

Predicting Short-time Traffic Flow Using Volterra Adaptive Model

Yumei Zhang Shiru Qu Kaige Wen

Department of Automatic Control, Northwestern Polytechnical University, Xi'an 710072, P. R. China

Abstract

An adaptive method for predicting short-time traffic flow, which is based on phase space reconstruction and Volterra series was proposed. We first employed Lyapunov exponent method based small data sets to validate that chaos exists in traffic flow. Then, phase space reconstruction for traffic flow data was performed. And we constructed Volterra adaptive prediction model, which coefficients were updated by LMS adaptive algorithm. We finally applied this model to execute simulations for chaotic time series and real measured traffic flow data. Experimental results show that Volterra adaptive model can effectively predict chaotic time series and traffic flow.

Keywords: Volterra series, Adaptive prediction, Phase space reconstruction, Traffic flow, Chaos

1. Introduction

Traffic guidance and control is usually considered as one of important components for intelligent transportation system (ITS). Real-time and exact prediction of traffic flow is the premise that realizes it. Because of strict real-time demands of traffic guidance and control, i.e. the maximum period of traffic control is usually about 3 minutes and the period of traffic guidance is usually 5 minutes, the key of ITS realization is to exactly and effectively predict traffic flow within 5 minutes. When the prediction period becomes small traffic flow usually shows very strong uncertainty and nonlinearity [1]-[2]. References [3]-[4] proved that traffic flow time series possesses some chaotic characteristic. Based on Takens' embedding theorem and phase space reconstruction [5]-[6] which provides theory foundation for predicting chaotic time series, many nonlinear prediction methods have been put forward. These prediction methods can be divided into global method, local method and adaptive method. Compared with global and local method, adaptive prediction method possesses many merits, in which Volterra adaptive method based on phase space reconstruction has been widely applied in some low-dimension chaotic time series predictions. This

adaptive method uses very small training samples to perform better predictions for chaotic series, and is therefore suitable for small data cases. This paper first applies Volterra series to traffic flow predictions and constructs Volterra adaptive prediction model for traffic flow. Subsequently, the adaptive model is applied to execute simulations for two classic chaotic time series generated by Lorenz and Rossler and real measured expressway traffic flow data.

This paper is formed as follows. Section 2 gives the flow chart of short-time traffic flow prediction; Section 3 describes phase space reconstruction for traffic flow time series; Section 4 develops the Volterra adaptive prediction model; Section 5 performs simulations for chaotic time series and the real measured expressway traffic flow data, and the last is conclusions.

2. Structure of short-time traffic flow prediction

Based on phase space reconstruction and Volterra series, an adaptive method for predicting short-time traffic flow is presented, as shown in Fig.1. At first, single-variable traffic flow time series $\{q(n)\}$ is acquired by observing the real traffic system. Then, phase space reconstruction for this time series is performed and we get a point in the reconstructed state space:

$$\vec{q}(n) = [q(n), q(n-\tau), \dots, q(n-(m-1)\tau)]^T$$

Subsequently, as long as proper embedding dimension m and delay time τ are selected, the geometrical construction of traffic dynamic system is equivalent to the one of m -dimensional reconstructed state space which can be used to make predictions for traffic flow time series. And, based on Volterra series expansion we construct Volterra adaptive prediction model, which coefficients are updated by LMS-type adaptive algorithm derived from least square error. We use this adaptive model to predict traffic flow in the reconstructed state space with input $\vec{q}(n)$ rather than the original traffic flow time series. The obtained reliable and exact short-time traffic flow prediction results from Volterra adaptive prediction method can

provide better reference and information for traffic guidance and control.

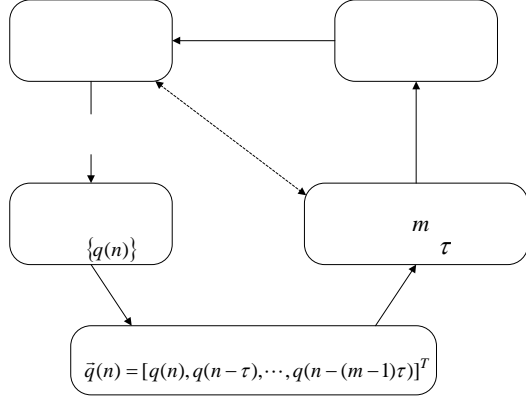


Fig. 1: Flow chart of short-time traffic flow prediction.

