The Fuzzy Lattice Order Decision-Making Method for Travel Mode Choice Based on The Travel Time Reliability

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Abstract

When the citizens make traffic mode choice, the bus and taxi are always abandoned for long waiting time, especially during the peak time. In view of this, this paper defines the concept of the travel time budget and the reliability of travel time. A traffic mode choice is proposed, considering four factors: budget travel time, reliability of travel time, travel costs and service quality. Based on the characteristics that the objective and subjective factors coexist in the travel traffic mode choice, the weight is determined by combining information entropy method and individual weight. The multi-objective lattice-order decision-making is put to solve the problem of traffic mode choice. The example implies that the method is effective.

Keywords: Traffic mode choice, Reliability of travel time, Information entropy, Lattice-order decision

1. Introduction

The problem of traffic mode choice is of great significance to the urban comprehensive transport planning management and forecast analysis to transport mode sharing and layout. Along with the development of urban economy and the process of urbanization intensifies, the city traffic demand rapidly increase. The alternative traffic modes for citizens have become diversiform. As the development of modern traffic information technology, the residents will take the factors such as travel time, costs, safety, comfort and the waiting time into the traffic mode choice consideration, and then make final decision.

A large number of domestic and foreign scholars [1, 2, and 3] use Logit Maximize utility theory to discuss the mode of traffic choice. The literature [4], [5] respectively give the traffic mode model and satisfaction criteria based on the value of travelers' behavior. All the traffic mode choice method above is based on the assumption which the travel time is reliable. Empirical studies show that the reliability of travel time between the OD pairs occupies important positions in the travelers' traffic mode choice. In order to meet the objectivity of the traffic mode choice better, this paper presents budget travel time and the travel time reliability concept. Both are taken into the factors of the choice besides the costs and service quality.

In 1930s, the theory of the lattice order was proposed. Yaohuang Guo [6] establishes the lattice-order decision-making theory which starts a new direction of decision-making. On the basis of these studies, the method of multi-objective lattice-order decision-making is put forward to solve the problem of the traffic mode choice.

2. The factors determination

The reliability of the travel time has played a very important role on the traffic mode choice. Different traffic mode has different reliability. For example, during the peak time the waiting time of taxi and bus are often long, which means the reliability is low. And then many travelers will choose the relatively high reliability traffic mode.

So the factors which affect the citizens' traffic mode choice can be divided into two main aspects: the traffic mode characteristic and the traveler' individual characteristic. The former include budget travel time, travel time reliability, budget costs and the service quality. The latter include the personal state of economy, tolerance and other factors. We use the individual weight to measure.

2.1. Travel time budget

Definition 1 when the citizens travel, the additional time is defined to be that the all travel time between the OD pairs detracts the running time of the traffic tools.

The additional time can not be ignored in the daily short travel. For example, the additional time of bus includes the time of walking to the bus station from the origin, waiting for the bus and walking from the station to the dictation. The additional time of taxi is mainly the time of waiting for the taxi. The ones of car and bike are the time of getting and taking time. Therefore, we consider that the budget travel time of choosing the k th traffic mode on the path p between the OD pairs rs, includes two parts: the running time of the traffic mode, namely T_{kp}, and the additional time, H_k.

Using the BPR function, the running time of the k th traffic mode on the link a, namely:

$$T_{ka}(x_{ka}, C_{ka}) = t_{ka}^0[1+\beta\left(x_{ka}/C_{ka}\right)^n] \quad (1)$$

t_{ka}^0, C_{ka}, T_{ka}, respectively are link a's free-flow travel time of the k th traffic mode (which is a deterministic parameter), capacity and travel time with flow x_{ka}; β, n are deterministic parameters.

The route travel time variable can thus be expressed by simply summing the corresponding link travel time variables:

$$T_{kp} = \sum_a (\delta_a^p \cdot T_{ka}) \quad (2)$$

where T_{kp} is the travel time of the k th traffic mode's traveling on the path p between the OD pairs rs. δ_a^p is the route-link incidence parameter whose value is one if a is on p; zero otherwise.

