DFSS of a diagonal impeller

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Abstract—A new methodology for DFSS suitable for the design of a complicated multidisciplinary system such as a gas turbine engine is proposed. There are four phases in this methodology: define, analysis, MDO and verify. More detailed information of each of the four phases was introduced. The design of a diagonal impeller which is used in an aero engine was taken as an example to demonstrate the implementation process of the methodology. In the phase of MDO, computational fluid dynamics, computational structural mechanics simulation methods were used. The mathematic model of DFSS of the diagonal impeller was established and self developed DFSS software was used to solve the model so as to get longer fatigue life, lighter weight and higher aerodynamic performance. The results of the example indicate that the proposed methodology is efficient.

Keywords- DFSS, methodology; diagonal impeller; MDO ; system engineering

I. INTRODUCTION

The great success of six sigma in some famous companies such as Motorola and GE aroused an upsurge in the world. From 1980's onward, a large number of companies and organizations practice six sigma[1-6] with methodology DMAIC (Define, Measure, Analyze, Improve, Control) which was developed by W. Edwards Deming . With DMAIC, these companies progress on their quality journey, attain three-sigma level and four-sigma level but finally ran into a five-sigma wall. DMAIC works very well when the process is repeatedly used. However, this approach is not appropriate to assure that new products meet or exceed customer requirements, are reliable and competitive because a majority of defects and failures are actually created during the design process and this approach does not offer any major tools to evaluate and improve reliability. DMAIC can only improve quality reactively. Design for Six Sigma (DFSS) [7-11] is more appropriate and offers more powerful tools better suited to introduce reliable new products which perform to customer expectations. DFSS comprises a list of advanced concepts, strategies, processes and tools; it avoids process problems at the outset by using advanced Voice of the Customer techniques and systems engineering techniques, robust design methodology, and tolerance design methodology. DFSS improve quality proactively. More and more attentions have been paid to DFSS not only by companies but also by researchers who have done academic investigations widely [12-15]. A series of DFSS methodologies have been proposed and used by many different companies and organizations. IDOV (Identify-Design-Optimize-Verify) which is one of famous methodologies was proposed by GE[16] and implemented in Seagate Technology. Since its adoption in late 1990's, successes have been reported by these corporations. Another famous methodology DMADV [17,18] which stands for Define, Measure, Analyze, Design, Verify, was developed by Motorola University within DFSS. Variations on DFSS methodology [19] also exist such as DCCDI (Define, Customer, Concept, Design and Implement), PIDOV (Plan, Identify, Design, Optimize, and Validate) and CDOV (Concept, Design, Optimize, and Verify). In these methodologies, optimization or robust design [20-22] takes important parts.

The design of a complicated system usually involves deferent disciplines which makes it extremely difficult task. A new DFSS methodology is proposed here into which multidisciplinary design optimization (MDO) [23-25] was incorporated so as to make DFSS a more powerful tool for the design of complicated multidisciplinary systems.

II. DFSS METHODOLOGY DAMV

The design of a complicated system like a new generation gas turbine engine usually involves several deferent disciplines such as computational fluid dynamics, computational thermodynamics, computational solid mechanics (CSM) and computer-aided design (CAD); and some of these disciplines interact from each other strongly. For such a complicated system with high performance, systems engineering techniques should be applied so as to ensure that the system will fulfill this initial idea as well as possible and as efficiently as possible. System engineering is an interdisciplinary process which recognizes that each system is an integrated whole even though composed of diverse subsystems. It also recognizes that any system has a number of objectives and that the balance between them may differ widely from system to system. The methods seek to optimize the overall system functions according to the weighted objectives and to achieve maximum compatibility of its parts. DFSS is closely related to systems engineering, but the feature of multi-disciplinary is not embodied in existed DFSS methodologies mentioned above.

From the view point of systems engineering a new DFSS methodology is proposed, and is expressed as DAMV which stands for Define, Analyze, MDO, Verify. This methodology is featured by MDO which means multidisciplinary design optimization. MDO is a methodology for the optimal design of complex, coupled engineering systems and subsystems that coherently exploits the synergism of mutually interacting disciplines using high fidelity analysis with formal optimization. The emphasis is on a systematic methodology and on exploiting interdisciplinary interactions to achieve a better overall system than can be achieved by ignoring the interactions. Since 1980's, a large amount of researches have been done in MDO techniques[26-27], such as approximation concepts, MDO architectures, sensitivity analysis, optimization algorithms and theory, software infrastructure, analysis methodology.