3. Phase space reconstruction for traffic flow time series

F. Takens [5] and R. Mane's [6] delay embedding theorem is the foundation for phase space reconstruction. This theorem indicates that so far as proper embedding dimension m and delay time τ are selected, the geometrical construction of the original chaos dynamic system is equivalent to the one of m -dimensional state space. So, they have the same topological structure, which means that prediction issue can be changed into traffic evolution process in the phase space so as to lay solid foundation for chaotic time series prediction [7].

Consider the observed single-variable traffic flow time series:

$$\{q(n)\}_{n=1}^N, q(n) = q(t_0 + n\Delta t) \quad (1)$$

where t_0 denotes initial time and Δt sample time interval. Select suitable embedding dimension m and delay parameter τ , and then reconstruct an m -dimensional state vector by equation (1):

$$\vec{q}(n) = [q(n), q(n-\tau), \dots, q(n-(m-1)\tau)]^T \quad (2)$$

$$n = N_0, N_0 + 1, \dots, N$$

From equation (2) we can get:

$q(N_0)$	$q(N_0+1)$	\dots	$q(N)$
$q(N_0-\tau)$	$q(N_0+1-\tau)$	\dots	$q(N-\tau)$
$q(N_0-2\tau)$	$q(N_0+1-2\tau)$	\dots	$q(N-2\tau)$
\vdots	\vdots	\vdots	\vdots
$q(N_0-(m-1)\tau)$	$q(N_0+1-(m-1)\tau)$	\dots	$q(N-(m-1)\tau)$
$\vec{q}(N_0)$	$\vec{q}(N_0+1)$	\dots	$\vec{q}(N)$

where $N_0 = (m-1)\tau + 1$, $\vec{q}(n)$ represents a phase space point. The m -dimensional sequence $\{\vec{q}(n) | n = N_0, N_0 + 1, \dots, N\}$ constructs a phase pattern which represents the instant state for traffic flow system. The states of $\vec{q}(n)$ in ascending time order can describe evolution trajectory of traffic flow system in m -dimensional phase space. The evolution progress of state space $\vec{q}(n) \rightarrow \vec{q}(n+1)$ reflects upon the traffic system evolution. So, we can use history traffic flow data to predict the future.

The selection of delay time τ and embedding dimension m has very important meaning in phase space reconstruction and also has relative difficulty. Reference [7] summarizes many methods, in which autocorrelation function method is a rather simple approach for deciding delay time τ , and mutual information method [8] broadly used in phase space reconstruction is an effective technique for estimating delay time τ . And the most employed technique for selecting embedding dimension m has false neighbor method [9] and prediction model. In this paper we use mutual information method to select delay time τ and employ false neighbor method to estimate embedding dimension m .

4. Volterra adaptive prediction model

Phase space reconstruction theory is the foundation of chaotic time series prediction. Assume that the observed traffic flow time series is $\{q(n)\}_{n=1}^N$, and then a point in the state space can be expressed as:

$$\vec{q}(n) = [q(n), q(n-\tau), \dots, q(n-(m-1)\tau)]^T$$

From Takens' theorem, the essence of prediction reconstruction for chaotic series is an inverse problem of dynamic system. This means that dynamic model is reconstructed by the condition of dynamic system, which is equivalent to constructing the model:

where $T > 0$ denotes the forward prediction step size. Because the observed sequence must contain some noise such as measure error and other disturbance, a practical prediction model can be written as:

$$q(n+T) = \hat{F}(\vec{q}(n)) + \varepsilon(t)$$

where $\varepsilon(t)$ is measurement noise or approximation error. So, this issue changes to the problem of constructing a prediction model.

