# **Closed-loop Virtual Reference Feedback Control System Design**

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*Abstract*—Firstly the controller using virtual reference feedback control design, at the same time considering the expectation of a given model match the closed-loop transfer function and sensitivity function, which need to be in the two objective function increases the expression in a model of the sensitivity function matching. Finally the controller parameters iteration algorithms are obtained by mathematical method. The asymptotic variance matrix can be applied to the control of virtual reference feedback correction virtual signal design.

Keywords-controller; virtual reference feedback; iteration; closed-loop

### I. INTRODUCTION

In this paper, on the basis of the adaptive control theory study a model reference adaptive control based on data driven virtual reference feedback corrective control. This method can effectively solve the unknown object in the closed-loop system model of parameterized design of the controller, the controller design problem is converted into parameter identification process optimization [1]. Presents a new adaptive control method, the basic idea, core technology and key points are as follows.

For the completeness of the narrative, this section in the appropriate place for the closed loop system the basic theory of control design method of two degree of freedom controller made simple introduction[2].

## II. THE ESTABLISHMENT OF THE VIRTUAL REFERENCE FEEDBACK CORRECTIVE CONTROL

First briefly introduce the idea of virtual reference feedback correction control. Hypothesis for a given two controller  $\{(C_1(z,\theta), C_2(z,\eta))\}$ , the control system is just meet r(t) and y(t) by the closed loop transfer function M(z), if the closed loop system is with arbitrary reference signal r(t), and that d(t) = 0, the output of the closed-loop system[3]

$$y(t) = M(z)r(t) \tag{1}$$

The idea of virtual reference feedback correction control including reasonably select virtual reference input signal  $\overline{r}(t)$  and the disturbance signal  $\overline{d}(t)$ , makes the controller

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design process becomes easier. Therefore respectively  $\overline{r}(t)$ 

and  $\overline{d}(t)$  introduces its construction process:

1) The structure of the virtual reference input signal  $\overline{r}(t)$ 



Figure 1 the structure of the virtual reference input signal

Given the observed output data y(t), define a virtual reference signal  $\overline{r}(t)$ :

$$y(t) = M(z)\overline{r}(t) \tag{2}$$

Because of the mathematical model of controlled object P(z) is unknown, but because of when P(z) input signal u(t) excitation, the output signal is y(t). Therefore, may be required by the proper choice of two controllers  $\{(C_1(z,\theta), C_2(z,\eta))\}$ , made when the closed loop system with virtual reference signal  $\overline{r}(t)$  and d(t) = 0, the closed-loop system can generate the desired u(t). Among them, the virtual reference input signal  $\overline{r}(t)$  structure as shown in figure 1.

From figure 1 reference tracking error can be defined as:

$$\varepsilon(t) = \overline{r}(t) - C_2(z,\eta) y(t) = (M^4 - C_2(z,\eta)) y(t)$$
(3)

Which are available from figure 2, when the closed loop system  $\overline{r}(t)$ , d(t) = 0 and y(t), the closed-loop system u(t) is generated:

$$u(t) = C_1(z,\theta)\varepsilon(t) = C_1(z,\theta) \left( M^{-1}(z) - C_2(z,\eta) \right) y(t)$$
(4)

2) The structure of the virtual disturbance signal  $\overline{d}(t)$ 



Figure 2 the structure of the virtual disturbance signal

A given output observation data y(t), define a virtual disturbance signal  $\overline{d}(t)$ , makes input signals of the closed-loop system is 0, that is r(t) = 0, the closed-loop system are only incentive  $\overline{y}(t)$  when the output of the disturbance signal  $\overline{d}(t)$ 

$$\overline{y}(t) = y(t) + \overline{d}(t) \tag{5}$$

The structure of the virtual disturbance signal  $\overline{d}(t)$  shown I figure 2, among them, the disturbance signal  $\overline{d}(t)$  there

shall ensure that[4]:

$$y(t) + \overline{d}(t) = S(z)\overline{d}(t)$$
(6)

Two transfer functions M(z) and S(z) expectations are known in advance, and no longer appear in this expression P(z), the mathematical model of controlled object.

#### III. ITERATIVE LEAST RECOGNITION

Considering virtual reference feedback correction control parameter optimization problems occurred in the iteration, and in-depth analysis of the iterative solutions converge to the global minimum in the degree of approximation, under the condition of this approximation formula measure the convergence rate of iterative approximate solution sequence[5].

$$r(\boldsymbol{\xi}) = \left[r_{f_{1}}(\boldsymbol{\xi}), \cdot; r_{f_{N}}(\boldsymbol{\xi}) \, r_{s_{1}}(\boldsymbol{\xi}), \cdot; r_{s_{N}}(\boldsymbol{\xi})\right]^{T}, N \ge n \tag{7}$$

It represents a minimization problem as you can see form as a nonlinear least squares problems, nonlinear least squares problem can get special circumstances as an unconstrained minimization, and can see as a solution of equations:

$$\begin{cases} r_{ft}(\xi) = 0\\ r_{st}(\xi) = 0 \end{cases}, t = 1, 2, \cdots; N$$
(8)

For target rule function,  $A(\xi)$  is a vector  $r(\xi) = [r_{f1}(\xi), \dots, r_{fN}(\xi), r_{s1}(\xi), \dots, r_{sN}(\xi)]^T$  of Jacobi matrix, which  $A(\xi)$  has the following structure<sup>[6]</sup>.

