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Analysis of Dynamic Behavior of a Class of Nonlinear System with Time Delay

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Abstract—Nonlinearity is an important characteristic of complex system, is generally believed that the nonlinearity is a necessary condition to produce complexity, no nonlinear no complexity. In addition, bifurcation, chaos in complex system, the dynamic behavior of the strange attractor, are inherently nonlinear. The nonlinear dynamic behavior, often by a certain amount (or large) nonlinear element (or subsystems) due to the combination and interaction, is essentially different from the linear element combination and interaction. In fact, the nonlinear factors in complex systems (internal and environmental) and their interactions is an important condition for the formation of complexity. Financial power system is a complex nonlinear system is a kind of typical, it may exhibit a variety of complex phenomenon and behavior, including chaos, many factors affecting the dynamic properties of the system, in which, the existence of time delays from the impact of various aspects of the stability of the whole financial system, and may even lead to financial crisis. In this paper, through the establishment of financial power system model with time delays, by using the theory of differential equations, characteristic of the system stability and chaos are discussed, from system to produce related behaviors and conditions, numerical simulation. Finally gives the conclusion of economic implications.

Keywords- The nonlinear;time delay; stability;chaos; financial

I. INTRODUCTION

Financial dynamic nonlinear system is a complicated, it may exhibit a variety of complex phenomenon and behavior, including chaos, delay is the financial system an important factor. As everyone knows, to be implemented from the development of regulatory policy and regulatory policy, the ultimate goal to produce completely effect, implementation of macroeconomic regulation and control, the entire process requires a certain time interval, the so-called policy lag. Because of the policy of the time-delay, may affect the effective policies on the economy, so in the policy formulation and operation, must consider the delay problem of macroeconomic regulation and control policy. Because of the very important complexity and stability of the financial system, financial system risk analysis is a challenging problem. The existence of time delays from all influence the stability of the financial system, and even the emergence of the financial crisis. Description and Analysis on the financial system with time delay is more beneficial to us to grasp the whole financial system operation, reveal the financial markets of various interaction between, is conducive to economic development and social stability.

In this paper, adding a time delay in the financial power system model, construct the financial system model with time delay, and discuss and chaos of the stability of the system, combining with the data of simulation under different parameters of the conclusion.

II. FINANCIAL POWER SYSTEM MATHEMATICAL MODEL

Literature [1] using system dynamics method to build and test a production that block, block, currency financial model composed of securities sub block and labor sub block, sensitivity to certain long-term behavior of this model is given with no rules and parameters of the initial state of the extreme, in order to further study the reason and mechanism the chaotic behavior patterns, people will be the key part in the model to be refined and simplified, after careful analysis and experiments, decided to model in the interest rate is represented by the variable x, investment demand is represented by the variable y, price index is represented by the variable Z. The three state variable and included in their information feedback loop as the key structure. By selecting the appropriate coordinate system and give the state variable to the appropriate dimension, obtained the following contains only 3 main parameters of the simpler model.

Financial power system model:

$$\begin{cases} \dot{x} = z + (y - a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} \qquad (1)$$

The type x is the rate of interest, y said that investment demand, says z price index, a savings, b is the unit investment cost, c for commodity demand elasticity (a, b, c were normal number).

Now consider the factors of the delay and the average rate of profit impact of interest rate changes, interest rate in the model with delay, the establishment of financial power system model with time delay. Here we consider the interest rate when the delay is a period of time, then the influence on the whole financial system, such as the decision delay interest rate policy is a continuous time.

Here introduced distributed delays

$$\int_{-\infty}^{t} F(t-\tau)x(\tau)d\tau$$

The F (T) meet $\int_{0}^{t} F(s)ds = 1$

The time delay dynamical system model for financial:

$$\begin{cases} \dot{x} = z + (y-a)x + mx + k\int_{-\infty}^{t} F(t-\tau)x(\tau)d\tau \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} \quad (2)$$

In it $\tau \geq 0$ Is a time delay, K is the feedback strength.

In order to facilitate the study of the influence of time delay for the system,let

$$\phi(t) = \int_{-\infty}^{t} F(t-\tau)x(\tau)d\tau$$

and $h \geq 0$, In this way, the system can be (2) into equivalent differential equations:

$$\begin{cases} \dot{x} = z + (y-a)x + mx + k\phi \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \\ \dot{\phi} = h(x-\phi) \end{cases} \quad (3)$$

Obviously, when h=0, the system (2) and (3) the same, here we only consider $h > 0$

