Synchronization Control between Two Different Chaotic Systems

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Abstract

In this paper, the problem of chaotic synchronization is studied. The systems under consideration have different structure and dynamical feature. Also the systems are subject to disturbances. The problem addressed is the design of a synchronization scheme such that, the response chaotic system has an adaptive controller which can synchronize the driving chaotic system. The nonlinear controller under consideration is composed of the estimations of the two systems' original parameters. Lyapunov function is developed to confirm the chaotic synchronization scheme. A numerical example is presented to show the effectiveness and efficiency of the proposed synchronization scheme.

Keywords: Chaotic synchronization, Nonlinear controller, Different structure

1. Introduction

In recent years, there has been intensive interest in the research of synchronizing chaotic dynamical systems[1]-[3]. In the 1990, Pecora and Carroll addressed the synchronization of chaotic systems using drive-response scheme. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system follows the output of the master system asymptotically. Many methods for handling chaos control and synchronization have been developed, such as PC method, OGY method , active control, adaptive method, impulsive control, and coupling control, etc. The afore mentioned methods and many other existing synchronization methods mainly concern the synchronization of two identical chaotic systems. In real-life applications, however, it is hardly the case that the structure of drive and response chaotic systems can be assumed to be identical. Moreover, the system's parameters are inevitably perturbed by external inartificial factors and cannot be exactly known in priori. Therefore, synchronization of two different chaotic systems in the presence of unknown parameters is more essential and useful in real-life applications. Recently, the chaos synchronization of different chaotic systems is studied [7]-[11].

In this article we deals with the problem of synchronization between two different chaotic dynamical systems with unknown parameters. Firstly we design a proper nonlinear feedback controller and then inject it to the slave system. It is found that the master system and the slave system can reach synchronization although we do not know the systems' parameters in priori. The synchronization stability is proved by Lyapunov function. Disturbances has been a subject of great practical importance, the system is subjected to disturbances can also realize synchronization.

This paper is arranged as follows. The synchronization scheme is designed in section 2, the Genesio system and the Lorenz system are studied. Section 3 numerical simulations are provided to demonstrate the synchronization scheme. Finally the conclusion is given in Section 4.

2. The synchronization scheme

We consider the drive chaotic system in the form of

$$\dot{x} = f(x,\alpha) \tag{1}$$

and the response system is assumed by

$$\dot{y} = g(y,\beta) + u \tag{2}$$

x, y are the state vectors, α, β are the system parameters.

Let $e = y - x$ is the synchronization error, our goal is to design controller u such that the trajectory of the response system (2) with initial conditions y_0 can approaches the drive system (1) with initial condition x_0, describe as follow:

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|y(t, y_0) - x(t, x_0)\| = 0$$

Where $\| \ \|$ is the Euclidean norm.

Firstly, we take Genesio system as master system and the Lorenz system with controller as slave system. The master system (Genesio system) is:

$$\dot{x} = y$$
$$\dot{y} = z$$
$$\dot{z} = -ax - by - cz + x^2$$

The slave system (Lorenz system) is:

$$\dot{x}_1 = -a_1 x_1 + a_1 y_1 + u_1(t)$$
$$\dot{y}_1 = -x_1 z_1 + c_1 x_1 - y_1 + u_2(t)$$
$$\dot{z}_1 = x_1 y_1 - b_1 z_1 + u_3(t)$$

a, b, c, a_1, b_1, c_1 are the two systems' parameters. Systems are chaotic if the parameters' values are follows:

$$a = 6, b = 2.92, c = 1.2,$$
$$a_1 = 10, b_1 = 8/3, c_1 = 28$$

$u_1(t), u_2(t), u_3(t)$ are active control functions to be determined. The parameters error are $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1$. $\hat{a}, \hat{b}, \hat{c}, \hat{a}_1, \hat{b}_1, \hat{c}_1$ are estimations of original corresponding parameters. We design the controller with the estimations.

