An Inventory Strategy with Emergency Order in Cluster Supply Chain

Benhe Gao [1]
Logistics Management Division
Graduate School at Shenzhen, Tsinghua University
Shenzhen, China
gaobh@sz.tsinghua.edu.cn

Yili Jiang [2]
Logistics Management Division
Graduate School at Shenzhen, Tsinghua University
Shenzhen, China
jiangyili1990@126.com

***Abstract*-The enterprises in the cluster supply chain network not only cooperate in a chain, but also compete and cooperate cross other supply chains. We focus on bidirectional replenishment policy for the enterprises in two chains so that both of the enterprises can reduce cost. Meanwhile, we provide a research method of horizontal cooperation for enterprises in the cluster supply chain network.**

Keywords- cluster supply chain; bidirectional emergency replenishment; replenishment cross the chains

I. INTRODUCTION

In some provinces of China like Guangdong and Zhejiang, the regional economy is treated as lump economy. There are many small and medium enterprises gathering in these places and they form a cluster supply chain network. Li Jizi(2004) says cluster supply chain is an organizational integration of industrial cluster and supply chain. Generally, the inventory system of small enterprise is small-scale, while sometimes they are faced with emergency conditions like some large order or delayed normal order, so emergency replenishment appears across supply chain. That is, some enterprises located in different supply chains ask for emergency order or provide replenishment with each other. Similar problem has been studied by many scholars.

In the early time, Barankin(1961), Daniel(1962) and Neuts(1964)do research on a periodic review model for an inventory system with normal and emergency orders. They suppose the normal orders need a lead time, while emergency orders can arrive at once.

Bulinskaya(1964), Fukada(1964) and Veinott(1966) extend the length of lead time. Whittmore (1978) constructs a dynamic model which has multi period and the lead times can be long or short. Blumenfeld et.al(1985) introduces an inventory strategy with emergency order. He assumes emergency replenishment quantity is big enough so that shortage can be avoided. Gross and Soriano(1972),Chiang and Gutierrez(1996) analyze a period review model for an inventory strategy with emergency order between two suppliers. Li Jizi (2005) provides a periodic review model with bidirectional replenishment for an inventory system. Moinzadch and Schmidt(1991) use the inventory policy --(S-1,S) to solve the problem, while Thorstenson(1998) has a further study on (Q,R) policy. Then Johansen and Thorstenson (1998) develop a model that (Q,R) policy is used for normal order,(S,s) policy is used for emergency replenishment.

II. COOPERATION FOR INVENTORY SYSTEM IN CLUSTER SUPPLY CHIAN

Li Jizi(2004) regards cluster supply chain as the coupling organizing form between supply chain and industrial cluster. The enterprises in the cluster supply chain network not only cooperate in a chain, but also compete and cooperate cross other supply chains, which can improve the enterprises' work efficiency and service quality, and circumvent the market risk in some extend. There are a variety of ways for inventory coordination in cluster supply chain. We divide it to the following 2 types:

A. Unidirectional /Bidirectional emergency replenishment for Homogeneous Enterprises at Single/Multi stage ,

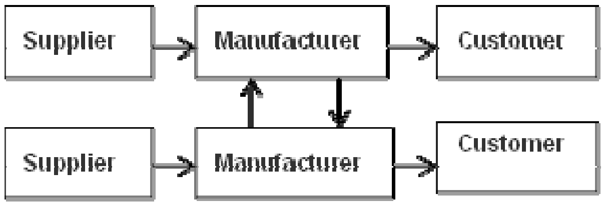

Figure1. Replenishment for Homogeneous Enterprises

B. Unidirectional /Bidirectional emergency replenishment for Inhomogeneous Enterprises at Single/Multi stage ,

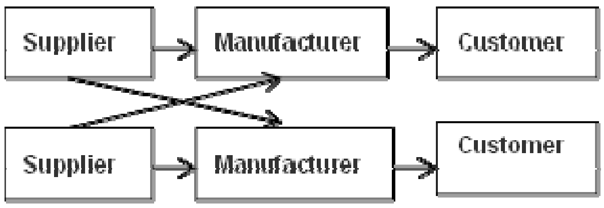

Figure2. Replenishment for inhomogeneous Enterprises

III. BASIC ASSUMPTIONS

Take the bidirectional emergency replenishment inventory system between two homogeneous enterprises in the cluster supply chain for example.

