A Stochastic Time-cost-quality Tradeoff Model for Discrete Project Scheduling Problem

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Abstract. In this paper, referred to the traditional discrete time-cost tradeoff problem, time-cost-quality balance combined with random environment is discussed. Moreover, in real projects, the uncertainty of the environment is considerable aspects for decision-makers. The goal of this paper is to reveal how to obtain the optimal balance among project completion time, total cost and overall quality in stochastic environments. A stochastic discrete time-cost-quality tradeoff model is presented for dealing with this problem. To solve the model, a hybrid intelligent algorithm integrating the stochastic simulations and genetic algorithm is developed. At the last part, a numerical experiment is conducted to testify the effectiveness of the hybrid intelligent algorithm.

Introduction

It is generally believed that there are three goals in Construction Project Management which are known as time, cost and quality. A project should be completed before the deadline to meet its time limit, within the budget to meet its cost limit and up to the project specifications to meet its quality requirement. However, sometimes the time, cost or quality requirement could not simultaneously be satisfied. Therefore, the project decision-makers should make a balance among them. In the previous literature, many works focused on the tradeoff problem between the project duration and cost, which is called time-cost tradeoff problem (TCTP) [1,2,3].

Some researchers suggested that the project quality would be influenced by the balance between project completion time and total cost and began to study on time-cost-quality tradeoff problem (TCQTP) [4,5,6,7,8]. In 1996, Babu and Suresh [4] took quality into consideration in TCTP by giving the relationships among project completion time, project cost and quality. Based on the research framework given by Babu and Suresh [4], Khang and Myint [5] applied it to a practical case study of a cement plant construction project. In 2005, El-Rayes and Kandil [6] proposed a multi-objective optimization model minimize project cost and duration and maximize quality. In 2012, Kim et al. [7] concentrated on minimizing project overall quality loss cost. Babu and Suresh [4] suggested that the three variables are continuous, but several researchers suggested that the three variables were discrete, such as Tareghian and Taheri [8]. The discrete time-cost-quality tradeoff problem (DTCQTP) suggested that any activity may be executed in several options (modes), and each option (T,C,Q) is completed with some deterministic time, cost and quality. Johnson and Liberatore [9] used an Analysis Hierarchy Method (AHP) to quantify activity quality.

Traditional researchers suggest that variables in project scheduling problem are deterministic. But uncertain factors always exist in project scheduling problem because of the external environment changes. Hence in real-life projects, the uncertain effects of the external environment should be of significant considerations for project decision-makers. Furthermore, in project scheduling problem, the uncertainty of project activity duration was traditionally supposed to be stochastic. Freeman [10,11] in 1960 first employed probability theory into project scheduling problem and founded PERT. In 2009, Ke et al. [12,13] developed a set of optimization models for stochastic TCTP. The authors suggested activity durations to be random variables and proposed two models which were solved by a hybrid intelligent algorithm combining stochastic simulations and genetic algorithm (GA).

In this paper, a new model for DTCQTP is developed. In section II we will describe the stochastic time-cost tradeoff problem. And the deduction of the formulation of the project completion time and the project cost introduced by Ke et al. [13] will be introduced in section II, either. Section III presents the new model for stochastic time-cost-quality tradeoff problem. In Section IV, we present our hybrid intelligent algorithm. In Section V, we discuss the results obtained from the proposed algorithm when applied to an example used in [9]. In the final section, we will draw the conclusions.

Problem description

Referred to Brucker et al. [14], a project can be depicted as a directed acylic activity-on-node *(AON)* network $G = (V, E)$ like Fig. 1, in which V is the set of nodes and E is the set of precedence relationships between the nodes. In the network, the real activities are numbered form 1 to N. Moreover, activity 0 represents the start of the project and activity $N+1$ represents the end. We use (i,j) to represent that activity i precedes activity j, which enforces that activity j may not be started before all its predecessors P_j are finished, $j = 2,. . .,N$.