MDO was incorporated into the proposed DFSS methodology DAMV in order to make it more powerful for the design of a complicated system. The implementation procedures of the four phases of DAMV are as follows:

1) Define phase: According to the characteristic of the system under consideration, define the object of the project, for example a component, an assembly of a gas turbine engine or a whole engine. And define characteristics that are critical to qualities (CTQs) for the customer and for the project by quality function deployment (QFD).

2) Analyze phase: Analyze similar systems in existence, come up with an initial design, analyze production process capacity and determine the control factors and uncertainty factors and transfer CTQ to CTP (critical to process).

3) MDO phase: Build an MDO mathematical model, that is to define design variables from control factors and uncertainty factors determined in the analyze phase, objective functions from CTQs defined in define phase and constraints. Carry out optimization which is based on multi-disciplinary simulation from the initial design.

4) Verify phase: Test the design resulted in MDO phase to ascertain that objective functions meet requirement as long as all tolerances are within their limits using DOE (design of experiments) and multi-disciplinary simulation techniques. Carry out geometrical tolerance and positional tolerance design to assure high multi-disciplinary performances at the lowest possible cost. Carry out conformal design to obtain optimal shape of the structure and position relations with other structures when it works.

The design of a diagonal impeller which is often used in a gas turbine engine as one of key components was taken as an example to demonstrate the implement of the DFSS methodology DAMV proposed here in details.

III. DEFINE

The project is to design a diagonal impeller using DFSS methodology DAMV. A diagonal impeller is a rotating component of a diagonal flow or mixed flow compressor, which transfers energy from the high pressure turbine that drives the rotor to the gas being compressed by accelerating the gas outwards from the center of rotation. The velocity achieved by the impeller transfers into pressure when the outward movement of the gas is confined by the compressor casing. A diagonal flow compressor is often used in a small or mid sized gas turbine engine. Fig.1 and Fig.2 show the 3D structure and meridian cross-section of a diagonal impeller with splitter discussed here. A diagonal impeller is a key rotating component in a gas turbine engine. The CTQs of the project are associated with those of the whole engine; and some important CTQs related to the project are listed in Table 1. Usually the power of a turbo-shaft engine or a turboprop engine and the thrust of a turbofan or turbojet engine are specified in overall design of the engine according to the requirement of customer. High aerodynamic efficiency or low aerodynamic losses are needed to obtain enough power or thrust and reduce fuel consumption and cost. Enough structural strength is helpful for high reliability, durability and safety. Higher power/weight and thrust/weight ratios require better aerodynamics performance and lighter structure weight. Table 2 shows the project CTQs which are transferred from customer CTQ listed in Tab. 1, the project Ys which are quantified project CTQs are also listed in Tab. 2. In order to get enough fatigue life of the diagonal impeller the stress of the blade and disk must be less than 780MPa and 720MPa respectively.

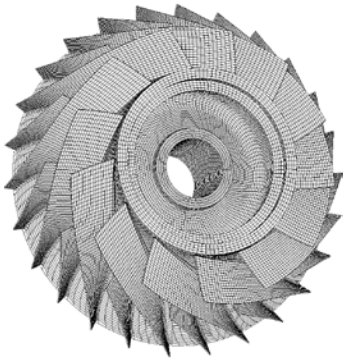

Figure 1. diagonal impeller

TABLE I. CUSTOMER CTQ

No.	CTQ
1	Specified power or thrust of the engine
2	High power/weigh or thrust/weigh ratio
3	Low specific fuel consumption
4	High reliability
5	High durability
6	high safety
7	Low cost

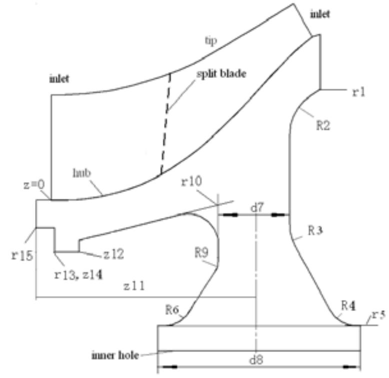

Figure 2. initial structure and design variables

IV. ANALYZE

The initial structure of the diagonal impeller is shown in Fig. 2 by referring other similar engines. Multidisciplinary analysis was carried and the results are listed in Tab. 3. From Tab. 3 can be seen that some of the CTQ items are not met the requirement listed in Tab. 2.

