

# Efficiency Optimization in the Efficiency Similarity Fan Stations

Fulai Yao

Department of Mechanical Engineering,  
University of Connecticut  
Storrs, Connecticut, USA  
e-mail: fulai.yao@engineer.uconn.edu

Chengyu Cao

Department of Mechanical Engineering,  
University of Connecticut  
Storrs, Connecticut, USA  
e-mail: ccao@uconn.edu

**Abstract**—This paper defines the normalization efficiency function, the efficiency similarity of different variable-speed fans, and the efficiency similarity fan station. We propose an efficiency optimization method for such fan stations. Based on the efficiency function characteristics which can be approximately considered a concave, non-negative function through the origin, it gives optimal control and optimal switch method and gives the minimum total power consumption. This method is independent of the linearity or nonlinearity of the system and has no need for the system's state equation or transfer function. In the constant output pressure fan station, the optimal control method is to keep the same flow rate for each fan, and the optimal switch points are at the same efficiency points of different operating methods. Since the shapes of efficiency functions in nature are similar, the method given by this paper can be used in other fields.

**Keywords**- variable speed; fan station; efficiency similarity; optimal control; efficiency optimization

## I. INTRODUCTION

According to statistics, the electricity consumption of fans is 10-15% of the world's total electricity consumption. With the development of energy-saving, many of speed-control equipment such as inverters have been widely applied to the fan stations. A traditional single-close loop control in a variable-speed fan station is shown in Fig. 1.

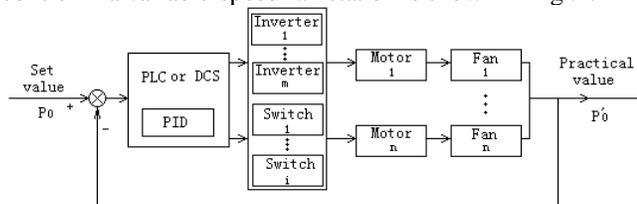


Figure 1. Block diagram of a single-close loop control in fan station

The traditional control is to measure the practical pressure  $P_0$  that is to be controlled, compare  $P_0$  with the set pressure  $P_0$  and feed the error into the PID, the PID changes a manipulated variable to control an inverter that regulates the speed of the motor and fan, and drives  $P_0$  back to  $P_0$ .

However, this control way doesn't consider the total power, so it can not realize the minimum power consumption. With increased awareness for environmental protection and the demand for energy efficient systems, this issue becomes

increasingly important. At the present, the studies of optimization in various fan stations are widely studied [1-4]. Almost such optimization approaches need to establish the system's exact model. However, as the exact model of an actual fan station is difficult to establish, these optimization methods can quickly become difficult to apply in an actual system. A scheme that could avoid these complications would be highly advantageous, especially to business and industry.

We can say that, at present, there still haven't optimal control methods which can accurately control energy-saving effect in fan stations [5].

In order to realize the minimum energy, we should decide how many fans are used and how much load each fan undertakes in this fan station.

By supposing the total flow fixed value, we propose an optimal control method akin to conventional static optimization [6], this method does not depend on the exact model of a fan station and is only based on the shape of the efficiency function. By changing the total flow, we also propose a method akin to conventional dynamic optimization [7], this method only depends on the efficiency function of the devices at constant output. These methods can be used for the systems which consist of the same model fans which also have the same efficiency functions.

However, in industrial applications, there still exist a lot of pumping stations in which all fans are not the same model. For solving the optimization of such systems, we expand on the discussion presented in [6, 7]. This paper defines a normalized efficiency function  $\eta_N(Q)$  of a fan and the efficiency similarity fan station consisting of multiple different pumps with the same normalization efficiency function. We are able to find an expression for an arbitrary fan station's power consumption and its minimum value in terms of function  $\eta(\beta)$ . Our method [6, 7] can still be employed when the normalized efficiency functions are the same for each fan. By using the normalized efficiency function determine the minimum power consumption and optimal switch points for such fan station.

We organize the paper in the following manner. Section II gives the motivation and background for the work. In section III we present the problem formulation and the optimization solution. We give a further analysis in section

IV and apply our results to an optimal switch problem in section V. We give a brief simulation of our work in section VI and conclude the paper in section VII.

## II. MOTIVE AND BACKGROUND

In general, the efficiency function of a centrifugal variable-speed fan in a output constant pressure  $P_0$  fan station has the shape shown in Fig .2.

