Algorithm Efficiency Research on Hanoi Problem Symmetric Solution

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Abstract—The tower of Hanoi problem in the design of computer algorithm is a classical problem. Generally people will use recursive algorithm to solve this problem. In this paper, we will use symmetric algorithms to solve Hanoi problem simply and efficiently. At the same time, this paper also detailed researches on the algorithm efficiency of traditional solution and symmetric solution. At the same time, through the control variable method to control the computer to the traditional algorithm and symmetric algorithm efficiency analysis.

Keywords- Hanoi problem; Recursive properties; Symmetry properties; algorithm efficiency; algorithm simplicity

I. INTRODUCTION

First invented Hanoi (also called the tower of Hanoi) problem is one of the French mathematician Edward Lucas . He wrote a India's ancient legend: in the center of the world Benares, in northern India) in the Confucian temple, a piece of brass insert on the three root stone needle. The Hindu god brahman in creating the world, in which a needle from down to up wear from big to small 64 pieces of gold, this is the so-called Hanoi. No matter day or night, always have a monk in accordance with the law to move the piece of gold: a move only a piece of, no matter where needle, small piece must be in the big top. The monks prophecy, when all the piece of gold from the brahma wear the good needle up to another on a needle, the world is in a loud bang destroyed, and the Vatican tower, temples and beings also will perish together[1].

Through in-depth study Hanoi problem can help us understanding the computer and mathematics of the recursive algorithm. Recursive itself including recurrence and regression thought, but Hanoi problem itself is to the idea of a perfect interpretation. At the same time, Hanoi problem is psychology experimental research one of the mission of the commonly used, because it not only helps to clarify the human problem solving process of the basic psychological mechanism, but also can promote Hanoi problem in clinical application, but the present domestic research in this field or start state[2].Overall, the study Hanoi problems a lot of practical use.

II. HANOI PROBLEM CHARACTERISTICS

For Hanoi problem, if I had discussed 64 disc of Hanoi problem without a doubt is very difficult, and our mathematical one of the most magical place is its change numerous for brief thought. So, we will assume that there are three root adjacent pillars, labeled A, B, C, A post from the bottom up according to the pyramid shape together with n A different size of the disk, now put all the dishes A move to their B, and every move the same post can appear big plate in the small plate.

A. The traditional Recursive properties

It is obvious that if only one plate, do not need to use B pillar, directly will plates from A move to C. If there are two dishes, may give a plate 1 plate 2 move to B; The plates 1 move to C; The plates move to 2 C. This shows that: can use B will two dishes from A move to C, of course, also can use C will two plates move from A to B. If there are three plates, then according to the two plates conclusion, can use c will plate 1 two plates move from A to B; The plates 1 from A move to C, A become empty pillars; With the aid of A seat, will B two plates move to C. This shows that: can use an empty pillar, three dishes from a post move to another. If there are four plates, so firstly by virtue of empty their C, will plate 1 three dishes from A move to B; The plates 1 move to C, A become empty pillars; With the aid of A vacant position, will B their three plates move to C. The above idea can has been extended to 64 plate: can use empty their C will plate 1 63 dishes from A move to B; The plates 1 move to C, A become vacant position; With the aid of vacant position A, B will seat 63 plates move to C. So, we put the mobile of H (n). First of all, we must be the top n - 1 dish move to their C, then put the biggest piece of in B, finally put on C all the plates move to B, from which we draw expression:

H(1) = 1

H(n) = 2*H(n-1） +1 (n>1）

Then we will soon get H (n) of the general type:

H(n) = 2^n - 1 (n>0)

See, in fact algorithm is very simple, when the number of plates for n, the number of mobile should be equal to 2 n- 1 time.

B. *Hanoi problem of symmetry properties*

We can see that the above method is the use of Hanoi problem itself recursive properties. In fact, if we consider this problem, we will find that the original Hanoi problem also has high symmetry. Below we will this N a dish from small to large in turn Numbers for 1, 2, 3,... N. In order to the N plate according to the rules from the needle A move to needle C, can be divided into 3 steps go:

Step 1: before N - 1 plate according to the rules from the needle A move to needle B;

Step 2: the first N plate directly from the needle A move to needle C;

Step 3: the former N - 1 plate according to the rules from the needle B mobile C[3].