III. ANALYSIS OF DYNAMIC SYSTEMS

A. Analysis of dynamic characteristics

In system (1) :When $c - b - abc \leq 0$,there is a only equilibrium point $P_0(0,1/b,0)$, and then $c - b - abc < 0$, $1 + ac - c/b > 0$, the equilibrium point P_0 is stable. When $c - b - abc < 0$ $c + a - 1/b < 0$, the equilibrium P_0 is a saddle point. When $c + a - 1/b = 0$, 0 <c <1, the equilibrium point P_0 is non-hyperbolic and unstable equilibrium point.When $c - b - abc \geq 0$, the system has three equilibrium points: $P_0(0,1/b,0)$, $P_{1,2}(\pm\theta,(1+ac)/c,\mp\theta/c)$,When $c - b - abc = 0$, 0 <c <1, P_0 is the unstable equilibrium point; When $c > b + abc$,when the parameters satisfy $bc^4 + b^2c^3 - 2ab^2c^2 + (2ab - 2 - 3b^2)c + 3b = 0$, the system (1) appears bifurcation, $P_{1,2}$ an unstable equilibrium point.

Take parameters a = 0.9, b = 0.2, c = 1.2, the initial conditions [3, 1, 5],to simulate by using Matlab software, when the simulation time was set to [0,10] and [0,1000], get the three-dimensional phase diagram of finance chaotic system (1) as shown in Fig .1, Fig .2, respectively. Fig .1, Fig .2 shows that as time extending, the chaotic behavior of the economic system become more and more prominent.

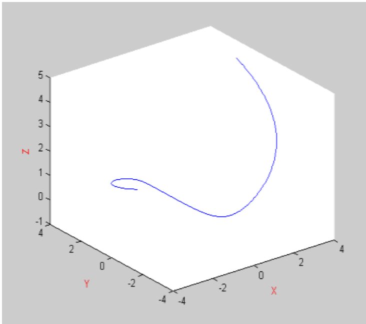

Figure 1. the three-dimensional phase diagram when simulation time is [0,10]

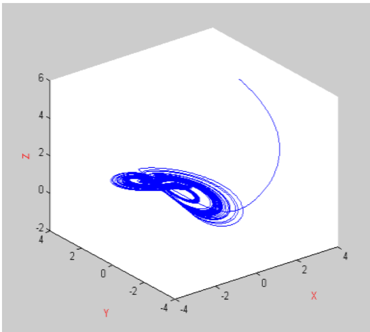

Figure 2. the three-dimensional phase diagram when simulation time is [0,1000]

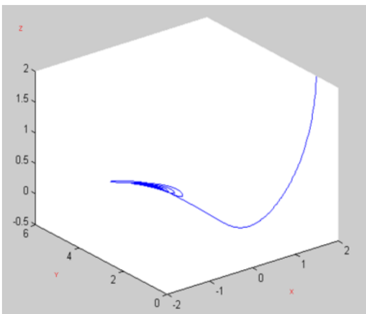

Figure 3. the initial value point (2,1,2), a = 3, b = 0.2, c = 2system x-y-z phase diagram

If we take a = 3, b = 0.2, c = 2, while at the same time to take the initial conditions [2, 1, 2], x,y,z, the phase diagram in Fig .3, numerical results show that:

Equilibrium point of the system to meet the conditions of the $c-b-abc>0$ is the exchange,fully consistent with the theoretical analysis.

By the above analysis and data simulation shows that the inappropriate combination of parameters in the system cause the system to the source of chaos, it is possible to make the system tend to chaotic and out of control, resulting in economic turmoil, when the parameters will appear in a different system state,analysis of changes in various parameters to get the kind of financial system of local conditions to produce complex behavior.

B. Analysis of Dynamic Systems with Time Delay

The system (3) is the symmetry of nature, In the transform $S:(x,y,z,\phi)\to(-x,y,-z,-\phi)$ all the parameters have invariance, The system is symmetric about the y axis, That Φ is the solution of thesystem, and $S\Phi$ the solution of the system.

The system (3) on the right is equal to zero, can obtain the equilibrium point.

When $\Delta=-b+c-abc+bck+bcm<0$,

The system has a unique equilibrium point:

$$p_0=(0,1/b,0,0)$$

When $\Delta\geq 0$ The system has 3 equilibrium points:

$$p_0=(0,1/b,0,0);$$

$$P_1=(\sqrt{\Delta/c},1+ac-ck-cm/c,-\sqrt{\Delta}/c^{3/2},\sqrt{\Delta/c});$$

$$P_2=(-\sqrt{\Delta/c},1+ac-ck-cm/c,\sqrt{\Delta}/c^{3/2},-\sqrt{\Delta/c}).$$

The following study when $\Delta<0$, at the equilibrium point of the system status. The system (3) is a linear transformation, let $X=x,Y=y-1/b,Z=z,\psi=\phi$,

Then (3) into:

$$\begin{cases}\dot{X}=(1/b-a+m)X+XY+Z+k\psi\\ \dot{Y}=-bY-X^2\\ \dot{Z}=-X-cZ\\ \dot{\psi}=h(X-\psi)\end{cases}\tag{4}$$