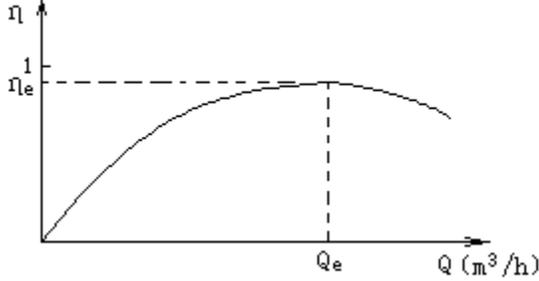


Figure 2. The efficiency function of a centrifugal fan

In Fig .2,  $Q$  is flow ( $m^3/hour$ ), and  $\eta(Q)$  is the efficiency,  $Q_e$  is the flow at  $\eta_e$  which is the maximum efficiency. We have

$$\begin{aligned} Q &\geq 0 \\ \eta(Q) &\geq 0 \\ \eta(0) &= 0 \end{aligned}$$

and

$$\eta''(Q) < 0$$

Approximately,  $\eta(Q)$  becomes

$$\eta(Q) = \sum_{i=0}^{\infty} a_i Q^i = Q \sum_{i=1}^{\infty} (a_i Q^{i-1}) \approx a_1 Q + a_2 Q^2 \geq 0$$

where

$$\begin{aligned} a_1 + a_2 Q &> 0 \\ a_1 &> 0 \\ a_2 &< 0 \end{aligned}$$

and

$$a_0 = 0$$

The fan's shaft power is  $N(Q)$  (kW), it has the form

$$N(Q) = \frac{QP_0}{3600000\eta(Q)} = \frac{N_0}{\eta(Q)}$$

and

$$N_0 = \frac{QP_0}{3600000}$$

where  $N_0$  is ideal work.

If a fan station has  $n$ -unit running fans, and the fan station is used to transfer air at the output constant pressure  $P_0$ , the total flow is  $Q_t$ , all fan's speed is variable, the  $i$ th fan's flow is  $Q_i$ , then total power consumption has the form

$$N_t(Q_t) = \frac{P_0}{3600000} \sum_{i=1}^n Q_i \frac{1}{\eta_i(Q_i)}$$

and

$$\sum_{i=1}^n Q_i = Q_t$$

where  $\eta_i(Q_i)$  is the  $i$ th fan efficiency at the point  $(Q_i, P_0)$ ,  $N_t$  is the total power consumption of the fan station.

The  $i$ th fan's power consumption is

$$N_i(Q_i) = \frac{P_0 Q_i}{3600000 \eta_i(Q_i)}$$

If  $Q_i$  equals zero,  $N_i(0)$  may be greater or equal zero. For most real fan station,  $N_i(0)$  is greater than zero because  $\eta_i(0)$  equals zero too. In order to avoid the confusion with a stopped fan, we assume  $Q_i$  is greater than zero for running fan. We then have

$$Q_i > 0$$

## III. THE OPTIMAL CONTROL METHOD OF FAN STATIONS WITH THE SAME FANS[6-7]

If a fan station has  $n$ -unit running fans which are the same model, then all  $\eta_i(Q_i)$  are the same function  $\eta(Q_i)$ , and the fan station is used to transfer air at the output constant pressure  $P_0$ , the  $i$ th fan's flow is  $Q_i$ ,  $Q_i$  is greater than zero, all fan's speed is variable, the output pressure of the  $i$ th fan can be kept a constant value  $P_0$ , the total flow is  $Q_t$ .

We assume  $Q_t$  is a fixed value. The optimal control method is to keep

$$Q_1 = Q_2 = \dots = Q_n = \frac{Q_t}{n}$$

The minimal value of the total power consumption is

$$\min N_t(Q_t) = N_0 \frac{1}{\eta\left(\frac{Q_t}{n}\right)}$$

The ideal work  $N_0$  is

$$N_0 = \frac{Q_t P_0}{3600000}$$

The total optimal efficiency is

$$\max \eta_t(Q_t) = \eta\left(\frac{Q_t}{n}\right)$$

## IV. THE EFFICIENCY SIMILARITY FAN STATION

We define the flow rate  $\gamma$  as

$$\gamma = \frac{Q}{Q_e}$$

We also define  $\eta_N(\gamma)$  to be the normalization efficiency function of a fan. The normalization efficiency function  $\eta_N(\gamma)$  has a shape shown in Fig .3.