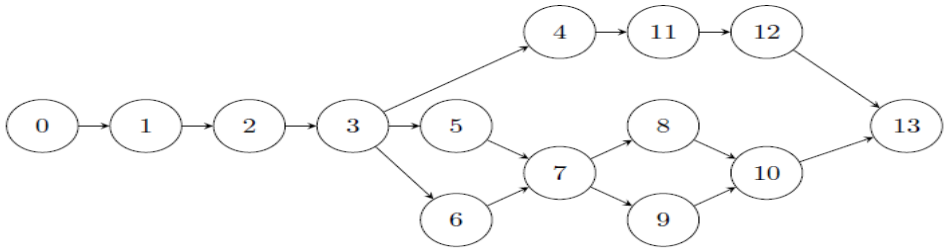

Fig. 1

As is mentioned above, each activity i in the project will be executed in several options. Let M_i be the set of options of activity i. For the k_ith ($k_i \in M_i$) option $(\xi_{ik_i}, c_{ik_i}, q_{ik_i})$, the activity is supposed to be completed in duration of ξ_{ik_i}, within cost c_{ik_i}, and on quality level q_{ik_i}. The duration ξ_{ik_i} is supposed to be a random variable with some given density function. However, c_{ik_i} and q_{ik_i} are considered to be constant as to the environment.

The stochastic durations of the activities are denoted as $\xi = \left\{ \xi_{ik_i} \middle| i \in V, k_i \in M_i \right\}$. The start time of activity i is written as $S_i(x,\xi)$, where $x = \left(x_{1k_1}, x_{2k_2}, \cdots, x_{Nk_N} \right)$ is the decision vector for which options to be selected. The start time of activity 0 $S_0(x,\xi)$ is suggested to be 0 for representing the beginning of the project. $S_{N+1}(x,\xi)$ can be used to represent the end of the project as well as the total duration of the project. Therefore, $S_i(x,\xi), i = 1,2,\cdots,N+1$ can be formulated as follows:

$$S_i(x,\xi) = \max_{(l,i)\in E} \left\{ S_l(x,\xi) + \xi_{lk_l} \right\} \tag{1}$$

if option $k_l \in M_l$ is selected for activity l.

The project completion time $S_{N+1}(x,\xi)$ can be calculated as follows:

$$S_{N+1}(x,\xi) = \max_{(l,N+1)\in E} \left\{ S_l(x,\xi) + \xi_{lk_l} \right\} \tag{2}$$

In consideration of the time value of the capital, the project total cost $C(x,\xi)$ can be calculated as follows:

$$C(x,\xi)=\sum_{i\in V}\sum_{k_i\in M_i} x_{ik_i} c_{ik_i} (1+r)^{S_{N+1}(x,\xi)-S_i(x,\xi)} \tag{3}$$

where

$$x_{ik_i}=\begin{cases}0, & \text{if option } k_i \text{ is selected for activity } i\\ 1, & \text{otherwise.}\end{cases}$$

and r is the loan interest rate.

Stochastic discrete time-cost-quality tradeoff model

In this section, a stochastic time-cost-quality tradeoff model for discrete project scheduling problem will be discussed. The form of the stochastic time-cost-quality tradeoff model is as follows:

$$\begin{aligned}&\min \quad E\left[C(x,\xi)\right]\\ &subject \quad to:\\ &\begin{cases}E\left[S_{N+1}(x,\xi)\right]\le D^0\\ \dfrac{1}{N}\sum_{i\in V}\sum_{k_i\in M_i} x_{ik_i} q_{ik_i}\ge Q^0\\ \sum_{k_i\in M_i} x_{ik_i}=1,\ for \quad i=1,2,3,\cdots,N\end{cases}\end{aligned} \tag{4}$$

where $D^{\wedge 0}$ is project duration bound and $Q^{\wedge 0}$ *is* project quality bound.

In the above model, $E\left[C(x,\xi)\right]$ is expected value of the project total cost. $E\left[S_{N+1}(x,\xi)\right]$ is the expected value of the project total duration and not permitted to be greater than the project duration bound $D^{\wedge 0}$. $\frac{1}{N}\sum_{i\in V}\sum_{k_i\in M_i} x_{ik_i} q_{ik_i}$ is the project average quality and not permitted to be less than the project quality bound $Q^{\wedge 0}$. x_{ik_i} is the decision variable and $\sum_{k_i\in M_i} x_{ik_i}=1$ is to make sure there is one and only one option to be selected for activity *i*.

