Robust Sliding Mode Control for Uncertain Time-Delay Systems

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Abstract

The problem of robust sliding mode control (SMC) for uncertain time-delay systems is investigated in this paper. Based on novel ideal of virtual state feedback control, a delay-independent sufficient condition is developed for the design of a robust stable sliding mode plane in term of linear matrix inequalities (LMI). A SMC controller is derived to ensure system trajectories starting from any initial state convergent to the sliding mode plane. The global stability of the closed-loop system is guaranteed. A numerical example with simulation results is given to illustrate the effectiveness of the methodology.

Keywords: Time-delay system, Sliding mode control, Uncertain system, LMI

1. Introduction

Time-delay is often encountered in various industrial systems, such as the turbojet engines, electrical networks, automotive systems, and chemical process, etc. Its existence is frequently a source of poor system performance, or instability. Hence, the control of time-delay systems has received considerable attention over the past two decades, and different design approaches have been proposed [1]. However, most of them are sensitive to the uncertainty, which directly affects the performances of the closed-loop systems.

The sliding mode control has attractive features to keep systems insensitive to the parameter uncertainties and external disturbances on the sliding mode plane [2]. SMC research has mostly focused on uncertain system without time-delay, but their methods are hard to apply indectly to uncertain time-delay system. Li and Yurkovich applied a linear nonsingular state transformation to time-delay systems to produce a delay-free system, which allows us to use existing techniques to design a control [3]. Gousisbaut et al. presented a sliding mode control methodology for a class of uncertain time-delay systems with matched external disturbances [4]. Yuanqin Xia et al. suggested a robust sliding mode control for uncertain time-delay system; a delay-independent sufficient condition for the existence of linear sliding plane is given in terms of LMI [5]. Said Oucheriah studied a continuous sliding mode controller to deal with the problem of robust exponential stabilizations of a class of time-delay systems [6]. But almost all designs of sliding mode controllers are depended on the nonsingular state transformation in order to get a regular form of system, which is easy to construct the sliding mode plane using the common method. Due to use of nonsingular state transformation, the results is more or less complicated and conservative. Xiaoqiu Li and Decarlo proposed a robust adaptive sliding mode controller in term of Lyapunov method [7]-[8], but the whole design is complicated and the range of switching controller is extravagant. Chien-Hsin Chou et al. developed a delay-independent adaptive SMC to overcome uncertainties and disturbances, but only suitable to the matched uncertainties [9]

In this paper, we consider how to design sliding mode plane and reaching motion controller for a class of time-delay system with mismatched uncertainties and matched external disturbances. The sliding mode plane is defined as a linear function of system state. Based on the Lyapunov method, a sufficient condition for the existence of linear sliding plane is derived in term of virtual state feedback controller without any state transformation, which can be relaxed to design the linear sliding mode plane. A simple and suitable sliding mode controller is adopted and the whole design is easy to implement since the given condition is represented by LMI, which can be very efficiently solved by LMI toolbox.

2. System description

Consider the uncertain time-delay system of the form

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-d) + B(u + f(t)) \tag{1}$$

where $x \in R^n$ is the system state, $u \in R^m$ is the control input, and A , A_d and B are constant matrices with appropriate dimension; $\Delta A(t)$ and $\Delta A_d(t)$ are unknown time-varying parameter uncertainties; $f(t)$ is a external disturbance. The initial condition is given as $x(t) = \Psi(t)$ for $t \in [-d \;\; 0]$.

We presume the following assumptions are valid.

1). The matrix B is assumed to have full column, and the pair (A, B) is stabilisable, i.e. there exist matrix K such that $\bar{A} = A - BK$ is stable.

2). The admissible uncertainties are assumed to be of the form

$$[\Delta A \;\; \Delta A_d] = DF(t)[E \;\; E_d] \tag{2}$$

where D, E and E_d are known constant matrices, and $F(t)$ is unknown time-varying matrices with Lebesgue measurable elements satisfying $F^T(t)F(t) \le I$.

3). The external disturbance is bounded as $\| f(t) \| \le \delta_f$.