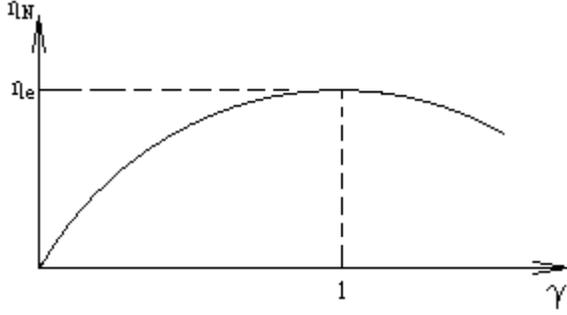


Figure 3. The normalization efficiency function  $\eta_N(\gamma)$

In Fig .3,  $\gamma$  is a variable and  $\eta_N(\gamma)$  is the normalized efficiency.  $\eta_N(\gamma)$  and  $\eta(Q)$  have the relation as following.

$$\eta(Q) = \eta(\gamma Q_e) = \eta_N(\gamma)$$

If the normalization efficiency functions of two different fans are the same, then

$$\eta_{N1}(\gamma) = \eta_{N2}(\gamma)$$

We define them as the efficiency similarity fans. The fan station which includes the efficiency similarity fans is called as the efficiency similarity fan station.

Suppose  $\gamma_i$  is  $i$ th fan's flow rate and has the form

$$\gamma_i = \frac{Q_i}{Q_{ie}}$$

Then for an efficiency similarity fan station, if the fan station is to keep  $P$  as a constant-value  $P_0$ , the total power consumption has the form

$$N_t(Q_t) = \frac{P_0}{3600000} \sum_{i=1}^n \frac{Q_i}{\eta_i(Q_i)} = \frac{P_0}{3600000} \sum_{i=1}^n \frac{\gamma_i Q_{ie}}{\eta_N(\gamma_i)}$$

## V. THE OPTIMAL CONTROL OF EFFICIENCY SIMILARITY FAN STATIONS

In this section, we use our derived expression for the total power consumption of the efficiency similarity system and establish its minimum power consumption.

*Theorem:* For the optimization problem of  $N_t(Q_t)$ , the minimization of the total power consumption

$$\begin{aligned} \min N_t(Q_t) \\ \text{s.t. } \gamma_i > 0, i = 1, 2, \dots, n \\ \sum_{i=1}^n \gamma_i Q_{ie} = Q_t \end{aligned}$$

is given by

$$\min N_t(Q_t) = \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^n Q_{ie}}\right)}$$

*Proof:* We begin our inductive proof by considering the case where  $n=2$ . Our equation for the total flow then becomes

$$\gamma_1 Q_{1e} + \gamma_2 Q_{2e} = Q_t$$

where

$$\gamma_1 > 0$$

$$\gamma_2 > 0$$

The objective function  $N_t(Q_t)$  is expressed as

$$N_t(Q_t) = \frac{P_0}{3600000} \left( \frac{\gamma_1 Q_{1e}}{\eta_N(\gamma_1)} + \frac{\gamma_2 Q_{2e}}{\eta_N(\gamma_2)} \right)$$

The optimal condition is given for

$$N_t'(\gamma_1) = 0$$

We have

$$\frac{Q_{1e} \eta_N(\gamma_1) - \gamma_1 Q_{1e} \eta_N'(\gamma_1)}{\eta_N^2(\gamma_1)} + \frac{-Q_{2e} \eta_N\left(\frac{Q_t - \gamma_1 Q_{1e}}{Q_{2e}}\right) + (Q_t - \gamma_1 Q_{1e}) \frac{Q_{2e}}{Q_{2e}} \eta_N'\left(\frac{Q_t - \gamma_1 Q_{1e}}{Q_{2e}}\right)}{\eta_N^2\left(\frac{Q_t - \gamma_1 Q_{1e}}{Q_{2e}}\right)} = 0$$

It is then easily verified that

$$\gamma_1 = \gamma_2 = \frac{Q_t}{Q_{1e} + Q_{2e}}$$

is a optimal point.

Checking the second derivative, we see

$$N_t''(\gamma_1) > 0$$

so the optimal point is only minimum.