Many methods exist for constructing a nonlinear function to approximate $F(\cdot)$. It has been proved that lots of nonlinearity system can be represented by Volterra series [10], which can be used to construct nonlinear prediction model $F(\cdot)$ for chaotic time series. Assume that the input of this model is:

$\vec{q}(n)=[q(n), q(n-\tau), \dots, q(n-(m-1)\tau)]$ and output is $y(n)=q(n+1)$. So, Volterra series expansion for chaotic time series can be expressed as:

$$\begin{aligned} q(n+1) &= F(\vec{q}(n)) = h_0 + \sum_{k=1}^{+\infty} y_k(n) \\ &= h_0 + \sum_{m_1=0}^{+\infty} h_1(m_1)q(n-m_1\tau) + \\ &\quad \sum_{m_1=0}^{+\infty} \sum_{m_2=0}^{+\infty} h_2(m_1, m_2) \cdot q(n-m_1\tau)q(n-m_2\tau) + \dots + \\ &\quad \sum_{m_1=0}^{+\infty} \sum_{m_2=0}^{+\infty} \dots \sum_{m_p=0}^{+\infty} h_p(m_1, m_2, \dots, m_p) \cdot q(n-m_1\tau) \dots q(n-m_p\tau) + \dots \end{aligned}$$

where $h_0, h_1, \dots, h_p, \dots$ are called the kernels of the system, y_k indicates the k -th order term which has the corresponding kernel h_k . In practical application, because this infinite series expansion is difficult to realize, we must take some strategies such as finite truncation and finite summation form. One broadly used is a second-order truncation form of the Volterra series:

$$\begin{aligned} \hat{q}(n+1) &= h_0 + \sum_{m_1=0}^{N_1-1} h_1(m_1)q(n-m_1\tau) + \\ &\quad \sum_{m_1=0}^{N_2-1} \sum_{m_2=0}^{N_2-1} h_2(m_1, m_2)q(n-m_1\tau)q(n-m_2\tau) \end{aligned}$$

From Takens' embedding theorem, a chaotic time series could completely describe dynamic action of the original dynamic systems if at least $m > 2d + 1$ (d is the dimension of dynamic systems) variables have been chosen. Let $N_1 = N_2 = m$, Volterra adaptive prediction model for traffic flow time series can be expressed as:

$$\begin{aligned} \hat{q}(n+1) &= h_0 + \sum_{i=0}^{m-1} h_1(i)q(n-i\tau) + \\ &\quad \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} h_2(i, j)q(n-i\tau)q(n-j\tau) \end{aligned} \quad (3)$$

Simultaneously, let:

$$H(n) = [h_0, h_1(0), h_1(1), \dots, h_1(m-1), h_2(0,0), h_2(0,1), \dots, h_2(m-1, m-1)]^T$$

$$Q(n) = [1, q(n), q(n-\tau), \dots, q(n-(m-1)\tau), q^2(n), q(n)q(n-\tau), \dots, q^2(n-(m-1)\tau)]^T$$

So, equation (3) can be rewritten as:

$$\hat{q}(n+1) = H^T(n)Q(n) \quad (4)$$

For Volterra prediction model described by equation (4), we use LMS-type adaptive algorithm derived from least square error to update its coefficients. When the vector of this model coefficient

uses $H(n)$ and the vector of input signal $Q(n)$ LMS algorithm can be described as follows [11]:

$$e^2(n) = d^2(n) - 2d(n)H^T(n)Q(n) + H^T(n)Q(n)Q^T(n)H(n)$$

$$H(n+1) = H(n) - 2\mu e(n) \frac{\partial e(n)}{\partial H(n)}$$

$$H(n+1) = H(n) + 2\mu e(n)Q(n)$$

where $\mu(0 < \mu < 2)$ is the convergence step size, $d(n)$ is the desired value and choose $d(n) = q(n)$. Fig.2 shows the structure of Volterra adaptive prediction model for traffic flow.

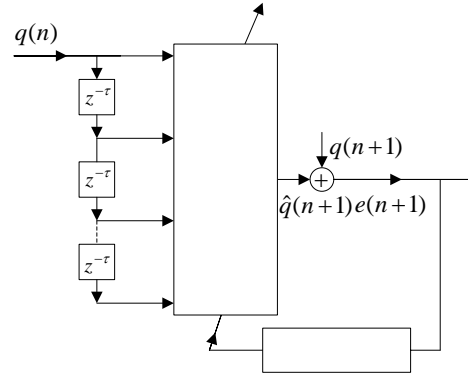


Fig. 2: Structure of Volterra adaptive model.

