

# Analysis and Reduction of Chattering in Sliding Mode Control for Wearable Robots

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**Abstract**— Considering the applications of human-computer interaction control, It is key problem to the forces provided by robot that make the movement between man and machine to be flexibility. So, the choices of control algorithms for robot, is not only to consider the real-time of control, but also to consider the stability of control. Sliding mode control is designed as a robust controller for wearable robots with nonlinearities, parameter uncertainties and bounded input disturbances. Considered the discontinuous nature of the control action, we introduced a scheme in some neighborhood of the switching surface to reduce the chattering. An effect of various control law within the boundary layer on chattering is studied and a function for chattering reduction and error convergence inside the boundary layer is proposed. As a result, a high-precision position tracking performance is obtained without any oscillatory behavior. The effectiveness of the proposed method is illustrated by simulations.

**Keywords**—wearable robots; sliding mode control; chattering reduction, exoskeletons robot, rehabilitation

## I. INTRODUCTION

As a class of human-like intelligence tools, robots are gradually developed into an important research area with the development of electronics, computer, communications, modern control, and other related machinery and materials science technology. Originally, robots were only intended for use in industrial environments to replace humans in tedious and repetitive tasks and tasks requiring precision, but the current scenario is one of transition towards increasing interaction with the human operator [1]. Today, application development in the field of robotics is more varied and specific. This also related closely to the pursuit of the security of life and the efficiency of daily behavior activities.

Wearable robots are person-oriented robots. They can be defined as intelligent equipment worn by human operators, which to supplement the function of a limb or to replace it completely. As a technology that extends or complements or substitutes or enhances human function, wearable robots are expected to work closely, to interact and collaborate with people in an intelligent environment [2]. In general,

the mechanical systems of wearable robot are designed around the shape and function of the human body, with segments and joints corresponding to those of the person it is externally coupled with [1].

The main purposes of developing exoskeletons are military or rehabilitation. The later is aimed to be used in clinical environments, where these devices can help stroke survivors to regain lost movements. In addition, rehabilitation exoskeletons can be applied to other types of patients, such spinal cord injury patients. One of the most used and studied device in rehabilitation exoskeletons is Lokomat [3, 4], shown by Fig .1. Developed by Hocoma, a Swiss company, the first version was strictly a position controlled device. Another typical exoskeleton rehabilitation device is eLEGS [5, 6], seen Fig .1, which is a exoskeleton lower extremity gait system developed by a North American company. In 2011 the device was renamed Ekso. Currently there are some clinical trials undergoing in rehabilitation centers in United States with Ekso.

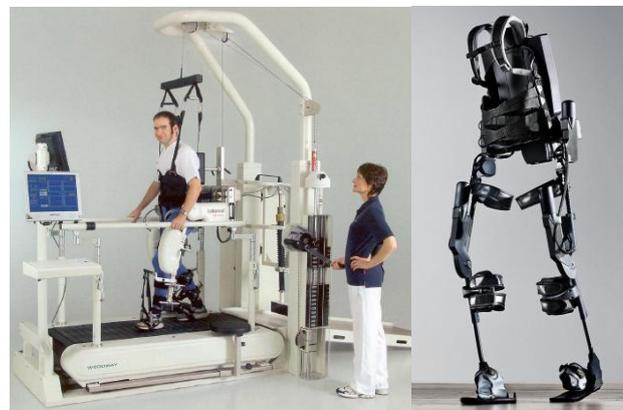


Figure 1. Lokomat and eLEGS developed by Hocoma and Ekso Bionics

As one of exoskeletons or orthotic robots, wearable robots may operate alongside human limbs or substitute for missing limbs, just for instance following an

amputation. We must take into account the function they perform in cooperation with the human actor. The new development of the exoskeleton control system has to meet the safety and admittance aspect demands in the control system [7-9]. It has to be able to assist the user as needed according to the physical condition of the users and the controller must be stable. Considered the instability is caused by high-frequency and high-amplitude external perturbation induced by robot-human interaction, we proposed the sliding mode control for lower limb exoskeleton and analyzed the reason of control law chattering. The incorporation of robot-human was considered in designing the control system.

