An Improved Genetic Algorithm for Multidimensional Assignment Problem

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Abstract

This paper mainly studies on the valid solution algorithm for multidimensional assignment problem of data association, proposes a heuristic searching algorithm for multidimensional assignment problem on the basis of order-searching algorithm and *m*-best algorithm, and presents a heuristic searching genetic algorithm for multidimensional assignment by fusing the heuristic searching algorithm and the genetic algorithm efficiently. The later algorithm can not only search the whole solution space quickly and efficiently by using the heuristic searching algorithm, but also can memorize and improve the better solution obtained in the initial step by using cross and mutation operations. So the convergence speed is accelerated and the accuracy of the convergence is improved. Simulation results show that the new algorithms are feasible and effective.

Keywords: Multidimensional assignment problem, Improved genetic algorithm, Heuristic searching algorithm, Solution space

1. Introduction

Multi-passive-sensor and multi-target data association problem can be converted into a multidimensional assignment problem by using different optimization principles [1]-[3]. Order-searching algorithm and *m*-best algorithm are searching algorithms for solving assignment problem [4]-[5], the above two algorithms are all suitble for better searching circumstance. In the simple circumstance, their convergence speed becomes slow. And in the dense target and clutter circumstance, both the two algorithms can find the suboptimal solution in a shorter time, but the solution maybe not the best. If we want a suboptimal solution of good quality, we need to search the solutions accurately by adding iteration number or using some heuristic information.

According to the characteristic of order-searching algorithm and *m*-best algorithm, order-searching algorithm is more suitable for the situation that there exist some rows (or planes) of data association cost matrix, in which there are a few elements allowed to enter the optimal assignment process, and in the other situation, the convergence speed of *m*-best algorithm is a little faster than the result of the order-searching algorithm. So, in order to enhance the adaptability of the searching algorithm, this paper firstly proposes a heuristic searing algorithm on the basis of the two algorithms.

Multidimensional assignment problem also can be solved by genetic algorithm, and because of the less information about the better solution in the initial stage, the convergence speed of the simple genetic algorithm is very slow [6]. If we use the heuristic searching algorithm in the initial stage of the genetic algorithm, we can find the better optimal solution quickly. And then we can make full use of the effective information contained in the better solution, i.e., memorize the optimal information snippet, and optimize the inferior information snippet by using the cross and mutation operations of genetic algorithm, thus, the procedure time can be reduced and the convergence accuracy can be increased as well. So, this paper further fuses the heuristic searching algorithm and the genetic algorithm, and proposes heuristic searching genetic algorithm which can find the feasible solution of good quality using the heuristic searching algorithm in the early stage, at the same time, it can fuse the useful information contained in the optimal solutions using the cross operation and optimize part of scattered solution component using mutation operation in the middle and late stage of the algorithm. Therefore, the heuristic searching genetic algorithm has not only strong ability of searching optimal solution in the whole solution space, but also high convergence accuracy.

2. Model of multidimensional assignment algorithm

In the multipassive-sensor multitarget location system, the likelihood function of S-set $\boldsymbol{Z}_{i_1 i_2 \cdots i_s}$ comes from target t can be denoted as [7]-[9]:

$$\Lambda(\boldsymbol{Z}_{i_1 i_2 \cdots i_S} | w_t) = \prod_{s=1}^{S} [P_{ds} \cdot p(\boldsymbol{Z}_{si_s} | w_t)]^{1-\delta_{0i_s}} [1 - P_{ds}]^{\delta_{0i_s}} \tag{1}$$

where, P_{ds} is the detection probability of the sensor s, δ_{0i_s} is an indicator function, when $i_s = 0$ (missed measurement), $\delta_{0i_s} = 1$. Otherwise, $\delta_{0i_s} = 0$. $p(\boldsymbol{Z}_{si_s} | w_t)$ is the probability density function of $\boldsymbol{Z}_{si_s}$ comes from target t.