Finite volume method was used in aerodynamic analysis, finite element method was used in structural analysis, meshes used in the analysis are shown in Fig. 3 and stress distribution is shown in Fig. 4.

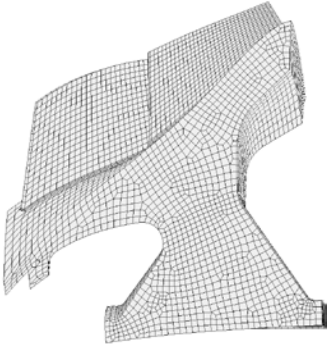

Figure 3. element mesh

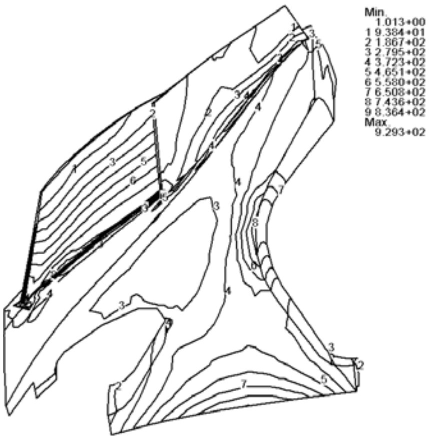

Figure 4. stress distribution

TABLE II. PROJECT CTQ AND PROJECT Y

No.	**project CTQ**	**Related items in Tab. 1**	**project Y**	
			requirement	**unit**
1	Low loss	1,2,3	<20	%
2	Enough pressure ratio	1,2,3	> 3	-
3	Blade stress	4,5,6	< 800	MPa
4	Disk stress	4,5,6	< 760	MPa
5	Enough resonant margin	4,5,6	>10	%
6	Light in weight	2	38	N
7	Small displacement at the tip of the blade	1,2,3	0. 6	mm

TABLE III. PROJECT YS OF THE INITIAL STRUCTURE

project CTQ	**project Y**	**Meet the requirement**
Loss (%)	9.7 (>9.0)	No
Pressure ratio	3	Yes
Max. disk stress (MPa)	827.3 (>730)	No
Max. blade stress (MPa)	929.3 (>780)	No
Resonant margin (%)	25 (<10)	Yes
Weight (N)	45.7	Yes
tip displacement (mm)	0.60	Yes

V. MDO

A. *Design variables*

The design variables of the disk are shown in Fig 2 where r is the radius coordinate of the control points, z axial coordinate, R the radius of a fillet, d distance. As shown in the Fig 2, at the blade root of inlet, it has z=0. The curve of hub and the radius of the inner hole are previously specified and can not be changed.

The blade mid-surface of the diagonal impeller discussed here is a ruled surface and can be determined by the tip and root two curves which is expressed by Bezier curve. As shown in Fig. 5, the tip and root curves are determined by a series of control points, where Pt for the tip curve and Pr for the root curve. A Bezier curve is expressed by the following equation:

$$x(s)=\sum_{i=0}^{n}P_i\frac{n!}{i!(n-i)!}s^i(1-s)^{n-i}\ \ (0\le t\le 1) \tag{1}$$

Where $x=(u,v,w)$, u,v,w are axial, radial and circumferential coordinates of the curve respectively, $P=(P_u,P_v,P_w)$, P_u,P_v,P_w are control parameters of axial, radial and circumferential coordinates respectively, n the number of control points. Because the flow path was determined at previous stage, only the control parameters P_w which control the circumference coordinate of the curves are taken as design variables. Usually the number of control points n=6-10. The thicknesses of the tip and root curves are also expressed by Bezier curve. The cone angle α, see Fig. 5, is also taken as a design variable.