## II. SYSTEM MODEL OF EXOSKELETON ROBOT

The system hardware for controlling the wearable exoskeleton robot is shown in Fig .1. Motor with drive unit is the actuator as a part of the system hardware. Torque and position data are taken by force or torque sensor and encoder emulation. First of all, we consider the rigid body dynamic  $n$ -link manipulator equations in the joint space as following.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(\dot{q})\dot{q} + G(q) = \tau - \tau_h \quad (1)$$

Where  $q \in \mathbb{R}^n$  denotes the coordinates of the robot manipulator and  $n$  is the DOF;  $M(q) \in \mathbb{R}^{n \times n}$  is the symmetric bounded positive definite inertia matrix;  $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$  denotes the Coriolis and Centrifugal force;  $F(\dot{q}) \in \mathbb{R}^{n \times n}$  denotes the viscous friction coefficients,  $G(q) \in \mathbb{R}^n$  is the gravitational force;  $\tau \in \mathbb{R}^n$  is the vector of control input; and  $\tau_h \in \mathbb{R}^n$  denotes the vector of active motor torque from human subject, which may be directly measured by the torque sensors mounted on the joints or indirectly calculated by the force sensors mounted on the interaction points.

Considered the inaccuracy of torque measurement, the following assumption is made.

Assumption 1: The torque measurement noise  $\tilde{\tau}_h$  is bounded, and  $\tilde{\tau}_h = \hat{\tau}_h - \tau_h$ .  $\hat{\tau}_h$  is the measurement of  $\tau_h$ . It is well-known that the model (1) has the following properties

Proposition 1: The matrix  $M(q)$  is symmetric and positive definite.

Proposition 2: The matrix  $2C(q, \dot{q}) - \dot{M}(q)$  is a skew symmetric matrix if  $C(q, \dot{q})$  in the Christoffel form, i.e.,  $x^T (2C(q, \dot{q}) - \dot{M}(q))x = 0, \forall x \in \mathbb{R}^n$ .

Proposition 3:  $\|M(q)\|, \|C(q, \dot{q})\|$  and  $\|G(q)\|$  are all bounded.

## III. SLIDING MODE CONTROL

### A. Problem Formulation

As a special version of an on-off control, the key idea of sliding mode control is to apply strong control action when the system deviates from the desired behavior. A

controllable  $n$ -th order system can be put in a standard format as follows [10]

$$\dot{x} = f(x) + B(x)u \quad (2)$$

Where  $\mathbf{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T$  is the state variable vector, both  $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^{n \times 1}$  and  $\mathbf{B}(\mathbf{x}) \in \mathbb{R}^{n \times 1}$  are unknown linear or nonlinear functions of  $\mathbf{x}$  and  $u \in \mathbb{R}^1$  is the control input. The block diagram of the sliding mode control is shown in Fig .2.

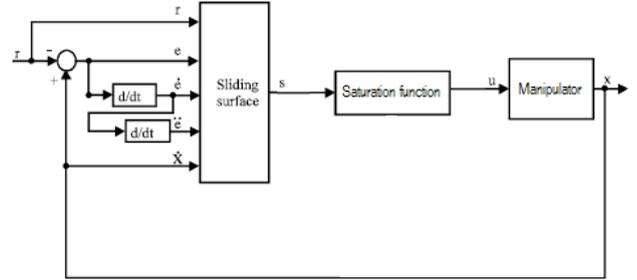


Figure 2. Block diagram of the sliding mode control

The output signal  $x$  is desired to converge to and follow a reference signal  $x_d$ . With this aim we can define a tracking error signal  $\sigma$  as

$$\sigma = x_d - x \quad (3)$$

The procedure for designing a sliding mode controller for such a system can be found in [1]. It is briefly reviewed as follows

1. Define a tracking error signal  $e(t)$  as in Equation 3, and a sliding variable  $s$

$$s = \dot{\sigma}(t) + \Lambda\sigma(t) \quad (4)$$

Where the value of the sliding mode parameter,  $\Lambda \in \mathbb{R}^+$ , needs to be chosen during the design of the controller.

2. Choose the candidate of the Lyapunov function

$$V = \frac{1}{2}s^2 \quad (5)$$

3. Design a controller  $u(t)$  such that

$$\dot{V} = s\dot{s} < 0 \quad (6)$$

The purpose of the sliding mode controller is to drive the system towards the sliding mode surface given by the equation  $s = 0$ , shown on the left by Fig .3. The asymptotic convergence of  $s$  to zero is assured by using Lyapunov stability theory and a controller designed in accordance with Equation 6.