5. Simulation and Discussion

In this section, we will apply the proposed Volterra adaptive prediction model to perform simulations for the simulated chaotic time series and the real measured traffic flow data. First we normalize chaotic time series by equation (5):

$$x(i) = \frac{y(i) - \frac{1}{N} \sum_{k=1}^N y(k)}{\max(y(i)) - \min(y(i))} \quad (5)$$

where $\{y(i)\}$ represents the original time series, $\{x(i)\}$ normalized time series and N series length. In this paper, we take mean square error (MSE) as the evaluation criterion, which definition is:

$$MSE = \frac{1}{N_p} \sum_{k=1}^{N_p} |x(k) - \hat{x}(k)|^2 \quad (6)$$

where $x(k)$ is the desired value, $\hat{x}(k)$ prediction value and N_p series length. Next we will give the simulations for chaotic time series and the real measured traffic flow using second-order Volterra adaptive prediction model based on chaos dynamics.

5.1 Predictions for Lorenz and Rossler chaotic time series

In the experiment chaos mapping is directly iterative according to the initial value. The integral step size of Lorenz series takes 0.01 and Rossler 0.05. Then use fourth-order Runge-Kutta integral method to compute simulated time series data including 8000 points. For the sake of reducing instant influence, we only employ the last 2000 data as the experimental data, in which the former 500 data are used as training sample and the other 1500 data as testing sample.

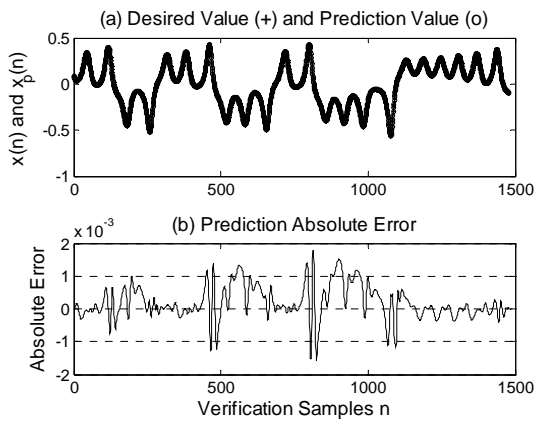


Fig. 3: Prediction results for Lorenz chaotic time series using second-order Volterra adaptive model. (a) A comparison between desired value $x(n)$ and prediction value $x_p(n)$; (b) Absolute error $e(n) = x(n) - x_p(n)$.

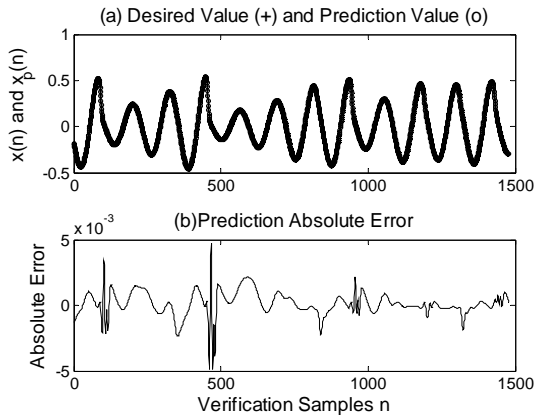


Fig. 4: Prediction results for Rossler chaotic time series using second-order Volterra adaptive model. (a) A comparison between desired value $x(n)$ and prediction value $x_p(n)$; (b) Absolute error $e(n) = x(n) - x_p(n)$.

Fig.3 and Fig.4 are prediction results for Lorenz and Rossler chaotic time series using Volterra adaptive prediction model, respectively. We can see that satisfying prediction results have been obtained. By computation from equation (6) we get the MSE for Lorenz time series is $5.9687e-006$ and Rossler time series $1.3401e-005$.

5.2 Traffic flow prediction

Traffic flow data used in the experiment come from the ring expressway in Xi'an city. Recording time is from April 2nd, 2006 on every Sunday, which covers consecutive ten weekday's morning traffic flow from 8:00 am to 11:00 am. Sample time interval is for 5 minutes. So, 360 data is obtained. Traffic flow data graph shows in Fig.5.