The likelihood function that the measurements are all spurious or unrelated to target, i.e., $t = \phi$ is:

$$\Lambda(\boldsymbol{Z}_{i_1 i_2 \cdots i_s} | t = \phi) = \prod_{s=1}^{S} (\frac{1}{\psi_s})^{n_s} \tag{2}$$

Let

$$J = \max(\frac{\Lambda(\boldsymbol{Z}_{i_1 i_2 \cdots i_s} | t)}{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_s} | t = \phi)}) \tag{3}$$

where, $\frac{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_s} | t)}{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_s} | t = \phi)}$ is the standard likelihood function, the partition that make it the maximum value is the optimal partition that based on measurement sample.
Let

$$J' = \min(-\ln \frac{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_S} | t)}{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_S} | t = \phi)} \tag{4}$$

then the cost of associating $Z_{i_1 i_2 \cdots i_s}$ with target t is

$$c_{i_1 i_2 \cdots i_S} = -\ln \frac{\Lambda(\boldsymbol{Z}_{i_1 i_2 \cdots i_S} | t)}{\Lambda(\boldsymbol{Z}_{i_1 i_2 \ldots i_S} | t = \phi)}$$

$$= \sum_{s=1}^{S} [(1 - \delta_{0i_s})(\ln \frac{\sqrt{2\pi} \cdot \delta_{0i_s}}{P_{ds} \cdot \psi_s}) + \frac{1}{2}(\frac{Z_{si_s} - \hat{\theta}_{st}}{\sigma_s})^2) - \delta_{0i_s} \cdot \ln(1 - P_{ds})] \tag{5}$$

where, σ_s is the detection error of the sensor s. Define the binary variable $\rho_{i_1 i_2 \cdots i_s}$ as:

$$\rho_{i_1 i_2 \cdots i_S} = \begin{cases} 1 & if\ \boldsymbol{Z}_{i_1 i_2 \cdots i_S}\ is\ from\ a\ true\ target \\ 0 & otherwise \end{cases} \tag{6}$$

Then, the multitarget data association problem can be converted into the following multidimensional assignment problem:

$$\min_{\rho_{i_1 i_2 \cdots i_S}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_s=0}^{n_S} c_{i_1 i_2 \cdots i_S} \rho_{i_1 i_2 \cdots i_S} \tag{7a}$$

s.t.

$$\begin{cases} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} \cdots \sum_{i_S=0}^{n_S} \rho_{i_1 i_2 \cdots i_S} = 1; & \forall i_1 = 1,2 \cdots n_1 \\ \sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \cdots \sum_{i_S=0}^{n_S} \rho_{i_1 i_2 \cdots i_S} = 1; & \forall i_2 = 1,2 \cdots n_2 \\ \vdots & \\ \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_{S-1}=0}^{n_{S-1}} \rho_{i_1 i_2 \cdots i_S} = 1; & \forall i_S = 1,2 \cdots n_S \end{cases} \tag{7b}$$

where, $c_{i_1 i_2 \cdots i_S}$ is the association cost of measurement $\boldsymbol{Z}_{i_1 i_2 \cdots i_S}$ comes from the same target. $\rho_{i_1 i_2 \cdots i_S}$ is a binary variable, if the S-set of measurement comes from the same target, it is 1. Otherwise, it is 0.

When all elements in a cost matrix are allowed to enter the optimal assignment process, the feasible solution number will explosion with the increase of the dimension of assignment problem and the size of data association cost matrix. Under this condition, if we search the whole solution space only use the order-searching algorithm or *m*-best algorithm, because of the limitation of the two algorithms of themselves, the convergence speed is very slow in some scenarios [3, 5, 9]. So, this paper will firstly propose a heuristic searching algorithm, which fuses the above two algorithms, in next section.

3. Heuristic searching algorithm

The heuristic searching algorithm does not use the order-searching algorithm and *m*-best algorithm by turns, but uses some heuristic operations according to the characteristic of the two algorithms. The main idea is that we find the row (column or plane) which has less elements allowed to enter the optimal assignment process, arrange the sequence number of the rows in order randomly, and sort the other row's sequence number randomly as well, then aiming at the two sequence, we use the order-searching algorithm and *m*-best algorithm to search the optimal solution, respectively. This algorithm can not only search the feasible solution quickly, but can also improve the data association accuracy.

The steps of the heuristic searching algorithm: (take two-dimensional assignment problem for example)

Step1 initialization:

$$ff = \infty, \quad \max iter = N, \quad k = 0, \quad l = 0 .$$

Step2 Count the member of the elements which enter the candidate association set for the row(or column) of the cost matrix of 2-D assignment problem, put the row's sequence number whose elements in the candidate association set are less than t (given in advance) into the set s_1 ,and put the rest into set s_2 . Use the randperm command in matlab to make an order for the elements of s_1 and an other order for elements from s_2 respectively, then put the order from s_1 in ahead, and the order from s_2 afterwards, so as to get an order from 1 to n . Repeat the operation a certain number to get more different orders.