According to Bell and Iida's study [7], it is generally believed that T_{kp} belongs to the normal distribution $T_{kp} \sim N(\mu_1, \sigma_1)$, where μ_1 and σ_1, respectively, are the mean and standard deviation, whose values can be expressed below [8]. We assumes that the link capacity C_{ka} follows a uniform distribution in the interval $[\theta_{ak}c_{ak}, c_{ak}]$, c_{ak} is constant, $0 \le \theta_{ak} < 1$. Then we have

$$\mu_1 = \sum_a \left\{ \delta_{ka}^p \cdot \left[t_{ka}^0 + \beta t_{ka}^0 x_{ka}^n \frac{(1-\theta_{ka}^{1-n})}{c_{ka}^n \phi(1-n)} \right] \right\}$$

$$\sigma_1 = \sigma(T_{kp}) = \sqrt{\sum_a \left[\varphi \left\{ \frac{1-\theta_{ka}^{1-2n}}{c_{ka}^{2n}\phi(1-2n)} - \left[\frac{1-\theta_{ka}^{1-n}}{c_{ka}^n \phi(1-n)} \right]^2 \right\} \right]}$$

$$\varphi = \delta_{ka}^p \cdot \beta^2 (t_{ka}^0)^2 x_{ka}^{2n}$$

$$\phi = 1-\theta_{ka}$$

The additional time H_k follows normal distribution $H_k \sim N(\mu_2, \sigma_2)$, which the probability density function can be obtained from the travelers' experience.

Definition 2 For different attitudes to the late risk, the travelers' travel time budget is different. We define the travel time budget as:

$$T_k = E(T_{kp}) + \lambda \cdot \sqrt{\sigma^2(T_{kp}) + \sigma^2(H_k)}$$
$$= \mu_1 + \lambda \cdot \sqrt{\sigma_1^2 + \sigma_2^2} \quad (3)$$

Where λ is the pessimistic coefficient of the travelers' attitudes to the late risk, $\lambda < 0$ means the travelers are optimistic to the travel time, so they give shorter budget time than the pessimists whose the value of λ are higher.

2.2. The reliability of travel time budget

A lot of random factors change the state of traffic network, such as traffic accidents, weather conditions, road maintenance and even traffic jams. These random factors will lead directly to the uncertainty of travel time.

Definition 3 between the OD pairs rs, the reliability of the travel time budget can be defined as: considering with the uncertainty and randomness of the travel time, the probability of the travel time budget to be less than the practical travel time:

$$P_k = P\left(T_{kp} + H_k \le T_k\right) \quad (4)$$

We know:

$$T_{kp} \sim N(\mu_1, \sigma_1), H_k \sim N(\mu_2, \sigma_2)$$

And the two distributions are independent. Let

$$Z_k = T_{kp} + H_k$$

So we have:

$$Z_k \sim N(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$$

Set its distribution function to be F_k, formula (5) can be changed into:

$$P_k = P(Z_k \le T_k) = F_k(T_k) \quad (5)$$

Where P_k denotes the travel time reliability of the k th traffic mode. The bigger the P_k, the higher the reliability is.

2.3. Travel costs budget

Travel costs budget includes the traffic mode costs and the punitive costs caused by delay and late risk. The lower reliability of the travel time, the higher the late risk is, and so the costs. So in this paper we use travel time reliability to measure the punitive costs.

$$C_k = \mu_k(G_k) + \nu(1-P_k) \quad (6)$$

Where C_k is travel costs budget, and μ_k is cost function of the k th traffic mode, v is the traveler's punitive cost function based on travel time reliability.

2.4. Service quality

For the k th traffic mode, the service quality includes three indexes: comfort s_{k1} , safety s_{k2} and convenience s_{k3}. So we have

$$S_k = \eta_{k1}s_{k1} + \eta_{k2}s_{k2} + \eta_{k3}s_{k3} \tag{7}$$

Where S_k is the value of the service quality of the k th traffic mode, and η_{k1} , η_{k2} η_{k3} respectively denotes the weight of s_{k1}, s_{k2}, s_{k3}; $0 \le s_{ki}, \eta_{ki} \le 1$.

3. The unification and weight determination

Given optional traffic mode sets $\mathrm{S} = \{S_1, S_2, \cdots, S_k\}$ and the index sets corresponding the k th traffic mode $V = \{V_1, V_2, V_3, V_4\} = \{T_k, P_k, C_k, S_k\}$.

where T_k, P_k, C_k, S_k are, respectively, the travel time budget, reliability of travel time, travel costs and service quality.

The weight is $w = \{w_1, w_2, w_3, w_4\}$

By the determining the factors we can get the index matrix:

$$F = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{k1} & x_{k2} & x_{k3} & x_{k4} \end{bmatrix} \tag{8}$$

where x_{ij} denotes the j th index value of the i th traffic mode.