$$u = -g(y, \hat{\beta}) + f(x, \hat{\alpha}) \qquad (3)$$

In the above synchronization system. The controller

$$u = -g(x_1, x_2, x_3, \hat{a}_1, \hat{b}_1, \hat{c}_1) + f(x, y, z, \hat{a}, \hat{b}, \hat{c})$$
$$= \begin{pmatrix} -\hat{a}_1 y + \hat{a}_1 x + y \\ x_1 z_1 - \hat{c}_1 x_1 + y + z \\ -x_1 y_1 + \hat{b}_1 z - \hat{a} x - \hat{b} y - \hat{c} z + x^2 \end{pmatrix}$$

u is

Next we get the error dynamical system:

$$\dot{V} = e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z + \tilde{a}\ \dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} +$$
$$\tilde{c}\dot{\tilde{c}} + \tilde{a}_1\ \dot{\tilde{a}}_1 + \tilde{b}_1\ \dot{\tilde{b}}_1 + \tilde{c}_1\ \dot{\tilde{c}}_1$$
$$= e_x(-a_1 e_x + \tilde{a}_1 y_1 - \tilde{a}_1 x_1) + e_y(-e_y + \tilde{c}_1 x_1)$$
$$+ e_z(-b_1 e_z - \tilde{b}_1 z + \tilde{a} x + \tilde{b} y + \tilde{c} z)$$
$$+ \tilde{a}\ \dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{a}_1\ \dot{\tilde{a}}_1 + \tilde{b}_1\ \dot{\tilde{b}}_1 + \tilde{c}_1\ \dot{\tilde{c}}_1$$
$$= -a_1 e_x^2 - e_y^2 - b_1 e_z^2 + \tilde{a} x e_z + \tilde{b} y e_z$$
$$+ \tilde{c} z e_z + \tilde{a}_1 (y_1 e_x - x e_x) - \tilde{b}_1 z e_z + \tilde{c}_1 x_1 e_y$$
$$+ \tilde{a}\ \dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{a}_1\ \dot{\tilde{a}}_1 + \tilde{b}_1\ \dot{\tilde{b}}_1 + \tilde{c}_1\ \dot{\tilde{c}}_1$$

$$V(e_x, e_y, e_z, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1)$$
$$= (e_x^2 + e_y^2 + e_z^2 + \tilde{a}^2 + \tilde{b}^2 +$$
$$\tilde{c}^2 + \tilde{a}_1^2 + \tilde{b}_1^2 + \tilde{c}_1^2)/2$$

$$\dot{e} = g(y, \beta) + u - f(x, \alpha)$$
$$= g(y, \beta) - g(y, \hat{\beta}) + f(x, \hat{\alpha}) - f(x, \alpha)$$
$$= \begin{pmatrix} a_1 y_1 - a_1 x_1 - u_1 - y \\ c_1 x_1 - x_1 z_1 - y_1 + u_2 - z \\ x_1 y_1 - b_1 z_1 + u_3 + ax + by + cz - x^2 \end{pmatrix}$$
$$= \begin{pmatrix} -a_1 e_x + \tilde{a}_1 y_1 - \tilde{a}_1 x_1 \\ -e_y + \tilde{c}_1 x_1 \\ -b_1 e_z - \tilde{b}_1 z + \tilde{a} x + \tilde{b} y + \tilde{c} z \end{pmatrix}$$

To verify the synchronization system's stability, we design the Lyapunov function:

And we take the adaptive laws of parameters :

$$\dot{\hat{a}} = \dot{a} - \dot{\tilde{a}} = x e_z$$
$$\dot{\hat{b}} = \dot{b} - \dot{\tilde{b}} = y e_z$$
$$\dot{\hat{c}} = \dot{c} - \dot{\tilde{c}} = z e_z$$
$$\dot{\hat{a}}_1 = \dot{a}_1 - \dot{\tilde{a}}_1 = y_1 e_x - x_1 e_x$$
$$\dot{\hat{b}}_1 = \dot{b}_1 - \dot{\tilde{b}}_1 = -z e_z$$
$$\dot{\hat{c}}_1 = \dot{c}_1 - \dot{\tilde{c}}_1 = x_1 e_y$$

And we obtain the derivative of V :

$$\dot{V} = -a_1 e_x^2 - e_y^2 - b_1 e_z^2 \le 0$$

Since V is positive function and $\dot{V}$ is negative, it follows that:

$e_x(t), e_y(t), e_z(t) \in L_\infty$ and

$\tilde{a}(t), \tilde{b}(t), \tilde{c}(t), \tilde{a}_1(t), \tilde{b}_1(t), \tilde{c}_1(t) \in L_\infty$

Based on the Barbalat's lemma and for any initial condition, $e_x(t), e_y(t)$ and $e_z(t) \to 0$ as $t \to \infty$. Thus in the close-loop system we can get:

$x_1(t) \to x(t),\ y_1(t) \to y(t),\ z_1(t) \to z(t)$ as $t \to \infty$.