The minimal value of the total power consumption is then

$$\min N_t(Q_t) = \frac{P_0 Q_t}{3600000} \frac{1}{\eta_N\left(\frac{Q_t}{Q_{1e} + Q_{2e}}\right)} = \frac{N_0}{\eta_N\left(\frac{Q_t}{Q_{1e} + Q_{2e}}\right)}$$

where  $N_0$  is the ideal work.

We then assume that this holds for  $n=k$ . The above conclusion os readily extended to the case of  $n=k$ , and the optimal point is then

$$\gamma_1 = \gamma_2 = \dots = \gamma_k = \frac{Q_t}{\sum_{i=1}^k Q_{ie}}$$

while the minimal value of the total power consumption is

$$\min N_t(Q_t) = \frac{P_0 Q_t}{3600000} \frac{1}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^k Q_{ie}}\right)} = \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^k Q_{ie}}\right)}$$

Our inductive case is then given by  $n=k+1$ . For the total power consumption we have

$$N_t(Q_t) = \frac{P_0}{3600000} \left( \sum_{i=1}^k \frac{\gamma_i Q_{ie}}{\eta_N(\gamma_i)} + \frac{\gamma_{k+1} Q_{(k+1)e}}{\eta_N(\gamma_{k+1})} \right)$$

and the minimum of the first item is

$$\min\left(\frac{P_0}{3600000} \sum_{i=1}^k \frac{\gamma_i Q_{ie}}{\eta_N(\gamma_i)}\right) = \frac{P_0}{3600000} \frac{Q_t - \gamma_{k+1} Q_{(k+1)e}}{\eta_N\left(\frac{Q_t - \gamma_{k+1} Q_{(k+1)e}}{\sum_{i=1}^k Q_{ie}}\right)}$$

where

$$\gamma_1 = \gamma_2 = \dots = \gamma_k$$

and

$$\sum_{i=1}^k (\gamma_i Q_{ie}) = \gamma_1 \sum_{i=1}^k Q_{ie} = Q_t - \gamma_{k+1} Q_{(k+1)e}$$

The expression for  $N_t(Q_t)$  becomes

$$N_t(Q_t) = \frac{P_0}{3600000} \left( \frac{\gamma_1 \sum_{i=1}^k Q_{ie}}{\eta_N(\gamma_1)} + \frac{\gamma_{k+1} Q_{(k+1)e}}{\eta_N(\gamma_{k+1})} \right)$$

Based on the above conclusion for  $n=2$ , the optimal point is

$$\gamma_1 = \gamma_{k+1}$$

Therefore the optimal point is then

$$\gamma_1 = \gamma_2 = \dots = \gamma_{k+1} = \frac{Q_t}{\sum_{i=1}^{k+1} Q_{ie}}$$

and the minimal value of the total power consumption is

$$\min N_t(Q_t) = \frac{P_0 Q_t}{3600000} \frac{1}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^{k+1} Q_{ie}}\right)} = \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^{k+1} Q_{ie}}\right)}$$

*Remark:* If all of the fans are the same model, the expression will become

$$\min N_t(Q_t) = \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^n Q_{ie}}\right)} = \frac{N_0}{\eta\left(\frac{Q_t}{n}\right)}$$

## VI. THE OPTIMAL RUNNING NUMBER OF FANS

Suppose a fan station has  $M$ -units in total,  $n$ -units are running,  $n$  is equal or less than  $M$ . If  $n$  is the optimal, there must be

$$\frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^n Q_{ie}}\right)} = \min\left(\frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^1 Q_{ie}}\right)}, \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^2 Q_{ie}}\right)}, \dots, \frac{N_0}{\eta_N\left(\frac{Q_t}{\sum_{i=1}^M Q_{ie}}\right)}\right)$$

Namely

$$\eta_N\left(\frac{Q_t}{\sum_{i=1}^n Q_{ie}}\right) = \max\left(\eta_N\left(\frac{Q_t}{\sum_{i=1}^1 Q_{ie}}\right), \eta_N\left(\frac{Q_t}{\sum_{i=1}^2 Q_{ie}}\right), \dots, \eta_N\left(\frac{Q_t}{\sum_{i=1}^M Q_{ie}}\right)\right)$$