We assume this cluster supply chain consists of two single chains. Each chain consists of three parts: Supplier(S), Manufacture(R), Customer(C), R_1 can order from upstream S_1, also can ask for emergency replenishment from R_2 .Similarly , R_2 can order from upstream S_2 or R_1 in the other supply chain. We assume inventory levels are reviewed periodically. When it drops to a certain value，it may be out of stock or has been out of stock, then the manufacturer will ask for emergency replenishment.

Table 1 and Table.2 are the summary of parameters.

Table 1. Decision variables in the model

Decision Variables	Explanation
$S^{(k)}$	Inventory level target under normal order ,use(up to S) policy, k=1，2
$S_0^{(k)}$	Inventory level target under emergency order ,use(up to S_0)policy, k=1，2
$Q^{(k)}$	Inventory level for deciding whether to provide emergency replenishment or not

Table 2. Other parameter variables in the model

Parameter Variables	Explanation
T	The cycle length for a periodic review.
L	The lead time that orders are placed under normal order
$D^{(k)}$	Expected demand in a cycle $D^{(k)} \sim F^{(k)}(\mu^{(k)}, \sigma^{(k)})$
t	The point that manufacturer has transverse periodic inventory review.
i	The i th day in a cycle.
$Q_e{}^{(k)}$	Emergency ordering quantity
C	Normal ordering cost
C_h	Holding cost
C_p	Shortage cost
C_e	The extra cost for emergency replenishment
$OH_i^{(k)}$	On-hand inventory of manufacturer on i th day in a cycle
$OH^{(k)}$	On-hand inventory of manufacturer on $(t+1)$ th day
$BO_i^{(k)}$	The number of units backordered on i th day in a cycle, k=1,2
$BO^{(k)}$	The number of units backordered in a cycle
$NS_i^{(k)}$	The inventory level on the i th day, i=1,2…T , k=1,2

IV. ANALYSIS AND CALCULATION

The model for the inventory cost is (1).

$$\mathrm{C} = CR + CH + CP + CE - RE \tag{1}$$

CR denotes the normal ordering cost, CH denotes the Holding cost, CP denotes shortage cost, CE denotes the extra cost for emergency replenishment, RE denotes the extra profit for emergency replenishment. Since CR is constant, we can write $\mathrm{E}(\mathrm{C}^{(k)})$ as following:

$$\begin{aligned}\mathrm{E}(\mathrm{C}^{(k)}) &= E(CH^{(k)} + CP^{(k)} + CE^{(k)} - RE^{(k)}) \\ &= c_h \times \sum_{i=1}^{T} E(OH_i^{(k)}) + c_p \times E(BO^{(k)}) \\ &+ c_e \times (E(Q_e^{(k)} \times I_0^{(3-k)}(Q_e^{(k)})) - E(Q_e^{(3-k)} \times I_0^{(k)}(Q_e^{(3-k)})))\end{aligned} \tag{2}$$

When $OH_t^{(1)} < S_0^{(1)}$, R1 asks for emergency replenishment from R2, the quantity $Q_e^{(1)} = S_0^{(1)} - OH_t^{(1)}$ .While R2 decides whether to provide replenishment by (3):

$$I_0^{(2)} = \begin{cases} 1 & OH^{(2)} - Q_e^{(1)} \geq Q^{(2)} \\ 0 & OH^{(2)} - Q_e^{(1)} < Q^{(2)} \end{cases} \tag{3}$$

The inventory level of R1 and R_2 can be wrote as (4):

$$NS_i^{(1)} = OH^{(1)} - D_{i-t}^{(1)},\ NS_i^{(2)} = OH^{(2)} - D_{i-t}^{(2)},$$
$$i = t+1, t+2, ..., T\ . \tag{4}$$

Let $E(C_0{}^{(1)})$ denote the expected cost of R1 under normal order , $E(C^{(1)})$ denote the expected cost under bidirectional emergency order .