So the objective criterion function  $J_{VR}^{N}(\xi)$  of quadratic model is:

$$m\xi = J_{R}^{N}(\xi) + g^{T}(\xi)\xi + \frac{1}{2}\xi^{T}(\xi)\xi = \frac{1}{2}\xi^{T}(\xi)\xi = \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}\xi^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}\xi^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) = \int_{-\infty}^{0} \frac{1}{2}f^{T}(\xi)h(\xi) + \frac{2}{N}A_{n}^{T}h(\xi) + \frac{2}$$

Thus the corresponding iteration formula is:

$$\xi_{k+1} = \xi_{k} - \left(\frac{2}{N}\mathcal{A}(\xi_{k})\mathcal{A}(\xi_{k}) + S(\xi_{k})\right)^{-1} \left(\frac{2}{N}\mathcal{A}(\xi_{k})r(\xi_{k})\right)$$
(10)

The main problem is the second order items  $S(\xi)$  in Hesse matrix, is often difficult to calculate or take larger workload, and the use of the secant  $G(\xi)$  approximation. So ignore the second order  $S(\xi)$  of information in the rule of target function  $G(\xi)$ , so the (10) can be rewritten as[7]:

$$\bar{m}(\xi_{\star}) = \frac{1}{N} r^{T}(\xi_{\star}) r(\xi_{\star}) + \left(\frac{2}{N} \mathcal{A}(\xi_{\star}) r(\xi_{\star})\right)^{T} \xi_{\star} + \frac{1}{2} \xi_{\star}^{\mathcal{F}} \left(\frac{2}{N} \mathcal{A}(\xi_{\star}) \mathcal{A}^{T}(\xi_{\star})\right) \xi_{\star}$$
(11)

So (11) can be written as:

$$\begin{aligned} & (12) \\ \xi_{\pm} = \xi \left( \frac{2}{N} \frac{3}{4} \frac{3}{4} \frac{3}{4} \right)^{-1} \left( \frac{2}{N} \frac{3}{4} \frac{3}{4} \frac{3}{4} \right) = \xi + \xi \\ & \xi = \left( A \xi \frac{3}{4} \frac{3}{4} \right)^{-1} \left( A \xi \frac{3}{4} \frac{3}{4} \right)^{-1} \left( A \xi \frac{3}{4} \frac{3}{4} \right) = \xi + \xi \end{aligned}$$

Therefore VRFT control parameter optimization problem in the k iteration of the process of iteration algorithm is:

$$\begin{cases} a : \text{resolve } A(\xi_k) A^T(\xi_k) s_k = -A(\xi_k) r(\xi_k) \\ b : \text{Assuming } \xi_{k+1} = \xi_k + s_k \end{cases}$$

 $r_{ft}(\xi)$  and  $r_{st}(\xi)$  the specific expression, gives its corresponding to the first or second order partial derivative of the results, thus we can see: multiple occurrences of zero

in the Jacobi matrix and Hesse matrix elements, it will greatly simplify the operation of the actual process.

However, the choice of initial ellipsoid for the convergence rate of the iterative algorithm is also very important. In Lyapunov function[8,9], on the basis of proof when the initial value to satisfy some symbols according to the constraint condition, the initial value after continually updated iteration to get the iterative parameter vector sequence.

#### IV. THE SIMULATION EXAMPLE

Consider the discrete time linear systems, the system model of the transfer function is[10]:

$$\xi_{k+1} = \xi_k - \left(\frac{2}{N}A(\xi_k)A^T(\xi_k) + S(\xi_k)\right)^{-1} \left(\frac{2}{N}A(\xi_k)r(\xi_k)\right)$$

 $\lceil n \rceil$ 

E 0.25

The controller is PID controller:

$$C_{1}(\theta) = \alpha^{T}(z)\theta = \begin{bmatrix} \frac{z^{4}}{z^{4}-z} & \frac{z^{3}}{z^{4}-z} & \frac{z^{2}}{z^{4}-z} & \frac{z}{z^{4}-z} & \frac{1}{z^{4}-z} \end{bmatrix} \begin{bmatrix} \theta_{1} \\ \theta_{2} \\ \theta_{3} \\ \theta_{4} \\ \theta_{5} \end{bmatrix}$$
$$C_{2}(\eta) = \beta^{T}(z)\eta = \begin{bmatrix} \frac{z^{4}}{z^{4}-z} & \frac{z^{3}}{z^{4}-z} & \frac{z^{2}}{z^{4}-z} & \frac{z}{z^{4}-z} & \frac{1}{z^{4}-z} \end{bmatrix} \begin{bmatrix} \eta_{1} \\ \eta_{2} \\ \eta_{3} \\ \eta_{4} \\ \eta_{5} \end{bmatrix}$$