3.1. Index unification

As the indexes are of different numerical units, they should be unified into [0, 1] before the decision-making and evaluation. The indexes are divided into three categories: income-type, cost-type and moderate-type. The unified method is different to each type. The travel time budget and travel costs are of cost-type, and the reliability and service quality belong to the income-type. We use the proportional transfer law [9] to unify:

For income – type index:

$$x'_{ij} = x_{ij} \big/ x_j^{\max} \tag{9}$$

For cost-type index:

$$x'_{ij} = x_j^{\min} \big/ x_{ij} \tag{10}$$

Use the unified method above to matrix (9), we can get the normalized matrix

$$F' = \left(x'_{ij}\right)^{k\times 4} \tag{11}$$

3.2. The determination of the index weights

The determination of the weight of the index which affects the choice of traffic mode is a very complex question. The reason is that the weights of the factors not only have the relation with the attributes of themselves, but also are connected with the individual character. So the final weight is a synthesis of objective and subjective weights. In this paper, we combine the information entropy method and the individual weight to determine the weight. First the traveler gives the subjective weight of each factor as his/her preference, w_j^0 , then make out the objective weight by the information entropy, w_j^1, and the final weight is:

$$w_j = w_j^0 w_j^1 \Big/ \sum_j w_j^0 w_j^1 \tag{12}$$

We introduce the information entropy method to determine the objective weight w_j^1.

First, choose the minimum of each column $x_j'^{\min}$ of F', and $x_j'^{\min}$ is the optimal value of the j th index.

Let $s_{ij} = x_j'^{\min} \big/ x_{ij}$, then for some fixed j , the larger difference of s_{ij} is, then the larger the relative intensity of the index value between different scheme is. The more effect of the index to the scheme, the more decision-making information the index has. We can use the information entropy to measure this.

For the j th index, define its property:

$$s_j = \sum_{i=1}^{k} s_{ij}$$

The entropy measure of the relative intensity of the index value is:

$$e(s_j) = (-1/\ln k)\sum_{i=1}^{k} (s_{ij}/s_j)\ln(s_{ij}/s_j)$$

where $0 \le s_{ij} \le 1$， $0 \le e(s_j) \le 1$。

The total Information Entropy is: $e = \sum_{j=1}^{4} e(s_j)$

the objective weight w_j^1 determined by Information Entropy is：

$$w_j^1 = (1 - e(s_j))/(4 - e) \quad (13)$$

4. Multi-objective lattice-order decision-making

In the lattice order decision-making, if the selected scheme can form limited lattice, the top factor will be the optimal scheme. If not, then take positive ideal solution (PIS) and negative ideal solution (NIS) as virtual schemes [10] and regard them as top factor s and bottom factors respectively. Delete the dominance scheme and construct a lattice. Choose the optimal solution or satisfying solution by comparing the closeness of scheme with positive ideal solution with that of scheme with negative ideal solution. The closer the scheme and the fuzzy positive ideal solution are, the farther the scheme and the fuzzy negative ideal solution are, and the better it is. In other words, the smaller the comprehensive difference between the scheme and the ideal solution is, the better it is. The algorithm in this paper is: Firstly, weigh index value and then establish positive and negative ideal solutions. Calculate the differences between every scheme and each ideal solution and make final choice.

Step 1: get index value matrix F and establish the normalized index value matrix F' by equation (11), make weight w_j using the method in 2.2, Construct weighted decision-making matrix $R = \left[r_{ij} \right]_{k \times 4}$, and $r_{ij} = w_j x'_{ij}, \forall i, j$;

Step 2: Every objective fuzzy weighted index is in corresponding with fuzzy maximum set M_i that forms $M^+ = \left(M_1, M_2, M_3, M_4 \right)$. Every objective fuzzy weighted index is in corresponding with minimum m_i that forms $M^- = \left(m_1, m_2, m_3, m_4 \right)$. Take positive ideal solution M^+ as top factor, negative ideal solution M^- as bottom factor. Identify the relation between schemes and draw Hasse figure.

Step 3: Establish the difference between scheme S_i ($i = 1, 2, \cdots, k$) and M^+, namely

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left[d\left(r_{ij}, M_j \right) \right]^2} \quad (14)$$

And the difference between scheme S_i and M^- , namely

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left[d\left(r_{ij}, m_j \right) \right]^2} \quad (15)$$

where d means the Minkowski distance.