In this way, N a plate of Hanoi problem is transformed into N - 1 a plate of Hanoi problem, and they only between the beginning and the end of the number and different just. And N - 1 a plate of Hanoi problem and can further into N - two dishes of Hanoi problem. Such transformation bottom go to, the end result is that: N a plate of Hanoi problem was converted into a sequence of a plate of Hanoi problem.

By the decomposition visible, N a plate of Hanoi problem can be transformed into two N - 1 a plate of Hanoi problem and a plate of Hanoi problem, and this one dish of Hanoi problem squarely in the two N - 1 a plate of Hanoi problem among. Therefore, to solve the N of Hanoi problem need at least operation steps should be odd, and all operation step operation object according to the order should be the center of symmetry, the center of symmetry is N number plate

Of course this kind of symmetry is only us from whole settle Hanoi problem abstracted, and then we'll use mathematical formula to prove it.

Set will N plate according to the rules from the first needle moved to the 3rd needle need at least for An operation steps, then according to Hanoi problem recursive sex and symmetry, sequence {An}. Meet the requirements:

An = 1, and when N > = 2 is An = 2 An - 1 + 1.By An = 2 An - 1 + 1, it can be led to:

An+1=2(An-1+1)

This shows that sequence {An + 1} is for 2 than by A1 + 1, namely led 2 item geometric sequence, the geometric series summation formula of solution can be a plate of Hanoi problem will need at least 2N - 1 step operation. This will give us the above symmetry a strong proof. From the above we draw the conclusion that, in the N a plate of Hanoi problem need 2N - 1 step operation, the center position of the step of the operation object is N number plate, the first half of the operation and operation after the operation of about half the object center of symmetry. So just make sure out of the top half of the operation of the operands, then half after operation object is then determined. And the first half before operation and solve the N - 1 a plate of Hanoi problem, so in this half of the center position of the step of the operation object should be N - 1 plate. And so it went on, every step of the operation object can be decided. The following is 1, 2, 3, 4, 5, every step of the operation object:

One disc：1

Two discs：1 2 1

Three discs：1 2 1 3 1 2 1

Four discs：1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

Five discs：1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

……

From this series can easily see that Hanoi problem itself contains strong symmetry properties, they are all about the center position of the highly symmetric.

III. ALGORITHM DESCRIPTION

Through the above analysis and research, we can use of Hanoi problem itself of these properties, compile corresponding algorithm. Of course, for recurrence and symmetric algorithm, for the nature of the use of different, so the train thought of algorithm are different.

A. *Recursive algorithm description*

First of all, we want to make sure that recursive thought, recursive is procedure call their programming skills. A process or function in the definition and description or directly or indirectly call itself a kind of method, it is usually a large complicated problem into a layer with the original problem similar smaller problems to solve. So to Hanoi problem, we can write this code to achieve its layers of recursive.

B. *Symmetric algorithm description*

So at first, we want to make sure that every step of the algorithm is the beginning and end of the problem. Because only know the starting point and end point we can step by step according to the request and moving plates. Then according to the mentioned Hanoi problem of symmetry and recursive sex, the N A plate of Hanoi problem need 2N - 1 step operation, the center position of the step of the operation object is N number plate, starting point and end point is A needle is needle C; First half operation is the first N - 1 A plate from the needle A move to needle B, therefore, in the half of the center position of the step of the operation object is N - 1 dish, starting point and end point is A needle is needle B; Half after the operation is the first N - 1 dish from needle B moved to needle C. Therefore, in this half of the center position of the step of the operation object is N - 1 dish, the starting point is the needle B, the end is needle C. In this way, start from the center position, and gradually expand to both ends, and finally to determine all the operation steps of the beginning and the end.