Hence the synchronization can be realized. With the nonlinear controller the slave system can synchronize the master system.

3. Numerical simulation

In this section, we give the scheme's simulation. Genesio system is the master system and Lorenz system is the slave system. And the parameters are:

$$a = 6, b = 2.92, c = 1.2$$

$$a_1 = 10, b_1 = 8/3, c_1 = 28$$

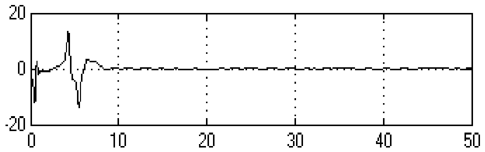
The systems' initial values are:

$$x = 3,\ y = -4,\ z = 2$$

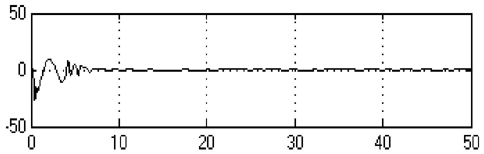
$$x_1 = -4,\ y_1 = 5,\ z_1 = 3$$

Thus the initial errors are –7, 9,1.

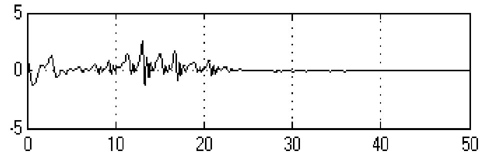
The simulation results are illustrated in fig.2. The corresponding error variations are demonstrated in fig1. One can find that the slave system traces the master system and finally becomes the same.



a: $x - x_1$



b: $y - y_1$



c: $z - z_1$

Fig1: Synchronization error (time: sec).

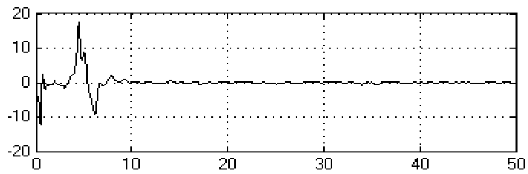
Noise may appear for different reasons, for instance measurement noise or uncertainties in the dynamics. In this case, synchronization becomes even more difficult to control. Here we explore the effect of external noise to the systems. Consider the genesio system (master system) with noise:

$$\dot{x} = y + \varepsilon_1$$

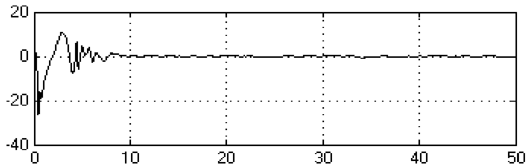
$$\dot{y} = z + \varepsilon_2$$

$$\dot{z} = -ax - by - cz + x^2$$

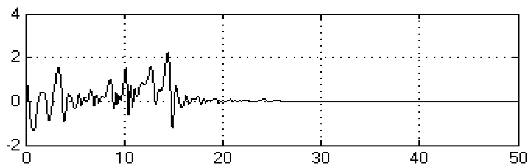
The white noise signal $\varepsilon_1 \sim N(0, 0.06)$ is added in the x channel and $\varepsilon_2 \sim N(0, 0.014)$ is added in the y channel. With the same systems parameters and initial values, numerical simulations demonstrate as fig2 show that the noise does not destroy the two systems' synchronization.



a: $x - x_1$



b: $y - y_1$



c: $z - z_1$

Fig2: Synchronization error under the noise (time: sec)

4. Conclusions

In this article, we demonstrate the problem of synchronization for a class of chaotic systems with different structure and unknown parameters. A nonlinear controller is developed. We estimate the parameters and construct the proper controller with these estimations. It is also found that this scheme has general meaning and is effective in synchronizing other chaotic systems.

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