Step 4: Establish the comprehensive difference of scheme S_i , namely

$$D_i = q\frac{D_\mathrm{i}^+}{D} + \left(1 - q\right)\left(1 - \frac{\mathrm{D}_\mathrm{i}^-}{D}\right) \quad (16)$$

Where $i = 1, 2, \cdots, k$, $q \in \left[0, 1\right]$ and D is the difference between positive ideal solution M^+ and negative ideal solution M^-.

Step 5: The decision-maker chooses the optimal scheme (or satisfying scheme) according to the comprehensive difference D_i ($i = 1, 2, \cdots, k$). The smaller the D_i value is, the better the scheme i is.

5. Case analysis

To demonstrate the method, we apply them to the travel mode choice in Chengdu from place A to B during peak time on the morning. The available traffic modes contain: car, taxi, bus and bike. The travel time on common route belongs to the normal distribution,

Car: $T_{1\overline{p}} \sim N(28, 2.5)$

Taxi: $T_{2\overline{p}} \sim N(28, 2.5)$

Bus: $T_{3\overline{p}} \sim N(45, 4)$

Bike: $T_{4\overline{p}} \sim N(70, 2)$

The additional time of car and bike are the taking and putting time, which can be ignored. The additional time of taxi and bus: H ~ $N(10, 5)$. The probability density distribution figure is presented in Fig. 1. (All the numerical calculating is obtained in MATLAB 7.0)

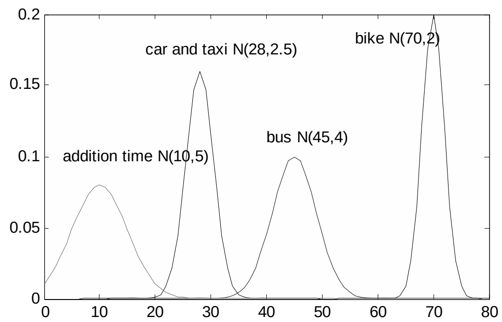

Fig. 1: the probability density distribution of each traffic mode

5.1. factors determination

We set the parameter value of λ for pessimistic strategy (long travel time budget, PE in short), medium strategy (medium travel time budget, ME in short), optimism strategy (short time budget, OP in short) to be, respectively, $\lambda=1$, $\lambda=0$ and $\lambda=-1$. From formula (3) ~ (7), we can get the value of travel time budget, reliability, cost and service quality for each travel mode. Fig.2 to Fig.5 schematically illustrates them respectively for three strategies above.

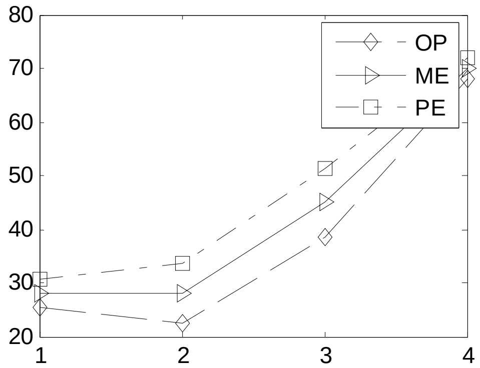

Fig. 2: the travel time budget of 4 traffic modes for 3 strategies.

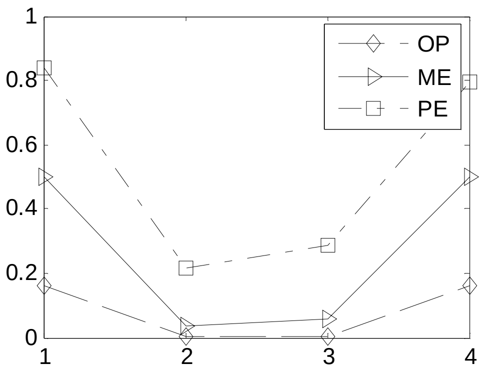

Fig. 3: the travel time reliability of 4 traffic modes for 3 strategies.

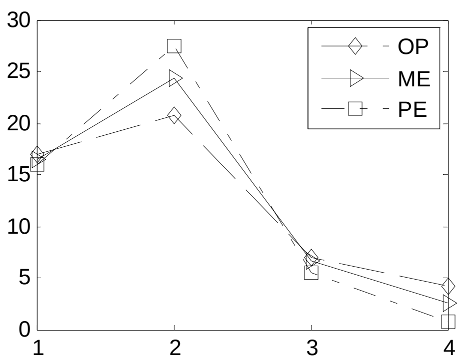

Fig. 4: the costs of 4 traffic modes for 3 strategies.