Remark1. In (2), the same matrix $F(t)$ appears in the uncertainties of system matrices. Some papers use independent uncertainties on e.g. A, A_d as follows

$$[\Delta A \;\; \Delta A_d] = [M_1 F_1(t) N_1 \;\; M_2 F_2(t) N_2] \tag{3}$$

Let us take

$$D = [M_1 \;\; M_2],\; E = [N_1^T \;\; 0]^T,\; E_d = [0 \;\; N_2^T]^T$$

$$F(t) = diag\{F_1(t), F_2(t)\}$$

This shows that uncertainties (3) can be always expressed as (2) with an appropriate structure of E, E_d and $F(t)$.

Let us choose the sliding mode plane

$$S = B^T P x(t) = 0 \tag{4}$$

where $P \in R^{m \times m}$ is a positive definite matrix to be chosen later. For convenience, usually, the following lemma is necessary [10].

Lemma 1. Given matrices $Y = Y^T$, D, E, and time-varying matrix $F(t)$ satisfies $F^T(t)F(t) \le I$, so the inequality $Y + DFE + (DFE)^T < 0$ is equivalent to the inequality $Y + \varepsilon DD^T + \varepsilon^{-1}E^T E < 0$ for some constant $\varepsilon > 0$.

3. Main conclusion

The aim is to design a sliding mode controller in two steps. First, we design a suitable controller to drive globally the system trajectory to the sliding mode plane. The second step is to design a robust sliding mode plane so that the system restricted to the sliding mode plane has insensitive to parameter uncertainties and external disturbances. We can obtain the following conclusions.

Theorem 1. Under assumptions (1-3), and the linear sliding mode plane is given by (4), the trajectories of the whole system starting from any initial state can be driven onto the sliding mode plane in finite time with the control

$$u(t) = u_{eq} + u_n \tag{5}$$

with the equivalent control

$$u_{eq} = -(B^T PB)^{-1}[B^T PAx(t) + B^T PA_d x(t-d)]$$

the switching control

$$u_n = -(B^T PB)^{-1}[\| B^T PD \| (\| Ex(t) \| + \| E_d x(t-d) \|) + \| B^T PB \| \delta_f + \varepsilon_0] \mathrm{sgn}(S)$$

where ε_0 is small positive constant.

Proof. Consider the Lyapunov function

$$V = 0.5 S^T S \tag{6}$$

which is positive-definite for all $S(x,t) \ne 0$. The derivative of the Lyapunov function respect to time is

$$\begin{aligned}
\dot{V} &= S^T \dot{S} \\
&= S^T B^T P \dot{x}(t) \\
&= S^T B^T P \Delta A x(t) + S^T B^T P \Delta A_d x(t-d) + S^T B^T PB(u_n + f) \\
&\le \| S \| . \| B^T PB \| \delta_f - S^T (\| B^T PB \| \delta_f + \varepsilon_0) \mathrm{sgn}(S) \\
&\le -\varepsilon_0 \| S \| \\
&\le 0
\end{aligned}$$

The last inequality is known to show that the trajectory of system can be driven onto the sliding mode plane in finite time. The proof is completed.

The next is to design the robust sliding mode plane such that the system trajectories restricted to the sliding mode plane are stable in the present of parameter uncertainties and external disturbances.

Theorem 2. The system is quadratically stable on the sliding mode plane described by (4) with $P = X^{-1}$ if exist symmetric positive-definite matrices X, V, general matrix L and positive constant ε such that the following LMI is held

$$\begin{bmatrix} H & A_d & XE^T & X \\ * & -V & E_d^T & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -V \end{bmatrix} < 0 \tag{7}$$

where $H = AX + XA^T - BL - L^T B^T + \varepsilon DD^T$.

Proof. Our idea is to design the robust sliding mode plane in term of virtual state feedback controller.

We consider the controller (5) expressed as

$$u(t) = -Kx + v(t) \tag{8}$$

where $v(t) = Kx + u_{eq} + u_n$. So the close-loop system can be obtain

$$\begin{aligned}
\dot{x}(t) &= (\bar{A} + \Delta A(t))x(t) \\
&\quad + (A_d + \Delta A_d(t))x(t-d) + B(v + f(t))
\end{aligned} \tag{9}$$

where $\bar{A} = A - BK$.