In fact, according to the mentioned analysis, we can get the following conclusion:

(1) solve a plate of Hanoi problem need at least operation steps of 2N - 1;

(2) in solving A plate of Hanoi problem need 2N - 1 step operation, the center position 2N - 1 step operation object is N number plate, and the starting point of the operation is A needle, the end is needle C;

(3) in solving a plate of Hanoi problem need 2N - 1 step operation, in addition to outside the center half before operation can be solve N - 1 a plate of Hanoi problem need

2N - 1 step operation get; Will solve N - 1 A plate of Hanoi problem need 2N - 1-1 step operation of B with C, C to B, but A remain unchanged.

(4) in solving N A plate of Hanoi problem need 2N - 1 step operation, in addition to outside the center half after operation can be made by first half operating get first half will be operating in A with B, B with C, C with A[4].

IV. ALGORITHM IMPLEMENTATION

A. *The traditional recursive algorithm*

C + + language core code is as follows:

void move(char one,char anoth)// Mobile function its function is to transfer to the value of one variable represents their the plate moves to the variable anoth value represented by the post

```
{
    cout<<one<<"--->"<<another<<endl;
}
```

void hannuota(int n,char no1,char no2,char no3)// Hanoi function the no1, no2, no3 represent respectively the three pillars of this function is the function of the realization of the use of the above mentioned recursive nature will Hanoi problem into three different simple operation (refer to 1.2[5])

```
{
    if(n==1)move(no1,no3);
    else
    {
        hannuota(n-1,no1,no3,no2);
        move(no1,no3);
        hannuota(n-1,no2,no1,no3);
    }
}
```

B. *The traditional recursive algorithm*

C + + language core code is as follows:

/* The code below is to use symmetry of Hanoi problem carries on the corresponding operation*/

```
for(j=1;j<=x-1;j++)
{
    if(s[j][1]=='B')
        s[j][1]='C';
    else if(s[j][1]=='C')
        s[j][1]='B';
    if(s[j][2]=='B')
        s[j][2]='C';
    else if(s[j][2]=='C')
        s[j][2]='B';
}
for(j=x+1;j<=2*x-1;j++)
{
    s[j][0]=s[j-x][0];
    if(s[j-x][1]=='A')s[j][1]='B';
    else if(s[j-x][1]=='B')s[j][1]='C';

    else s[j][1]='A';
    if(s[j-x][2]=='A')s[j][2]='B';
    else if(s[j-x][2]=='B')s[j][2]='C';
    else s[j][2]='A';
}
```

V. ALGORITHM PERFORMANCE ANALYSIS

Through the in my computer program testing, and separately calculated when plate number phase at the same time, the recursive algorithm and symmetric algorithm to Hanoi problem running time. (this lab use computer for Y460 association CPU: Intel pentium dual-core memory: 2 GB P6100 DDR3) :

Disc number (a)	10	12	14	16	18	20
Recursive solution(s)	0.4	0.8	1.79	6.22	22.89	63.25
Symmetric solution (s)	0.65	0.94	2.21	6.43	22.94	67.45

It can be seen from the above data, when the number is small, the two algorithm's running time are almost the same, or very close to. But once the plate number is more and more big, the traditional recursive method of computing time is better than symmetric algorithm of computing time shorter. However, this does not mean that symmetric algorithm than recursive algorithm good, because the computer algorithm many times consider system implementation program of the allocated memory space. In other words, the efficiency of the algorithm is not only to consider the running time of the algorithm, but also consider the algorithm of space overhead. From this point of view, in fact, in order to solve the N of Hanoi problem, array S only need 2 N - 1 line can: first find out N - 1 a plate of Hanoi problem solution, and then, according to the content of the S output N a plate of Hanoi problem solution. So, the space was reduced spending half. So, we can see that the advantages of recursive algorithm is run time symmetric algorithms compared to short, but in space overhead, symmetric algorithm than recursive algorithm saves half of the space overhead.

VI. CONCLUSION

Through the study of Hanoi problem of symmetry properties, we designed a different way from traditional symmetric algorithms to solve the problem. At the same time, through the control variable method to control the computer to the traditional algorithm and symmetric algorithm efficiency analysis, we can see, recursive algorithm in speed than symmetric algorithm is fast, but symmetric algorithm of space spending is half of the recursive algorithm, it is to solve large Hanoi problem of computer implementation process to save a lot of space.

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