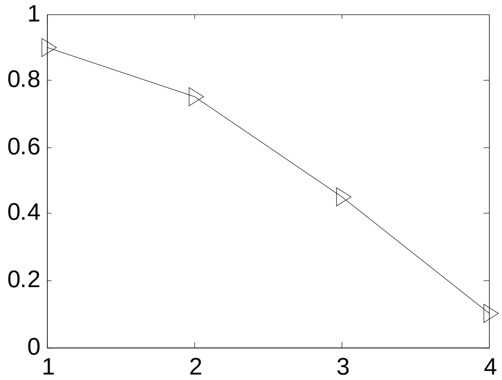

Fig. 5: the service quality of 4 traffic modes for 3 strategies.

5.2. Lattice-order decision-making

① Establish the index value matrix F .

For different pessimistic coefficient λ , $F(\lambda)$ is different. We can get three index value matrixes to the three types ($\lambda=-1,0,1$) through formula (8). Both travel time and cost belong to cost's index, so we use formula (9) to normalize; and the other two factors use the formula (10) to get normalized matrix F' .

② Weight decision

For each $F(\lambda)$, by using the information entropy method we can get the weight $w^1(\lambda)$ from formula （13）. We give five representative individual weights:

$$w^{01}=\{0.35,0.05,0.05,0.55\}$$

$$w^{02}=\{0.1,0.1,0.45,0.35\}$$

$$w^{03}=\{0.55,0.35,0.05,0.05\}$$

$$w^{04}=\{0.1,0.2,0.6,0.1\}$$

$$w^{05}=\{0.45,0.05,0.35,0.15\}$$

The final weight is determined by substituting $w^1(\lambda)$ and w^{0l} $(l=1,\cdots,5)$ into the formula (12)

③ Final decision-making

Construct weighted decision-making matrix $R=\left[r_{ij}\right]_{k\times 4}$ as Step 1in part 3. The comprehensive difference is determined from Step3 to Step5. Table1 shows the comprehensive difference of the 5 individual for three strategies: pessimism (PE), medium (ME), optimism (OP).

From the table2, we can see that people who take the optimism strategy always use the traffic mode with high reliability, and with the pessimism strategy take taxi and bus which are with low reliability.

Comprehensive Difference	1 PE ME OP	2 PE ME OP	3 PE ME OP	4 PE ME OP	5 PE ME OP
Car	0.21 0.08 0.01	0.89 0.71 0.42	0.22 0.06 0.02	0.95 0.75 0.40	0.93 0.80 0.56
Taxi	0.23 0.26 0.28	0.92 0.81 0.79	0.83 0.91 0.92	0.98 0.96 0.95	0.95 0.87 0.72
Bus	0.60 0.58 0.28	0.39 0.69 0.74	0.86 0.93 0.96	0.88 0.73 0.75	0.16 0.38 0.69
Bike	0.83 0.86 0.28	0.10 0.22 0.36	0.19 0.09 0.08	0.02 0.04 0.07	0.25 0.32 0.41

Table 1: the comprehensive difference of each traffic mode

	1	2	3	4	5
PE	car taxi	bike bus	bike car	bike	bus
ME	car	bike	car bike	bike	bike bus
OP	car	bike car	car bike	bike	bike car

Table 2: the final traffic mode choice of each individual

In other words, the travelers with high late risk will abandon the traffic mode with low reliability during the peak time. Travelers' traffic mode choices are different to distinct individual weight. We can get this obviously from table 2. The individual 1 who emphasizes the travel time and service quality choose the mode of car and taxi when the pessimistic strategy is taken, only car for others two. Individual 4 choose the bike as the best mode for the high weight to the travel costs. We can get the point that the lattice-order decision-making method has strong reasonability in traffic mode choice problem.

6. Conclusions

Recently the urban traffic jams phenomena have become more and more common. Considering with the travel time reliability into the traffic mode choice has great practical significance. Combining the information entropy weight and individual weight to determine the final weight can effectively show the difference of traffic mode choice behavior for different individual character and preference. The case make out that the lattice-order decision-making method considering both the reliability and individual character can impersonality describe the truth of traffic mode choice and have important practical significance.

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