## VII. EXTENSION TO THE OPTIMAL SWITCH POINT

Now we consider the optimal switch point for a efficiency similarity pumping station. Suppose the pumping station has  $M$ -units in total. Then we take  $n$  to be less than or equal to  $M$  and the optimum. The total optimal efficiency is then

$$\eta_N\left(\frac{Q_t}{\sum_{i=1}^n Q_{ie}}\right) = \max\left(\eta_N\left(\frac{Q_t}{\sum_{i=1}^1 Q_{ie}}\right), \eta_N\left(\frac{Q_t}{\sum_{i=1}^2 Q_{ie}}\right), \dots, \eta_N\left(\frac{Q_t}{\sum_{i=1}^M Q_{ie}}\right)\right)$$

We define  $\gamma(Q_t, n)$  as

$$\gamma(Q_t, n) = \frac{Q_t}{\sum_{i=1}^n Q_{ie}}$$

For a variable total flow  $Q_t$ , we see that the optimal switch point is dependent upon whether  $Q_t$  is increasing or decreasing.

In Fig .4 and Fig .5, the  $\eta_e$  is the maximum efficiency, 1 is the flow rate at  $\eta_e$ . The  $\eta_N(Q_t, n_1)$  is less than  $\eta_N(Q_t, n)$  and greater than other efficiency value on the right side of  $\gamma=1$ , the  $\eta_N(Q_t, n_2)$  is less than  $\eta_N(Q_t, n)$  and greater than other efficiency values on the left side of  $\gamma=1$ .

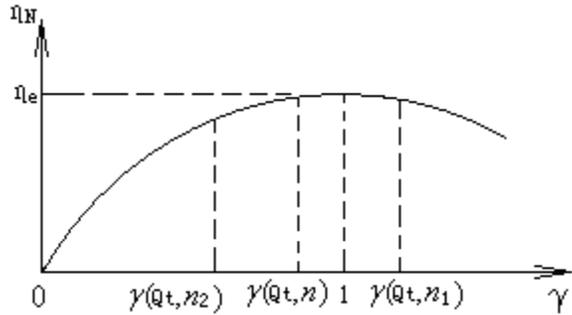


Figure 4.  $\eta_N(\gamma)$  curve when  $\gamma(Q_t, n)$  is equal or less than 1

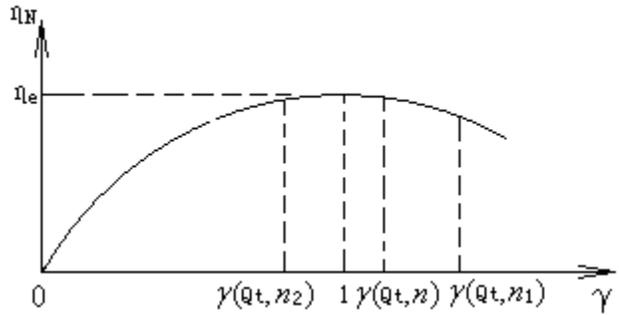


Figure 5.  $\eta_N(\gamma)$  curve when  $\gamma(Q_t, n)$  is greater than 1

*Theorem:* The optimal switch point for an  $M$ -unit system is at  $\eta_N(Q_t, n) = \eta_N(Q_t, n_2)$  if  $Q_t$  is increasing. The optimal switch point for an  $M$ -unit system is at  $\eta_N(Q_t, n) = \eta_N(Q_t, n_1)$  if  $Q_t$  is decreasing.

*Proof:* From Fig .4 and Fig .5, we see that

$$\max(\eta_N(Q_t, n_1), \eta_N(Q_t, n), \eta_N(Q_t, n_2)) = \eta_N(Q_t, n)$$

As seen in Fig .4, if  $\gamma(Q_t, n) < 1$  and  $Q_t$  is increases, then  $\eta_N(Q_t, n)$  will continue to increase until  $\gamma(Q_t, n)$  reach 1, at which point immediately after it will begin to decrease. At the same time,  $\eta_N(Q_t, n_2)$  will increase until it will eventually become greater than  $\eta_N(Q_t, n)$  and will be the new maximum. This change of the maximum efficiency defines the optimal switch point at  $\eta_N(Q_t, n) = \eta_N(Q_t, n_2)$ .

If  $\gamma(Q_t, n) > 1$ , then  $\eta_N(Q_t, n)$  will decrease with the increase of  $Q_t$  while  $\eta_N(Q_t, n_2)$  will increase as can be

inferred from Fig .5. Ultimately,  $\eta_N(Q_t, n_2)$  will increase to such a point that it will become the new maximum and  $\eta_N(Q_t, n) = \eta_N(Q_t, n_2)$  is the optimal switch point.