### B. Control Law Design.

The motivation of this controller is to introduce the Lyapunov function

$$V(x) = \frac{s^2(x)}{2} \quad (7)$$

where  $s(x)$  is the switching surface of the system. A controller will be designed in a way such that

$$\dot{V}(x) < 0 \quad (8)$$

for all  $t$ . By then, the controlled system response will then be guaranteed to reach the switching surface, where  $s(x) = 0$ , in finite value of  $t$ . The switching surface is then defined by

$$s = \dot{e} + \Lambda e, \quad \Lambda = \text{diag}(\lambda_1, \dots, \lambda_n), \quad \lambda_i > 0$$

where  $\Lambda$  is known positive matrix and  $e = q_d - q$ . We need to design a variable structure controller such that  $e = 0$  is an asymptotically stable solution. Considered (1) and (7), we choose the following Lyapunov function

$$V = \frac{1}{2} s^T M s, \quad (9)$$

Then,

$$\begin{aligned} \dot{V} &= \frac{1}{2} s^T \dot{M} s + s^T M \dot{s} \\ &= s^T [B(\dot{q}_d + \Lambda e) + M(\ddot{q}_d + \Lambda \dot{e}) + Kq - \tau_h - \tau] \end{aligned} \quad (10)$$

We choose the control law

$$\begin{aligned} \tau &= M_0(\ddot{q}_d + \Lambda \dot{e}) + B_0(\dot{q}_d + \Lambda e) + K_0 q \\ &\quad - \tau_{h0} + \Gamma \text{sgn}(s) \end{aligned} \quad (11)$$

Where  $M_0, B_0, K_0$  and  $\tau_{h0}$  are the nominal value of  $M, B, K$  and  $\tau_h$  respectively.  $\Delta M = M - M_0$ ,  $\Delta B = B - B_0$ ,  $\Delta K = K - K_0$ ,  $\Delta \tau_h = \tau_h - \tau_{h0}$ . Then

$$\begin{aligned} \dot{V} &= s^T [\Delta B(\dot{q}_d + \Lambda e) + \Delta M(\ddot{q}_d + \Lambda \dot{e}) + \Delta K q - \Delta \tau_h] \\ &\quad - \Gamma |s| \end{aligned}$$

Where  $\Gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n), (\gamma_i > 0)$ . If

$$\begin{aligned} \gamma_i &> |\Delta B|_{\max} |\dot{q}_d + \Lambda e| + |\Delta M|_{\max} |\ddot{q}_d + \Lambda \dot{e}| + |\Delta \tau_h|_{\max} \\ &\quad + |\Delta K|_{\max} |q| \end{aligned}$$

Then,  $\dot{V}$  is negative, that is

$$\dot{V} \leq 0. \quad (12)$$

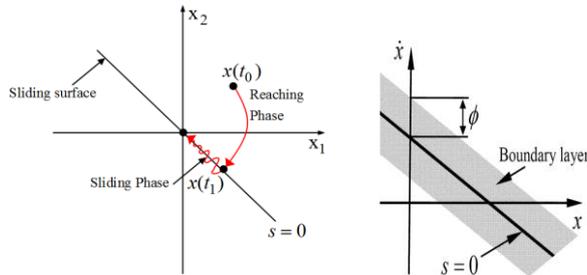


Figure 3. Scheme of the sliding mode control

### C. Chattering Reduction

As we known, the control has a high-frequency component. So the problem of robustness between an ideal sliding mode and real-life processes must be

analyzed, especially at the presence of unmodeled dynamics. Unlike continuous systems that the solution depends on the small parameters continuously, the switchings in sliding mode control excite the unmodeled dynamics, which leads to oscillations in the control vector at a finite frequency. The oscillations, usually called as chattering, are known to result in low control accuracy. Chattering is a drawback of sliding modes, which caused by the high-frequency control switching between positive and negative values in the vicinity of the surface. In addition, chattering can excite unmodeled high-frequency modes and may lead to degraded system performance, system instability and high wear of mechanical parts. The chattering caused by unmodeled dynamics can be eliminated in systems with chattering reduction methods. One way to avoid this is to design the switching function with smoother characteristics.