Because of machining error, the sizes of the actual structure are different from those marked on the technical drawings. The design variables of the diagonal impeller are mean sizes which are defined as control factors. The variations of the actual sizes to the mean sizes are defined as uncertainty factors and the design variables are assumed to be distributed normally. In the analyze phase, production process capacity was analyzed from which the variance of each design variables was get. The initial values, upper and lower boundaries and variances of the design variables of the disk are listed in Tab. 4.

TABLE IV. PARAMETERS OF DESIGN VARIABLES OF THE DISK

No.	**design variable**	**initial value**	**Upper boundary**	**lower boundary**	**variance**
1	r1	108	114	102	0.00083
2	R2	15	20	10	0.00053
3	R3	1.	2	0.5	0.0003
4	R4	10	30	8	0.0012
5	R5	8	20	5	0.0012
6	r6	32	33	28	0.0012
7	d7	22	20	10	0.0012
8	d8	66	75	45	0.0012
9	R9	8	30	5	0.0012
10	r10	72.4	90	60	0.0012
11	z11	68	75	60	0.0012
12	z12	9	18	4	0.0012
13	r13	55	58	48	0.0012
14	z14	1	10	-2	0.0012

B. *Objective functions*

From Tab. 2 it can be seen that the loss, blade stress, disk stress are not met the requirements, they will be included in objective function. Although the weight in Tab. 3 is met the requirement, it also included in objective function for the lighter the structure the better. The objective function is expressed as:

$$L(\{x\}) = \sum_{i=1}^{4} \left[W_i f_i(\{x\}) + \sigma_i^2 \right] \qquad (2)$$

Where $\{x\}$ is the vector of design variables which are listed in Tab. 4, f_1 is the aerodynamic loss, f_2 the blade maximum stress, f_3 the disk maximum stress, f_4 the weight of the impeller, W the weight coefficient; σ is the standard deviation which can be obtained by:

$$\sigma_i^2 = \sum_{j=1}^{n} (\partial f_i / \partial x_j)^2 \sigma_{x_j}^2 \text{ , (i=1,2,3,4)} \qquad \text{(3a)}$$

where is n number of design variables, x_j the jth design variable, σ_{x_j} the standard deviation of the jth design variable listed in Tab. 4.

Because function f_i is calculated by digital simulation techniques instead of analytical ones, in the actual calculating process, derivatives are approximated by differences:

$$\sigma_i^2 \approx \sum_{j=1}^{n} (\Delta f_i / \Delta x_j)^2 \sigma_{x_j}^2 \text{ , (i=1,2,3,4)} \qquad \text{(3b)}$$

σ is included in the objective function so as to increase robustness of the structure.

C. Constraints

Boundary constraints include upper and lower limits of the design variables. The performance constraints are:

pressure ratio $R > 3$,

Vibration resonance margin

$$k_i = \left| f_i - f_{ej} \right| / f_{ej} \geq 10\%$$

where k_i is the ith resonance margin, f_i is the ith order natural frequency of the rotating impeller, f_{ej} is the jth exciting frequency.

D. Mathematic model

Find $\{x\}$, $\{x\} \in D \subset R^n$,satisfying

$$D: \{x\} - \{x_{Low}\} \geq \{0\}, \{x_{Up}\} - \{x\} \geq \{0\},$$

$$R - 3 \geq 0,\ k_i - 0.1 \geq 0;$$

so that project $Y = \min\ L(\{x\})$. (4)

E. Results

We solved (4) using a DFSS software developed. The results are listed in Tab. 5. Genetic algorithm was used in the optimization process. It can be seen from Tab. 5 that the weight of the diagonal impeller is reduced by 5.16(N), (13.6%), the maximum stress of the disk is reduced by 101.3MPa, 12.2%, the maximum stress of the blades is reduced by 234.8MPa, 24.5%, the maximum displacement of the blades is reduced by 0.29mm, 35.2%. These results are satisfactory. The outline and the stress distribution of the impeller after MDO are shown in Fig. 6.