In a manner akin to our arguments for an increasing  $Q_t$ , we begin by looking at  $\gamma(Q_t, n) > 1$ . If  $Q_t$  is decreasing, then Fig .5 shows that  $\eta_N(Q_t, n)$  will increase until  $\gamma(Q_t, n)$  reaches the value of 1, where it will immediately begin to decrease. Simultaneously,  $\eta_N(Q_t, n_1)$  will increase. As before, there will be point in which  $\eta_N(Q_t, n_1)$  overtakes  $\eta_N(Q_t, n)$  as the new maximum. This is our optimal switch point and is given by  $\eta_N(Q_t, n) = \eta_N(Q_t, n_1)$ .

As seen in Fig .3, for a given  $\gamma(Q_t, n) < 1$ ,  $\eta_N(Q_t, n)$  will decrease as  $Q_t$  decreases. We also have that  $\eta_N(Q_t, n_1)$  will increase to the point where it will be the maximum efficiency. The optimal switch point is then  $\eta_N(Q_t, n) = \eta_N(Q_t, n_1)$ .

### VIII. SIMULATION

If a fan station is used to transfer air at the output constant pressure 0.6Mpa, and the station consists of two different fans, A and B, each with adjustable speed.

Fan A is smaller and its efficiency function about the flow  $Q$  with its out at 0.6Mpa is given by

$$\eta_1(Q) = 0.0168Q - 0.000084Q^2$$

where

$$Q_{1e} = 100(m^3/h)$$

and

$$\eta_{1e} = 0.84$$

Fan B is larger, and its efficiency function about the flow  $Q$  at 0.6Mpa is given by

$$\eta_2(Q) = 0.0084Q - 0.00002Q^2$$

where

$$Q_{2e} = 200(m^3/h)$$

and

$$\eta_{2e} = 0.84$$

We see that A and B have the same normalization efficiency function, given by the following

$$\eta_N(\gamma) = 1.68\gamma - 0.84\gamma^2$$

Based on our previous conclusions, the optimal switch point between A and B is at

$$1.68\left(\frac{Q_t}{Q_{1e}}\right) - 0.84\left(\frac{Q_t}{Q_{1e}}\right)^2 = 1.68\left(\frac{Q_t}{Q_{2e}}\right) - 0.84\left(\frac{Q_t}{Q_{2e}}\right)^2$$

Solving this equation for  $Q_t$ , we see that the optimal switch point between pumps A and B is

$$Q_t = 133.33 = Q_{A-B}$$

Thus, when the total flow  $Q_t$  is less than  $Q_{A-B}$ , only fan A should run, and when  $Q_t$  is greater than  $Q_{A-B}$  only fan B should run. Therefore,  $Q_{A-B}$  is the optimal switch point.

Similarly, we can extend this and determine when the system should switch from the large fan B and run both fans simultaneously. The optimal switch point between B and A+B is found by solving the following for  $Q_t$

$$1.68\left(\frac{Q_t}{Q_{2e}}\right) - 0.84\left(\frac{Q_t}{Q_{2e}}\right)^2 = 1.68\left(\frac{Q_t}{Q_{1e} + Q_{2e}}\right) - 0.84\left(\frac{Q_t}{Q_{1e} + Q_{2e}}\right)^2$$

Our optimal switch point between B and A+B is then

$$Q_t = 240 = Q_{B-(A+B)}$$

We see that when the total flow  $Q_t$  is less than  $Q_{B-(A+B)}$  and greater than  $Q_{A-B}$ , only fan B should run. When  $Q_t$  becomes greater than  $Q_{B-(A+B)}$ , both fans should run together, and thus,  $Q_{B-(A+B)}$  is the optimal switch point. The optimal control method is to have fan A and B keep the same flow rate

$$\gamma_1 = \gamma_2 = \frac{Q_t}{Q_{1e} + Q_{2e}} = \frac{Q_t}{300}$$

### IX. THE CONCLUSION

The Proof of the optimal control and switch method is given by this paper is mainly based on the efficiency function characteristics which can be approximately considered a concave, non-negative function through the origin. Thusly, the optimal method is independent of the linearity or nonlinearity of the system and has no need for the system's state equation or transfer function. Since the shapes of efficiency functions in nature are similar, the method given by this paper can be used in other fields. To resolve the optimization problems of general devices, the authors have done extensive research [8-11].

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