Step3 In sight of an order from s_1 , we use the order-searching algorithm to find an optimal solution component, and aiming at an sequence from s_2 , use the *m*-best algorithm to search an optimal solution component, repeat above operation till we get n_1 feasible solution of the assignment problem. Then we calculate the n_1 suitble function value, if $\min f_i < ff, i = 1,2 \cdots n_1$; then let $ff := \min f_i, l = 0, \quad A = A_{ff}$, turn to the next step. Otherwise, $l := l + 1$, if $l = 5$, stop, put out the optimal solution of the assignment problem $A^* = A$, and put out the optimal value $z^* = ff$. Otherwise, turn to the next step.

Step4 If $k \neq N, let \quad k := k + 1$, return to Step2. Otherwise, put out $z^* = ff, A^* = A$.

The above algorithm can be extended to solve multidimensional assignment problem. For example, with a problem of 3-D assignment, we should arrange the cost plane's serial number in different order. The difference is that we find the minimum or the second minimum value in all of the elements of a cost plane, in the meantime, we must ensure that the solution is feasible.

In the above searching process, if we can memorize some information snippet contained in the solutions of good quality, and optimize the other information snippet, so as to get the better solution, it certainly can increase the efficiency of the algorithm. In next section, we will study this problem and propose a heuristic searching genetic algorithm.

4. Heuristic searching genetic algorithm

4.1. Genetic algorithm

When we use genetic algorithm to solve the multidimensional assignment problem, in consideration of a great deal of encoding and decoding operations when we adopt the binary encoding way, this paper adopts a decimal notation way to encode. For example, a feasible solution of a 3-D assignment problem which have $n \times n \times n$ cost matrix can be composed of 3 coding series. For conveniently represent, we juxtapose the 3 coding series, and this corresponds to a gene of genetic algorithm, i.e., a feasible solution of the assignment problem.

A 3-set of element from different row but the same column is a gene of the chromosome, i.e., one component of the feasible solution. For example, when $n = 8$, the code of a chromosome A can be presented as:

$$A = \begin{pmatrix} 4 & 1 & 5 & 2 & 3 & 7 & 8 & 6 \\ 7 & 2 & 1 & 4 & 3 & 6 & 8 & 5 \\ 7 & 4 & 1 & 2 & 3 & 6 & 8 & 5 \end{pmatrix}$$

In order to develop the advantages of the heuristic searching algorithm and the genetic algorithm, we propose a heuristic searching genetic algorithm based on fusing the two algorithms. It is described in the next section.

4.2. Heuristic searching genetic algorithm

The steps of the heuristic searching genetic algorithm:

Step1Initialization:

$$ff = -\infty, \quad \max iter = N, \quad k = 0, \quad l = 0 .$$

Step2 Produce M chromosomes by using the heuristic searching algorithm, from them, select m chromosomes with larger suitable functions as the initial population, and construct n_1 groups with m_1 chromosome.

Step3 Carry out the cross and mutation operations to the chromosomes in each group, the cross here is to extract the same gene snippet from the chromosomes in each group, and remember and denote the optimal gene from the snippet information in solution matrix. Furthermore we eliminate the serial number of the row (or column or plane), which contains the elements from the optimal gene, from the solution matrix, and then we can get a lower dimensional assignment problem.

Step4 If the dimension of the lower dimensional assignment problem is still larger than s (given in advance), repeat step 3, till the dimension of the current lower dimensional assignment problem is less than s. Then we carry out the searching for a certain number to the lower dimensional assignment problem to get some variant gene snippet, and denote the corresponding sequence number in the solution matrix respectively, so we can get a better feasible solution of the assignment problem through comparison.

Step5 Repeat step3-step4 for a certain number till we get n_1 better feasible solution of the assignment problem, on the other hand, we get n_2 feasible solution by using heuristic searching algorithm. Compare the suitble function value of the $n_1 + n_2$ feasible solution, if $\max f_i > ff, i = 1,2 \cdots n_1 + n_2$; Let $ff = \max f_i$, the optimal solution $\boldsymbol{A} = \boldsymbol{A}_{\max f_i}$, $l = 0$,turn to Step5. Otherwise, $l := l + 1$ when $l = 5$, stop, put out the optimal solution $\boldsymbol{A}^* = \boldsymbol{A}$, optimal value $z^* = -ff$. Otherwise, turn to Step6。

Step6 If $k \neq N$, put the n_1 evolution solution, n_2 feasible solution and m initial solution together, and select m feasible solution with larger suitble function value from them as the population of the next iteration. let $k := k + 1$, repeat Step2-Step5. Otherwise, put out the optimal solution $\boldsymbol{A}^*$ and the optimal value z^*.