TABLE V. RESULTS OF MDO

	Before MDO	After MDO	deduction	Ratio (%)
Loss (%)	9.7	7.5	2.2	2.3
Max. blade stress (MPa)	929.3	731.4	197.9	21.3
Max. disk stress (MPa)	827.3	736.9	90.4	10.9
Weight(N)	37.94	32.78	5.16	13.6
Displacement (mm)	0.825	0.535	0.29	35.2

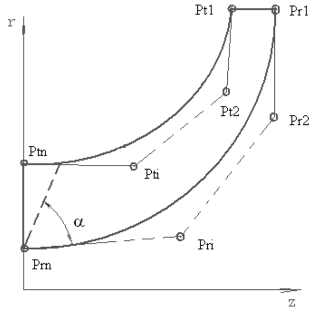

Figure 5. design variables of the blade

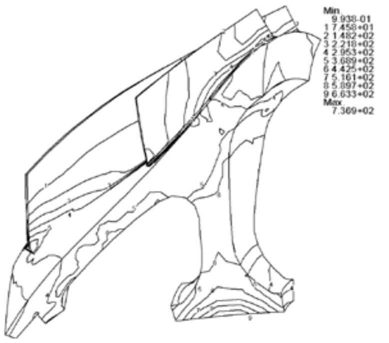

Figure 6. outline and the stress distribution after MDO

VI. VERIFY

In this example multi-disciplinary simulation techniques were used to carry out tolerance design and conformal design for the optimized diagonal impeller.

A. Tolerance design

By using simulation techniques and Eqn. 2 and 6， $f_i(\{x\})$ and deviation σ_i of the five performances: the aerodynamic loss, the maximum blade stress, the maximum disk stress, the maximum blade tip displacement and the weight of the impeller, can be obtained. The range of $f_i(\{x\})$ caused by uncertainty of the design variables can be obtained by following equations：

$$\underline{f}_i(\{x\}) = f_i(\{x\}) - c \times \sigma_i$$

$$\overline{f}_i(\{x\}) = f_i(\{x\}) + c \times \sigma_i \qquad (5)$$

where $\underline{f}$ and $\overline{f}$ are the lower and upper boundary of f ， C=3-6 is the coefficient. For C=3, it means that the P($\underline{f} \leqslant f \leqslant \overline{f}$)=99.73％ and for C=6， P($\underline{f} \leqslant f \leqslant \overline{f}$)=99.9997％.

The upper boundaries of the aerodynamic loss, maximum blade stress, the maximum disk stress, maximum blade tip and the weight of the impeller are listed in Tab. 6. From Tab. 2 and Tab. 6 it can be seen that all 5 performances exceeded the requirements.

TABLE VI. UPPER BOUNDARIES OF THE PERFORMANCES

$\underline{f}_1$	$\underline{f}_2$	$\underline{f}_3$	$\underline{f}_4$	$\underline{f}_5$
7.9(%)	723.3(MPa)	727.7(MPa)	33.02(N)	0.551(mm)

B. Conformal design

The purpose of conformal design of a structure is to assure to obtain desired shape and position relation with other parts when the structure works. In this example conformal design for structure shape was carried out. If a diagonal impeller is machined according to the sizes obtained by previous process, when it works, it deforms under the application of centrifugal load and thermal load. The deformed shape of the blade is different from that of undeformed. The aerodynamic loss of the deformed structure is changed from 7.5(%) to 9.3(%) and the differences are remarkable and must be considered in the design process.

Finite element method was used to carry out conformal design. The mathematical model of conformal design is as follows:

Find $\{y\}$ so that

$$\max\left|\{y\}-\{z\}-\{\delta\}\right| \le \{\varepsilon\} \qquad (6)$$

where $\{y\}$ is the coordinate vector of nodes which are on the surface of the blades, $\{z\}$ is the coordinate vector of nodes which are determined by solving Eqn. (2), $\{\delta\}$ is the displacement vector, N is the number of finite element nodes.

For a linear elasticity problem it has:

$$\{y\}=\{z\}-\{\delta\} \qquad (7)$$

For a nonlinear problem iterative method should be used.

VII. CONCLUSIONS

A new methodology of DFSS was proposed in this paper. Some new components such as MDO and conformal design were incorporated into the methodology, making it suitable for a very complex system with high performance as all advanced gas turbine engines. The example presented here indicates that the proposed methodology is efficient; although it only involves a diagonal impeller, one component of an engine, this methodology can be used to the design of an assembly such as a compressor, a combustor, a turbine or a whole engine. The contents in each phase still can be changed or improved to make it more suitable for different projects.

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