
X_2	2.5	8	80	3	5
X_3	1.8	20	50	5	11
X_4	2.2	12	70	5	9

First, we should make sure whether the scheme(X1) is an effective DEA or not. The linear programming model is established as follows.

$$\min E$$
$$\begin{cases} 100\lambda_1 + 80\lambda_2 + 50\lambda_3 + 70\lambda_4 \geq 100 \\ 7\lambda_1 + 3\lambda_2 + 5\lambda_3 + 5\lambda_4 \geq 7 \\ 7\lambda_1 + 5\lambda_2 + 11\lambda_3 + 9\lambda_4 \geq 7 \\ 3\lambda_1 + 2.5\lambda_2 + 1.8\lambda_3 + 2.2\lambda_4 \leq 3E \\ 10\lambda_1 + 8\lambda_2 + 20\lambda_3 + 12\lambda_4 \leq 10E \\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \\ \lambda_j \geq 0 (j=1,2,3,4) \end{cases} \quad (3)$$

We solve the problem by optimization software, such as Matlab or Excel. The results are just as follows: E=1, $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$. This shows that the plan(X1) is effective with DEA. Similarly, the other three schemes are also effective.

We get the weight by using analytic hierarchy process (AHP) in the second step.

After the comparison of each attribute, supposing that the judgment matrix is just as follows:

TABLE III.   JUDGMENT MATRIX OF AHP ANALYSIS

	Price	Distance from the working place	Usable floor area	Facilities	Environment
Price	1	1/2	1/3	1/4	1/5
Distance from the working place	2	1	1/2	1/2	1/2
Usable floor area	3	2	1	1	1/2
Facilities	4	2	1	1	1
Environment	5	2	2	1	1

We can notice that this two-two judgment matrix can satisfy $a_{ij} = 1/a_{ij}$, but can't always satisfy $a_{ik} \times a_{kj} = a_{ij}$. This point declares that the two-two judgment matrix has no complete consistency.

Find out the eigenvalue of the two-two judgment matrix by using Matlab.

λ_1 =5.0508, λ_2 =-0.031821 - 0.50649i, λ_3 =-0.031821 + 0.50649i, λ_4 =0.012812, λ_5 =-2.2151e-019. We can know that the biggest eigenvalue is λ_1 =5.0508.

For consistency checking,

$$CI = \frac{\lambda_{\max} - n}{n-1} = \frac{5.0548 - 5}{5-1} = 0.0137,$$ and look at

RI=1.12 of two-two judgment matrix in table.

TABLE IV.   THE EIGENVECTORS NORMALIZED RESULT TABLE

	Price	Distance from the working place	Usable floor area	Facilities	Environment	Summation
Weight (k_i)	0.066303285	0.131724	0.218657	0.263449	0.319867	1

Place the data of Table-1 at standardization level.

It's not better for decision makers to invest more, therefore, adopt formula $z_{ij} = \dfrac{y_j^{\max} - y_{ij}}{y_j^{\max} - y_j^{\min}}$ to standardize the input, including the price and the distance from the working place. In a similar way, adopt formula

$$z_{ij} = \frac{y_{ij} - y_j^{\min}}{y_j^{\max} - y_j^{\min}}$$ to standardize the output, which

includes the usable floor area, facilities and environment.

After the standardization, the data is just as follows (z_{ij}):

TABLE V.   THE NORMALIZED DATA TABLE

Scheme (Place)	Price	Distance from the working place	Usable floor area	Facilities	Environment
X_2	0	0.8333333	1	1	0.333333
X_2	0.416667	1	0.6	0	0
X_3	1	0	0	0.5	1
X_4	0.666667	0.6666667	0.4	0.5	0.666667

Finally, by using the linear weighting method, the weight of each scheme is in the list below. The formula is: $u(X_i) = \sum\limits_{j=1}^{5} k_j z_{ij}$

TABLE VI.   THE WEIGHT TABLE OF EACH SCHEME

Scheme(Place)	X_2	X_2	X_3	X_4
Final Weight	0.698498	0.290545	0.517895	0.56445

Therefore, the first scheme(X1) is the best one.

IV.   CONCLUSION

Multi-objective decision-making is a method that makes a more scientific and reasonable selection for many conflicting goals and then finds out a decision. It developed rapidly after1970s and it's a branch of management science. Multi-objective decision-making is different from the general decision, which only selects the best scheme from many feasible schemes in order to achieve a fixed goal. In the multi-objective decision-making, multiple objectives will be considered. However, these objectives are often difficult to compare with each other and even contradictory sometimes. Generally it is difficult to achieve the optimal state for every objective and make a decision that can satisfy all aspects finally. Therefore, multi-objective decision-making is essentially a reasonable compromise among the various targets. This is the process of multi-objective optimization.

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