Choose a candidate Lyapunov function

$$V(x,t) = x^T P x + \int_{t-d}^{t} x^T(s) R x(s) ds \qquad (10)$$

where P and R is the symmetric positive-definite matrices. Then the derivative of $V(x,t)$ is

$$\begin{aligned}\dot{V}(x,t) &= 2x^T(t)P\dot{x}(t) + x^T(t)Rx(t) - x^T(t-d)Rx(t-d) \\ &= 2x^T P[(\bar{A}+\Delta A)x(t) + (A_d + \Delta A_d)x(t-d)] \\ &\quad + x^T(t)Rx(t) - x^T(t-d)Rx(t-d) \\ &\quad + 2x^T PB(v + f(t))\end{aligned}$$

Once on the sliding mode plane, the derivative of $V(x,t)$ can be reduced to the following simple quadratic form according to (4)

$$\dot{V}(x,t) = [x^T(t) \quad x^T(t-d)]M \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} \qquad (11)$$

where

$$M = \begin{bmatrix} (\bar{A}+\Delta A)^T P + P(\bar{A}+\Delta A) + R & P(A_d + \Delta A_d) \\ * & -R \end{bmatrix} \qquad (12)$$

By substituting (2) into (12), one can be obtained

$$M = Y + \begin{bmatrix} PD \\ 0 \end{bmatrix} F[E \quad E_d] + [E \quad E_d]^T F^T \begin{bmatrix} PD \\ 0 \end{bmatrix}^T \qquad (13)$$

where

$$Y = \begin{bmatrix} \bar{A}^T P + P\bar{A} + R & PA_d \\ 0 & -R \end{bmatrix}$$

From Lemma1, inequality $M < 0$ is equivalent to

$$Y + \varepsilon \begin{bmatrix} PD \\ 0 \end{bmatrix} \begin{bmatrix} PD \\ 0 \end{bmatrix}^T + \varepsilon^{-1}[E \quad E_d]^T [E \quad E_d] < 0 \qquad (14)$$

where constant $\varepsilon > 0$. Inequality (14) can be transformed into LMI

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \varepsilon PDD^T P + R & PA_d & E^T \\ * & -R & E_d^T \\ * & * & -\varepsilon I \end{bmatrix} < 0 \qquad (15)$$

Pre-multiplying and post-multiplying (15) by matrix $diag\{P^{-1} \quad I \quad I\}$, and defining $X = P^{-1}$, the following LMI can be obtained using Schur complement theorem

$$\begin{bmatrix} X\bar{A}^T + \bar{A}X + \varepsilon DD^T & A_d & XE^T & X \\ * & -R & E_d^T & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -R^{-1} \end{bmatrix} < 0 \qquad (16)$$

Pre-multiplying and post-multiplying (16) by $diag\{I \quad R^{-1} \quad I \quad I\}$ again, and defining $V = R^{-1}$, $L = KX$, the LMI (7) can be obtained using Schur complement theorem. The proof is completed.

So we can draw a conclusion that the system trajectories starting from any initial state will asymptotically convergent the sliding mode plane and system trajectories restricted to the sliding mode plane are stable from theorem 1 and theorem 2.

Remark 2. The state feedback is virtual controller in term of (8), which is only help to design the stable sliding mode plane. Obviously, the whole design is considerable brief because any state transformation is not needed. Furthermore, although systems only with a state delay are considered, it is straightforward to extend the method to systems with finite time delays.

4. Number example

Consider the uncertain time-delay system (1) of the form with

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1.75 & 0.25 & 0.8 \\ -1 & 0 & 1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 & 0 \\ 0.1 & 0.25 & 0.2 \\ -0.2 & 0.4 & 0.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0 & 0.2 & 0.2 & 0 & 0.2 & 0.2 \\ 0 & 0 & 0.3 & 0 & 0 & 0.3 \end{bmatrix},$$

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.33 & 0 & 0 & 0 \end{bmatrix}^T, \quad f = \begin{bmatrix} 0 \\ 0.3\sin(2t) \\ 0 \end{bmatrix},$$

$$E_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0.3 & 0.2 & 0.33 \end{bmatrix}^T, \quad F(t) = \frac{1}{\sqrt{3}} \begin{bmatrix} \sin(t) I_{3\times 3} & \\ & \cos(t) I_{3\times 3} \end{bmatrix}.$$