Two real controller is respectively:

$$C_{1}(\theta) = \alpha^{T}(z)\theta = \begin{bmatrix} \frac{z^{4}}{z^{4}-z} & \frac{z^{3}}{z^{4}-z} & \frac{z^{2}}{z^{4}-z} & \frac{z}{z^{4}-z} & \frac{1}{z^{4}-z} \end{bmatrix} \begin{bmatrix} 0.35 \\ 0.24 \\ 0.11 \\ 0\\ -0.02 \end{bmatrix}$$
$$C_{2}(\eta) = \beta^{T}(z)\eta = \begin{bmatrix} \frac{z^{4}}{z^{4}-z} & \frac{z^{3}}{z^{4}-z} & \frac{z^{2}}{z^{4}-z} & \frac{z}{z^{4}-z} & \frac{1}{z^{4}-z} \end{bmatrix} \begin{bmatrix} 0.28 \\ 0.1 \\ 0.02 \\ 0.01 \\ 0.00 \\ -0.01 \end{bmatrix}$$

Two PID controller of the communist party of contains 10 unknown parameter values need to identify an estimate. Using the iterative least recognition algorithm to identify parameters of the simulation results are shown in figure 3. The figure 4 shows: although both methods can obtain the final convergence parameters, but the method more smoothly, and conjugate gradient method in the process of iteration is undulating, unable to give him a correct prediction.







In order to show to the image, the identification results of controller in the closed-loop transfer  $P(z)C_1(z,\theta)$ function to the expectations of  $1+P(z)C_1(z,\theta)C_2(z,\eta)$ proximity between transfer real the closed-loop function M(z)and also need to test 1

 $\overline{1+P(z)C_1(z,\theta)C_2(z,\eta)}$  with real proximity between the desired sensitivity function S(z) [11].

Figure 5 shows the real closed-loop system transfer function and the design method of this paper the VRFT obtained closed-loop system transfer function of the Bode plot comparison graph, figure 6 shows the real closed-loop system sensitivity function and the design method of this paper the VRFT get bird figure contrast sensitivity function of the closed-loop system. The figure 6 shows the closedloop transfer function identification performance at low frequency have larger deviation, only when demand is gradually increasing in frequency identification accuracy are improved greatly.



Figure5 real closed-loop transfer function

(—real closed-loop system; o identify the closed-loop system)



Figure6 real closed-loop sensitivity function

(-real closed-loop sensitivity; o identification of closed loop sensitivity)

### V. CONCLUSIONS

Virtual reference feedback corrective control method is used for unknown parameters calibration of two degrees of freedom closed loop controller. At the same time the equations of the filter used for pretreatment of observation data expression, thus ensuring model reference control. When the objective function is derived by means of the jacobian matrix and hessian matrix in the global minimum, the iterative least squares identificate algorithm of the iterative sequence generated by approximation with global optimal value relation. Also the asymptotic variance matrix can be applied to the control of virtual reference feedback correction virtual signal controller design and inspection process.

#### REFERENCES

- [1] Alexandre S. Bazanella. Iterative minimizaation of H2 control performance criteria[J]. Automatica, 2008, 44(3): 2549-2559.
- [2] Cristian R. Rojas, Märta Barenthin, James S. Welshb and Håkan Hjalmarsson. The cost of complexity in system identification: The Output Error case[J]. Automatica, 2011,47(9):1938-1948.
- [3] G. Yin, Yu Sun and Le Yi Wang. Asymptotic properties of consensustype algorithms for networked systems with regime-switching topologies[J]. Automatica, 2011,47(7):1366-1378.
- [4] G. Yin, Le Yi Wang and Shaobai Kan. Tracking and identification of regime-switching systems using binary sensors[J]. Automatica,2009,45(4):944-955.
- [5] Henrik Ohlsson ,Lennart Ljung. Identification of switched linear regression models using sum-of-norms regularization[J]. Automatica,2013,49(4):1045-1050.
- [6] Lennart Ljung , Adrian Wills. Issues in sampling and estimating continuous-time models with stochastic disturbances[J]. Automatica, 2010, 46(5): 925-931.
- [7] Henrik Ohlsson ,Fredrik Gustafsson and Lennart Ljung. Smoothed state estimates under abrupt changes using sum-of-norms regularization[J]. Automatica, 2012, 48(4):595-605.
- [8] A.S.Bazanella, X.Bombois and M.Gevers. Necessary and sufficient conditions for uniqueness of the minimum in Prediction Error Identification[J]. Automatica, 2012, 48(8):1621-1630.
- [9] L. Campestrini, D. Eckhard and M. Gevers. Virtual Reference Feedback Tuning for non-minimum phase plants[J]. Automatica,2011,47(8):1778-1784.
- [10] M. C. Campi ,M. Vidyasagar. Learning With Prior Information[J]. IEEE Transanctions on automatic control,2011,46(11):1682-1695.
- [11] Maciej Niedźwiecki, Marcin Ciołek. Elimination of Impulsive Disturbances From Archive Audio Signals Using Bidirectional Processing[J]. IEEE Transanctions on Audio, Speech, and Language Processing, 2013, 21(5):1046-1059.