$$\begin{aligned} &E(C^{(1)}) \\ &= E(C_0{}^{(1)}) + c_h \sum_{i=1}^{T} \Delta E(OH_i^{(1)}) + \\ &c_p \Delta E(BO^{(1)}) + c_e E(Q_e^{(1)} I_0^{(2)}) - c_e E(Q_e^{(2)} I_0^{(1)}) \\ &= E(C_0{}^{(1)}) + c_h \sum_{i=t+1}^{T} \Delta E(OH_i^{(1)}) + \\ &c_p \Delta E(OH_T^{(1)}) + \left(c_e - c_p\right)\left(E(Q_e^{(1)} I_0^{(2)}) - E(Q_e^{(2)} I_0^{(1)})\right)\end{aligned} \tag{5}$$

When i=t+1,t+2,…,T, the increments of on –hand inventory of R1 and R2 can be wrote as (6) :

$$\begin{aligned} &E(Q_e^{(1)} I_0^{(2)}) = \int_{S^{(1)} - S_0^{(1)}}^{+\infty} \left(S_0^{(1)} - S^{(1)} + y\right) F_{t+L}^{(2)}(S^{(2)} + S^{(1)} - S_0^{(1)} - Q^{(2)} - y) f_{t+L}^{(1)}(y) dy \\ &E(Q_e^{(2)} I_0^{(1)}) = \\ &\int_{S^{(2)} - S_0^{(2)}}^{+\infty} \left(S_0^{(2)} - S^{(2)} + y\right) F_{t+L}^{(1)}(S^{(1)} + S^{(2)} - S_0^{(2)} - Q^{(1)} - y) f_{t+L}^{(2)}(y) dy \end{aligned} \tag{6}$$

So according to the above equations,

$$\begin{aligned} E(C^{(1)}) &= E(C_0{}^{(1)}) + c_h \sum_{i=t+1}^{T} \Delta E(OH_i^{(1)}) + c_p \Delta E(OH_T^{(1)}) \\ &+ \left(c_e - c_p\right) E(Q_e^{(1)} I_0^{(2)}) - \left(c_e - c_p\right) E(Q_e^{(2)} I_0^{(1)}) \end{aligned} \tag{7}$$

Similarly,

$$E(C^{(2)}) = E(C_0^{(2)}) + c_h \sum_{i=t+1}^{T} \Delta E(OH_i^{(2)}) + c_p \Delta E(OH_T^{(2)}) + (c_e - c_p) E(Q_e^{(2)} I_0^{(1)}) - (c_e - c_p) E(Q_e^{(1)} I_0^{(1)}) \quad (8)$$

This problem can be solved as a two-objective optimization problem：

$$\begin{aligned} &\min \quad E(C^{(1)})、E(C^{(2)}) \\ &\qquad S^{(1)} \geq 0, S^{(2)} \geq 0 \\ &S.t. \quad S_0^{(1)} \geq 0, S_0^{(2)} \geq 0 \\ &\qquad Q^{(1)} \geq 0, Q^{(2)} \geq 0 \end{aligned} \quad (9)$$

A. emergency order analysis

We assume that the emergency ordering quantity of R_1 is $Q_e^{(1)}$, the inventory level on t th day is m. $\Pi_0^{(1)}$ denotes the cost of R_1 when it gets emergency replenishment .

$\Pi_{\text{non}}^{(1)}$ denotes the cost of R1 when it does not get emergency replenishment.

$$\Pi_{\text{non}}^{(1)} = c_h \sum_{i=t+1}^{T} \left[\int_0^m F_{i-t}^{(1)}(x)dx \right] + c_p \left[\int_0^m F_{T-t}^{(1)}(x)dx + \mu^{(1)}(T-t) - m \right] \quad (10)$$

$$\begin{aligned} \Pi_0^{(1)} &= c_h \times \sum_{i=t+1}^{T} E(OH_i^{(1)}) + c_p \times E(BO^{(1)}) + c_e \times Q_e^{(1)} \\ &= c_h \sum_{i=t+1}^{T} \left[\int_0^{m+Q_e^{(1)}} F_{i-t}^{(1)}(x)dx \right] + c_p \left[\int_0^{m+Q_e^{(1)}} F_{T-t}^{(1)}(x)dx + \mu^{(1)}(T-t) - m - Q_e^{(1)} \right] + c_e Q_e^{(1)} \end{aligned} \quad (11)$$