5. Simulation and analyses

5.1. Model of simulation

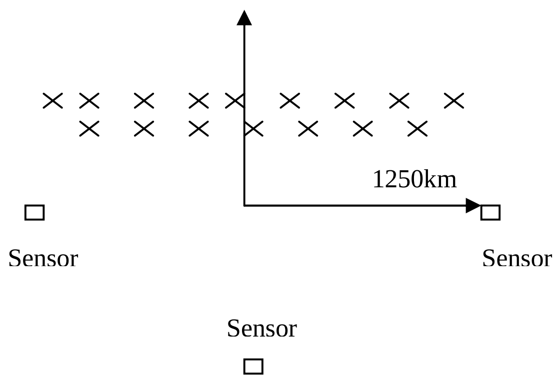

Fig.1: Location of three sensors

When the sensors and the targets are in the same plane, it is difficult to distinguish the true target point and the false point if we locate the multitarget using two bearing-only passive sensors. In order to get a better data association effect, we need to deal with measurements from multisensor synthetically and effectively. Suppose three bearing-only passive sensors are used to locate targets as illustrated in Fig.1.

In the simulation, let the false alarm rate and the detection rate be 0 and 1, respectively. Suppose the bearing measurement error of various sensor is the same value, it is taken as $\pi/360, \pi/180$ *and* $\pi/90$, respectively. The number of target is 15, the distance between targets is 250km. the heuristic searching algorithm uses a multigroup iterations, the scale of each group is 3, the maximum iterations of each group is taken as 400, 300, 300 corresponding to the various measurement error. The population scale of the genetic algorithm and heuristic searching genetic algorithm is taken as 18, and $m_1 = 3$, $n_1 = 6, l = 2$. The maximum iteration N of 3-D assignment algorithm is used as 80, 60, 60 in the three conditions of various measurement errors.

5.2. Analyses of simulation

Form table1, we can see that the data association accuracy of the heuristic searching genetic algorithm is far exceed the result of the simple genetic algorithm. The reason is mainly about that in the initial period of the simple genetic algorithm, there has no heuristic information, and the patter centralizes in the individuals that have lower adaptability, so the convergence speed is slower. The method that can increase the accuracy of data association is to increase the number of the iteration of the algorithm.

In addition, the data association result of the heuristic searching genetic algorithm excels the effect of the simple heuristic searching algorithm, and the procedure time of the former algorithm is also less than the result of the later algorithm accordingly. This mainly because that the operation of the multi-operator recombination and mutation makes the searching result near to the better solution, so the convergence speed is accelerated. On the other hand, because that the algorithm avoids the shortcoming of difficulty to get the feasible solution for the actual require of the simple heuristic searching, the procedure time further decreases correspondingly.

		Data association accuracy	Procedure time (s)	Average error in position estimate (km)
$\pi/360$	Genetic algorithm	62.4%	3.13	20.76
	Heuristic osearching algorithm	82.7%	6.92	3.29
	Heuristic searching genetic algorithm	87.3%	3.61	2.24
$\pi/180$	Genetic algorithm	53.6%	3.08	23.48
	Heuristic searching algorithm	55.6%	5.97	20.47
	Heuristic searching genetic algorithm	71.7%	3.49	9.68
$\pi/90$	Genetic algorithm	48.7%	3.43	29.78
	Heuristic searching algorithm	52.6%	4.48	25.91
	Heuristic searching genetic algorithm	64.6%	2.92	17.87

Table 1: Results of 3-D assignment problem solved by the three algorithms (average of 50 runs)

6. Conclusions

This paper studies the searching algorithm for multidimensional assignment problem deeply, proposes the heuristic searching algorithm used to solve the multidimensional assignment problem, and it also fuses the idea of heuristic searching algorithm and genetic algorithm, and gives the heuristic searching genetic algorithm. Simulation experiment shows that compared with the result of using one simple algorithm, the fused algorithm can not only increase the accuracy of data association effectively, and increase the precision of the target location, but also can save the procedure time greatly, therefore, the algorithms presented in this paper are feasible and effective.

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