The initial condition is given as

$$x(t) = [1 \quad 1 \quad 1]^T, \ t \in [-0.5 \ \ 0]$$

Corroding to theorem2, the following feasible solutions can be obtained

$$X = \begin{bmatrix} 434.9 & 192.1 & -1309.5 \\ 192.1 & 124.6 & -801.1 \\ -1309.5 & -801.1 & 6038.9 \end{bmatrix} V = \begin{bmatrix} 4410.6 & 67 & -1205.1 \\ 67 & 3849.4 & -450.2 \\ -1205.1 & -450.2 & 7337.3 \end{bmatrix},$$

$L = \begin{bmatrix} 66 & 465 & 1366.1 \end{bmatrix}$, $\varepsilon = 1926.4$.

So we can obtain the sliding mode plane described by (4) with $P = X^{-1}$. Fig1-3 are simulation results when choosing corresponding parameters $\delta_f = 0.3$, $\varepsilon_0 = 0.01$. Obviously, the system is asymptotically and sliding mode motion trends to the origin in finite time in spite of time-delay and uncertainties.

5. Conclusions

In this paper, the problem of designing robust sliding mode planes has been considered for a class of time-delay systems with unmatched uncertainties based on quadratic stability. The sliding mode planes are defined as a linear function of current state, and corresponding sufficient conditions are derived to guarantee quadratic stability of sliding mode motion in term of the virtual state feedback controller. A simple controller is proposed to guarantee the trajectory of the closed-loop systems can convergent to the sliding mode plane in finite time. Finally, a numerical example is given to illustrate the effectiveness of our method.

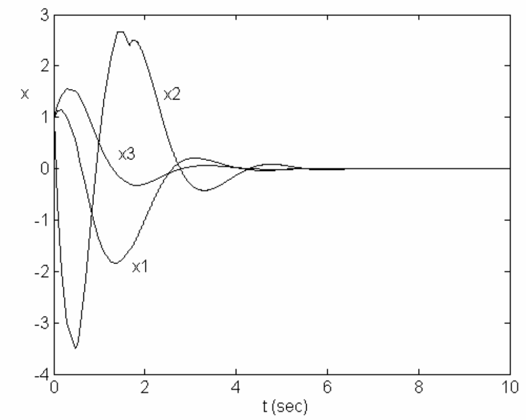

Fig. 1: Trajectories of system states.

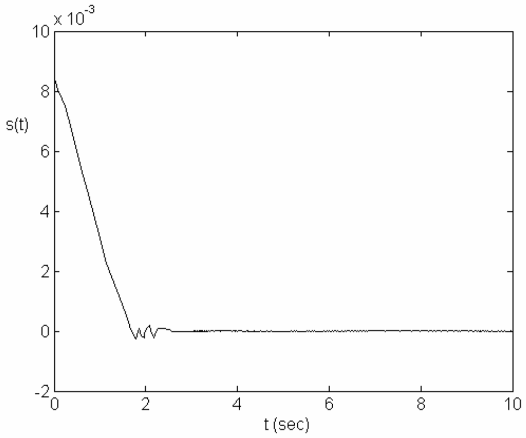

Fig. 2: The sliding mode.

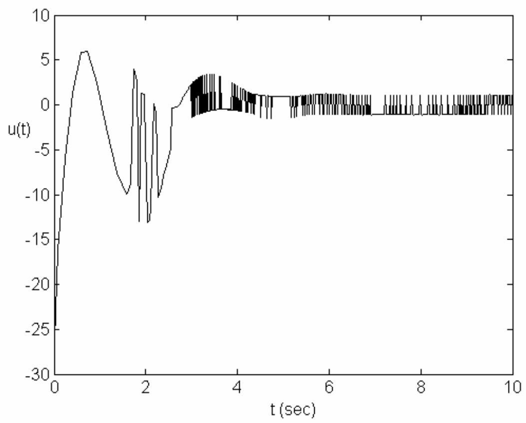

Fig. 3: The proposed control.

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