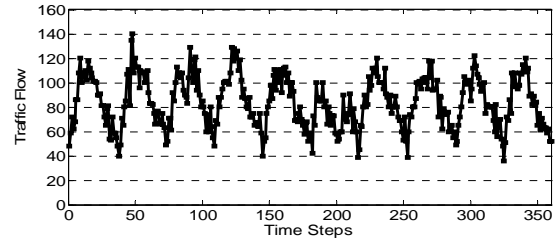


Fig. 5: Time series graph for traffic flow.

First we decide that whether chaos exists in traffic flow data. By computing Lyapunov exponent based small data sets we get the maximum value $\lambda_1 = 0.610$ that is positive, which means that chaos exists in traffic flow system. Then, we figure out the best delay time $\tau = 7$ and the least embedding dimension $m = 4$ for traffic flow time series $\{q(n)\}_{n=1}^{220}$ respectively using mean mutual information method [8] and false nearest neighbor method [9], as shown in Fig.6 and Fig.7. So, phase

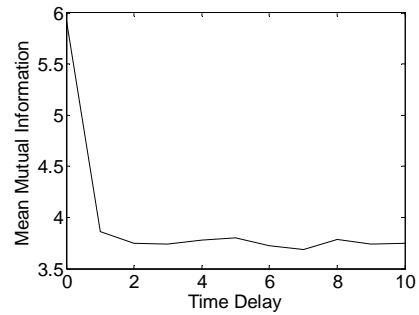


Fig. 6: Graph for delay time using mean mutual information method.

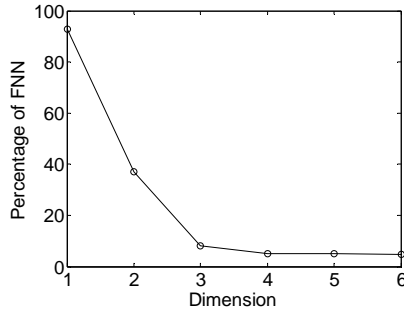


Fig. 7: Graph for embedding dimension using false nearest neighbor method.

Space reconstruction for traffic flow time series is:

$$\vec{q}(n) = (q(n), q(n-7), q(n-14), q(n-21))$$

Let $\{q(n)\}_{n=1}^{220}$ as training sample to train the second-order Volterra adaptive model. Then predict the last 140 traffic flow data $\{q(220+p)\}_{p=1}^{140}$, which prediction results are shown in Fig.8. Simulation results show that tendency for prediction value is consistent with the desired value. Prediction MSE is $2.332e-003$, which shows that prediction results are capable of reflecting upon the tendency and rule for traffic flow. And, prediction precision is rather high, which can completely satisfy high precision demand for traffic guidance and control.

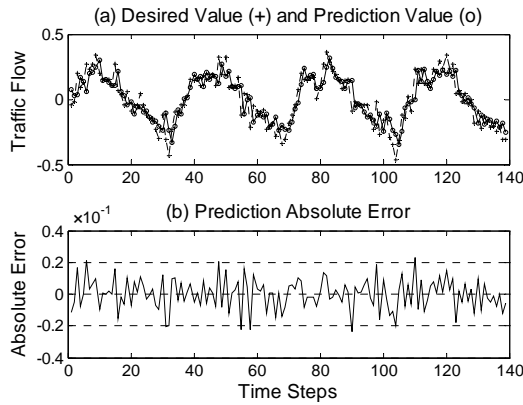


Fig. 8: Prediction results for traffic flow time series using second-order Volterra adaptive model. (a) A comparison between desired value $x(n)$ and prediction value $x_p(n)$; (b) Absolute error $e(n) = x(n) - x_p(n)$.

6. Conclusions

We employed Volterra series expansion to construct a nonlinear adaptive prediction model, which is based on phase space reconstruction theory, and applied it to predict two chaotic time series and the real measured traffic flow. Simulations show that the second-order Volterra adaptive prediction model proposed in this

paper is capable of effectively predicting chaotic time series and traffic flow series. And prediction precision is rather high. This is because that the proposed Volterra adaptive prediction model can effectively make use of linear and nonlinear factors and high moments of chaotic time series. So, for traffic guidance and control, the proposed method provides meaningful attempts for exactly predicting short-time traffic flow from the aspect of nonlinear time series analysis.

Acknowledgement

This paper is supported by the National Nature Science Foundation of China (Grant No. 60134010).

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