Hybrid intelligent algorithm

To solve the proposed stochastic time-cost-quality tradeoff model, a hybrid intelligent algorithm is developed integrating stochastic simulation and GA. There are two stochastic functions in the model, i.e. $E\left[C(x,\xi)\right]$ and $E\left[S_{N+1}(x,\xi)\right]$ which need to be simulated. Moreover, a decoding process of the chromosome is embedded into the simulation process.

Stochastic simulation

The detailed process of stochastic simulation [15] can be described as follows:

Step 1: Set $p=0, C=0, S=0$;

Step 2: Let $p=p+1$;

Step 3: Generate $\xi^p=\left(\xi_1^p,\xi_2^p,\cdots,\xi_N^p\right)$ according to the distribution of ξ

Step 4: Set $i=0,\ s^p=0,\ c^p=0,\ s_i^p=0,\ c_i^p=0$;

Step 5: Let $i=i+1$;

Step 6: Decoding the chromosome to obtain the duration ξ_i^p and cost c_i^p for activity *i*;

Step 7: Let $c = c + c_i^p (1+r)^{S_h - \max_{(l,i)\in E}\{s_l^p + \xi_l^p\}}$, where S_h is a hypothetical completion time;
Step 8: Repeat Steps 5 to 7 until i=N+1 to obtain s_{N+1}^p, c_{N+1}^p;
Step 9: Let $c_{N+1}^p = c_{N+1}^p (1+r)^{s_{N+1}^p - S_h}$;
Step 10: Let $S = S + s_{N+1}^p$, $C = C + c_{N+1}^p$;
Step 11: Repeat Steps 2 to 10 until p=M, where M is a sufficiently large integer number;
Step 12: Return S/M and C/M as the expected value of $C(x,\xi)$ and $S_{N+1}(x,\xi)$, respectively.

Genetic algorithm

Developed by Holland [16], GAs have been applied successfully to solve hard optimization problems such as TCTP [13,17] and resource-constrained TCTP [18]. Referred to the mode list developed by Hartmann [17], we take the mode list ($k_1, k_2,...,k_N$) as the chromosome if the option $k_i \in M_i$ is selected for activity i. The structure of chromosome is shown in Table 1.

Table 1

1	2	3	4	…	N	activity list
1	1	3	3	…	2	option list

The chromosomes in the population are allowed to be unfeasible. Then we will introduce a penalty function to reduce their probability to be selected. The fitness function can be set as follows:

$$FIT(chromosome_i)=E[C(x, \xi)]+Py_i \tag{5}$$

where P is sufficiently large positive number and

$$y_i = \begin{cases} 1, & \text{if chromosome } i \text{ is unfeasible} \\ 0, & \text{otherwise.} \end{cases}$$

The detailed process of GA applied in this paper can be described as follows:
Step 1: Initialize *popsize* chromosomes as the initial population pop_{ini};
Step 2: Carry out crossover operation to pop_{ini} to get population pop_c ;
Step 3: Carry out mutation operation to pop_c to get population pop_{cm} ;
Step 4: Calculate $E\left[S_{N+1}(x,\xi)\right]$ and $E\left[C(x,\xi)\right]$ of each chromosome in pop_{cm};
Step 5: Calculate fitness of every chromosome, i.e. *FIT*;
Step 6: Select the chromosomes through wheel spinning to get the population pop_{cms};
Step 7: Repeat Steps 2 to 6 for a given number of times;
Step 8: Take the best chromosome as optimal solution.

Hybrid intelligent algorithm

Referred to [9], consider a construction project with twelve activities. The project network is illustrated in Fig.1 and the time-cost-quality options of the activities are shown in Table II. And the duration times are supposed to be random variables with normal distribution denoted as $N(\mu,\sigma^2,)$. The quality of each activity is an integer and limited in the interval [50,100].