So the inventory cost of R_1 drops to $H^{(1)}(Q_e^{(1)})$.

$$\begin{aligned} &H^{(1)}(Q_e^{(1)}) \\ &= (c_p - c_e)Q_e^{(1)} - c_h \sum_{i=t+1}^{T} \left[\int_m^{m+Q_e^{(1)}} F_{i-t}^{(1)}(x)dx \right] - c_p \left[\int_m^{m+Q_e^{(1)}} F_{T-t}^{(1)}(x)dx \right] \end{aligned} \quad (12)$$

Corollary1: we can prove, there is a value , $M^{(2)}$, greater than 0,①when $m \geq M^{(1)}$, R1 doesn't need replenishment ;②when $m < M^{(1)}$, the emergency ordering quantity $Q_e^{(1)} = M^{(1)} - m$, $H^{(1)}$ peaks .

And $Q_e^{(1)} = S_0^{(1)} - OH_t^{(1)}$.

We can obtain $S_0^{(1)}$ by the equation :

$$c_p - c_e - c_h \sum_{i=t+1}^{T} F_{i-t}^{(1)}(S_0^{(1)}) - c_p F_{T-t}^{(1)}(S_0^{(1)}) = 0 \quad (13)$$

B. Emergencyr replenishment analysis

t days earlier, the holding cost of R_2 does not matter to emergency replenishment. Let $\Pi_0^{(2)}$ denote the cost of R_2 that gets emergency replenishment, $\Pi_{\text{non}}^{(2)}$ denote the cost of R_2 that does not get emergency replenishment.

When $NS_t^{(1)} < S_0^{(1)}$, $Q_e^{(1)} = S_0^{(1)} - NS_t^{(1)}$.

$$\Pi_{\text{non}}^{(2)} = c_h \sum_{i=t+1}^{T} \left[\int_0^m F_{i-t}^{(2)}(x)dx \right] + c_p \left[\int_0^m F_{T-t}^{(2)}(x)dx + \mu^{(2)}(T-t) - m \right] \quad (14)$$

$$\begin{aligned} \Pi_0^{(2)} &= c_h \times \sum_{i=t+1}^{T} E(OH_i^{(2)}) + c_p \times E(BO^{(2)}) - c_e \times Q_e^{(1)} \\ &= c_h \sum_{i=t+1}^{T} \left[\int_0^{m-Q_e^{(1)}} F_{i-t}^{(2)}(x)dx \right] + c_p \left[\int_0^{m-Q_e^{(1)}} F_{T-t}^{(2)}(x)dx + \mu^{(2)}(T-t) - m + Q_e^{(1)} \right] - c_e Q_e^{(1)} \end{aligned} \quad (15)$$

When $\Pi_{non}^{(2)} > \Pi_0^{(2)}$, which means

$$H^{(2)}(Q_e^{(1)}) = (c_e - c_p)Q_e^{(1)} + c_h \sum_{i=t+1}^{T} \left[\int_{m-Q_e^{(1)}}^{m} F_{i-t}^{(2)}(x)dx \right] + c_p \left[\int_{m-Q_e^{(1)}}^{m} F_{T-t}^{(2)}(x)dx \right] > 0 \quad (16)$$

Then , R_2 will provide emergency replenishment.