Chattering can be reduced by introducing a regulation scheme in some neighbourhood of the switching surface [10]. In the simplest case, this is achieved using the boundary layer technique by replacing the  $\text{sign}(s)$  function with a saturation function  $\text{sat}(s)$

$$\text{sat}(s) = \begin{cases} \text{sign}(s) & \text{for } |s| \geq \delta \\ \frac{s}{\delta} & \text{for } |s| < \delta \end{cases} \quad (13)$$

## IV. SIMULATION

This paper also focuses on the modeling and control of the wearable robot. A kinematic model of wearable robot has been developed based on D-H notations. The dynamic can be described by equation (1). Without going into the details, and with the aim of outlining the interest of sliding mode controls in trajectory following, let us consider a manipulator system described as equation (1). The detail parameters of system can refer to [10], and some of them are specified as follows:

$$M(q) = \begin{bmatrix} v + q_{01} + 2q_{02} \cos(q_2) & q_{01} + q_{02} \cos(q_2) \\ q_{01} + q_{02} \cos(q_2) & q_{01} \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -q_{02} \dot{q}_2 \sin(q_2) & -q_{02} (\dot{q}_1 + \dot{q}_2) \sin(q_2) \\ q_{02} \dot{q}_1 \sin(q_2) & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 15g \cos q_1 + 8.75g \cos(q_1 + q_2) \\ 8.75g \cos(q_1 + q_2) \end{bmatrix}$$

$$f(t) = 3 \sin(2\pi t)$$

Where  $v = 13.33, q_{01} = 8.98, q_{02} = 8.75, g = 9.8$ .

$$q_{1r}(t) = 2 + 2 \sin(2t)$$

$$q_{2r}(t) = \cos(2t)$$

The boundary layer function  $\text{sat}(s)$  is then defined specifically as following

$$sat(s) = \begin{cases} sign(s) & \text{for } |s| \geq 1.5 \\ \frac{s}{1.5} & \text{for } |s| < 1.5 \end{cases}$$

Fig .4, Fig .5 and Fig .6 show the tracking performance, control signal and error signal resulting from the controller designed respectively. It is observable from Fig .6 that the error converges to zero within one second and good tracking performance has been obtained. The chattering of control signal is eliminated, as shown in Fig .3.

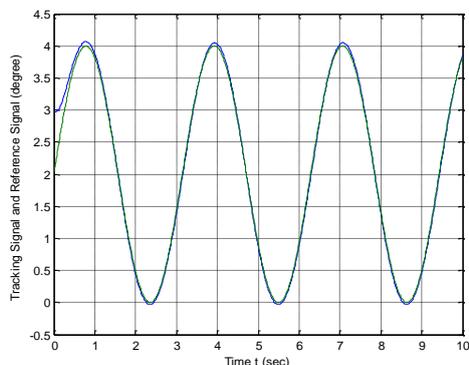


Figure 4. Tracking Signal and Reference Signal

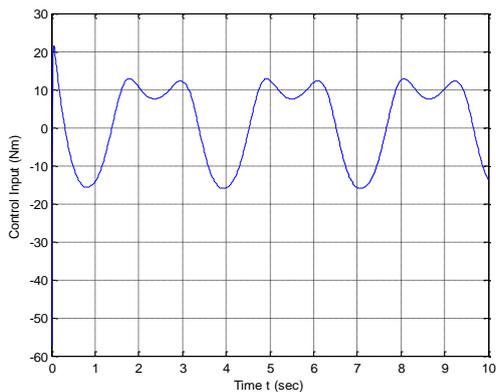


Figure 5. Control Signal  $u$

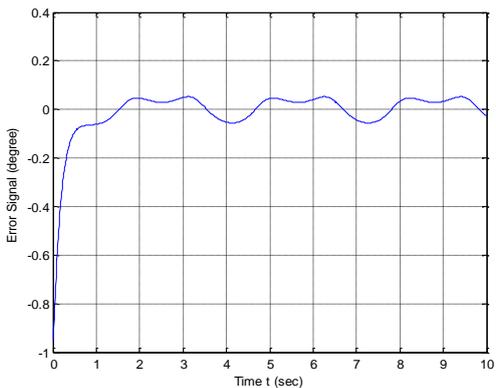


Figure 6. Error Signal  $e(t)$

## V. CONCLUSIONS

The design and simulation of a sliding mode controller for a wearable robot is presented in this report. Simulation results of sliding mode control based *sat* switching function have shown that the chattering is eliminated. The control scheme prescribed in this paper is better choices to the wearable robot, which can increase the comfort of co-operating process.

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