Let us consider an instance of the stochastic discrete time-cost-quality tradeoff model. The decision-maker may want to minimize the expected project total cost, and simultaneously meet constraints of the total completion time for 24 days and the overall quality for 820/12. Then the following stochastic minimization model can be built as equation (6).

In the hybrid intelligent algorithm, stochastic simulations will be executed for 1000 times and GA will be executed for 300 generations. In the algorithm, there are three parameters given beforehand, namely the population size of one generation *popsize*, the crossover probability P_c, and the mutation

$$
\min \quad E\left[C\left(x,\xi\right)\right]
$$
$$
subject \quad to:
$$
$$
\begin{cases}
E\left[S_{13}\left(x,\xi\right)\right] \le 24 \\
\dfrac{1}{12}\sum_{i\in V}\sum_{k_i\in M_i} x_{ik_i} q_{ik_i} \ge 820 \\
\sum_{k_i\in M_i} x_{ik_i} = 1, \; for \quad i = 1,2,3,\cdots,12
\end{cases}
\tag{6}
$$

probability P_m. To testify the effectiveness of the algorithm, the above three parameters will be given in several values and the corresponding quasi-optimal results will be compared and evaluated. From Table 3, we find that the 'Error's, calculated by the formula (actual value - optimal value)/ optimal value $\times 100\%$, do not exceed 0.47%, which does not exceed the general project demand. Therefore, the designed hybrid algorithm is effective for solving the stochastic time-cost-quality tradeoff model.

Table 2

ACTIVITY(i)	n_i	$(\xi_{i1}, c_{i1}, q_{i1})$	$(\xi_{i2}, c_{i2}, q_{i2})$	$(\xi_{i3}, c_{i3}, q_{i3})$
1	1	(N(3,0.25),21600,70)	--	--
2	1	(N(1,0.16),7200,70)	--	--
3	3	(N(4,0.25),28800,80)	(N(3,0.16),39600,70)	(N(4,0.36),35000,90)
4	3	(N(6,0.49),28800,70)	(N(3,0.25),43200,70)	(N(4,0.49),36000,60)
5	3	(N(3,0.16),14400,70)	(N(2,0.16),19200,60)	(N(3,0.36),12000,60)
6	2	(N(4,0.25),9600,70)	(N(2,0.09),16800,60)	--
7	2	(N(5,0.49),12000,70)	(N(4,0.25),14400,80)	--
8	2	(N(6,0.81),28800,70)	(N(3,0.16),57600,60)	--
9	3	(N(3,0.25),10800,70)	(N(2,0.04),16800,70)	(N(3,0.16),9000,60)
10	1	(N(3,0.16),7200,70)	--	--
11	3	(N(2,0.25),4800,60)	(N(1,0.04),6000,70)	(N(2,0.16),1800,50)
12	3	(N(3,0.25),14400,70)	(N(2,0.16),19200,70)	(N(3,0.49),16000,80)

Table 2

popsize	P_c	P_m	quasi-optimal solution	COST	Error(%)
40	0.3	0.4	(1,1,1,1,3,1,2,1,3,1,3,3)	184591	0.45
40	0.4	0.2	(1,1,1,1,3,1,2,1,3,1,3,1)	184626	0.47
50	0.4	0.2	(1,1,1,1,3,1,2,1,3,1,3,1)	184584	0.45
50	0.2	0.3	(1,1,1,1,3,1,1,1,3,1,3,3)	183766	0.00
60	0.2	0.2	(1,1,1,1,3,1,2,1,3,1,3,1)	184563	0.44
60	0.3	0.1	(1,1,1,1,3,1,2,1,3,1,3,1)	184544	0.43

Conclusion

It is not realistic to assume quality to be insusceptible in the tradeoff between project total completion time and total cost. Hence, it is significant for decision-makers to consider the tradeoff among the three variables. Furthermore, the uncertainty of the external environment is also considerable. In this paper, we presented a stochastic discrete time-cost-quality tradeoff model for dealing with this problem. To solve the model, we then provided a hybrid intelligent algorithm integrating stochastic simulations and GA. The effectiveness of the proposed hybrid intelligent algorithm was illustrated by a numerical experiment at the last part.

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