Corollary 2: we can prove, when $OH_t^{(2)} > M^{(2)}$ and $Q_e^{(1)} \leq OH_t^{(2)} - M^{(2)}$, then $H^{(2)} > 0$ 。

It follows that

$$I_0^{(2)}(Q_e^{(1)}, OH_t^{(2)}) = \begin{cases} 1 & OH_t^{(2)} - Q_e^{(1)} \ge Q^{(2)} \\ 0 & otherwise \end{cases} \quad (17)$$

$$(c_e - c_p) + c_h \sum_{i=t+1}^{T} F_{i-t}^{(2)}(Q^{(2)}) + c_p F_{T-t}^{(2)}(Q^{(2)}) = 0$$

V. HEURISTIC POLICY

According to the above equations,

$$\begin{aligned} & E(C) = E(C^{(1)} + C^{(2)}) = \\ & E(C_0{}^{(1)}) + E(C_0{}^{(2)}) + \\ & c_h \sum_{i=t+1}^{T} \left[\Delta E(OH_i^{(1)}) + \Delta E(OH_i^{(2)}) \right] \\ & + c_p \left[\Delta E(OH_T^{(1)}) + \Delta E(OH_T^{(2)}) \right] \end{aligned} \quad (18)$$

Since the relationship between R_1 and R_2 is mutual, we can regard them as two independent systems. And the cost does not matter to c_e .First, we calculate the inventory policy under normal order, then we obtain $S^{(1)}, S^{(2)}$ by the following equations:

$$\begin{aligned} & c_h \sum_{i=1}^{T} F_{i+L}^{(1)}(S^{(1)}) + c_p \left[F_{T+L}^{(1)}(S^{(1)}) - 1 \right] = 0 \\ & c_h \sum_{i=1}^{T} F_{i+L}^{(2)}(S^{(2)}) + c_p \left[F_{T+L}^{(2)}(S^{(2)}) - 1 \right] = 0 \end{aligned} \quad (19)$$

VI. . EXAMPLE ANALYSIS

The parameter settings used were T=7, L=3, t=5, $c_h = 1$, $c_p = 50$, $c_e = 20$, $\mu^{(1)} = 40$, $\mu^{(2)} = 60$, $\sigma^{(1)} = 20\sqrt{0.4}$, $\sigma^{(2)} = 20\sqrt{0.6}$.

By using the MATLAB, we obtain

$$Q^{(1)} = S_0^{(1)} = 83, Q^{(2)} = S_0^{(2)} = 123$$

The results under normal order, unidirectional order and bidirectional order are as following:

Table3. The results of costs under 4 situations.

	$S^{(1)}$	$S_0^{(1)}$	$S^{(2)}$	$S_0^{(2)}$	$E(C^1)$	E(C2)
normal	444	--	654	--	1288.2	1808.8
unidirectional	440	84	654	124	1266.0	1802.8
bidirectional	444	84	659	124	1247.78	1773.23
heuristic policy	444	84	654	124	1250.0	1773.98

VII. SUMMARY

1. By using this policy will bring down costs for the two companies that cooperate with each other.

2. Under bidirectional order and replenishment, the sum of the cost of R_1 and R_2 drops to minimize. Meanwhile, they both cost less.

3. The heuristic algorithm is a feasible solution which can reduce costs.

REFERENCES

[1] Barankin, E. W. 1961. A delivery-lag inventory model with an emergency provision. Naval Res. Logist. Quart. 8 285-311.

[2] Blumenfeld, D. E., R. W. Hall, W. C. Jordan. 1985. Trade-off between freight expediting and safety stock inventory costs. J. Bus. Logist. 6 79-100.

[3] Moinzadeh, K., S. Nahmias. 1988. A continuous review model for an inventory system with two supply modes. Management Science. 34 761-773

[4] C. P. Schmidt. 1991. An (S-1, S) inventory system with emergency orders. Operational Research 39 308-321.

[5] George Tagaras. 1998. Pooling in multi-location periodic inventory distribution. Omega, Int. J. Mgmt. Sci. 27 (1999) 39-59

[6] George Tagaras, Dimitrios Vlachos. 2001. A Periodic Review Inventory System with Emergency Replenishments. Management Science, 47 415-429

[7] Axa¨ ster, S., 1990. Modelling emergency lateral transshipments in inventory systems. Management Science 36, 1329–1338.

[8] Axa¨ ster, S., 2003. Evaluation of unidirectional lateral transshipments and substitutions in inventory systems. European Journal of Operational Research 149, 438–447.

[9] Lee, H.L., 1987. A multi-echelon inventory model for repairable items with emergency lateral transshipments. Management Science 33, 1302–1316.