All equilibrium points of the system is P= (0,0,0,0). The system (4) in the P matrix of linear Jacobin:

$$J=\begin{bmatrix}1/b-a+m+Y & X & 1 & k\\ -2X & -b & 0 & 0\\ -1 & 0 & -c & 0\\ h & 0 & 0 & -h\end{bmatrix}$$

Then, the characteristic equations:

$$\begin{aligned}|\lambda I-J|&=(\lambda-1/b+a-m)(\lambda+b)(\lambda+c)(\lambda+h)\\ &+(\lambda+b)(\lambda+h)-kh(\lambda+b)(\lambda+c)\\ &=(\lambda+b)(\lambda^3+E_1\lambda^2+E_2\lambda+E_3)\\ &=(\lambda+b)G(\lambda)=0\end{aligned}\tag{5}$$

Type in:

$G(\lambda)=\lambda^3+E_1\lambda^2+E_2\lambda+E_3$; $E_1=c+h+u$; $E_2=1+ch-kh+u(c+h)$; $E_3=ch(1/c+u-k)$; $u=-1/b+a-m$.

In order to make the system (4) is stable at point P, then the equation (5) eigenvalues have negative real parts. Apparently (5) of the 1 negative eigenvalues, therefore, only need to discuss the value of the. By the following lemma:

lemma: By Routh-Hurwitz conditions, $G(\lambda)=\lambda^3+E_1\lambda^2+E_2\lambda+E_3$ All the eigenvalues have negative real part if and only if

$$E_1>0,E_2>0,E_3>0,E_1E_2>E_3$$

Only when these conditions are established, P is locally asymptotically stable.

Proposition: Equation (5) has 2 real roots $\lambda_1=-b$, $\lambda_2=-(u+c+h)$ And a pair of pure imaginary conjugate $\lambda_{3,4}=\pm i\sqrt{1+ch+(c+h)u-kh}$,

If and only if $1+ch+(c+h)u-kh>0$,and

$$k=(c+u)[1+(c+h)(u+h)]/[h(h+u)]$$

Prove. Only that $G(\lambda)=0$ There is a real root and a root can. The first hypothesis λ_2 and $\lambda_{3,4}=\pm i\omega$ is a real root and a pair of pure imaginary roots of $G(\lambda)=0$,then

$$(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)=\lambda^3+E_1\lambda^2+E_2\lambda+E_3=0$$

That is $\lambda_3-\lambda_2\lambda^2+\omega^2\lambda-\lambda_2\omega^2=\lambda^3+E_1\lambda^2+E_2\lambda+E_3$

Coefficients on both sides, can be obtained:

$$\lambda_2=-E_1=-(u+c+h)$$

$$\omega^2=E_2=1+ch-kh+u(c+h)$$

$$\lambda_2\omega^2=E_3=uch+h-khc$$

That is

$$E_2=1+ch-kh+u(c+h)>0$$

$$k=(c+u)[1+(c+h)(u+h)]/[h(h+u)]$$

$$\lambda_2=-E_1=-(u+c+h)$$

$$\lambda_{3,4}=\pm i\sqrt{1+ch+(c+h)u-kh}$$

At this time, propositions syndrome.Therefore, when the system is not satisfied

$$E_1>0,E_2>0,E_3>0,E_1E_2>E_3$$

may produce chaotic.

Below we to calculate Lyapunov index. The Lyapunov index is close to each other in the phase space of the two trajectories over time according to the average change rate index separation or polymerization, is one of the important parameters for the quantitative description of chaos dynamics. Gerry bases that: when a system Lyapunov index appears positive, this system is chaotic, means that in the system in phase space, regardless of the initial two line spacing is so small, the difference will be as time evolved exponential increase rate to achieve the impossible to predict, this is the chaos phenomenon.

Grippo Ki proved that: (steady state and periodic motion and quasi periodic motion) may not have a positive Lyapunov exponent, Lyapunov exponent stability are negative, the Lyapunov index of periodic motion has a 0, the rest is negative. We use the method of calculation in literature [11] here.

Let a=3,b=0.2,c=1,m=0.5,h=2,k=0.6, The system (4) of the 4 Lyapunov index respectively:

$$\lambda_{L1} = 0.0312616, \lambda_{L2} = -0.0512554$$

$$\lambda_{L3} = -0.568041, \lambda_{L4} = -2.35516$$

One of the largest Lyapunov exponent, so the system has a chaotic attractor, explains the system chaos.

IV. NUMERICAL SIMULATION

According to the above analysis, dynamical behavior and numerical simulation system, A=3, b=0.2, c=1, m=0.5, k=0.6, h=2, the system has a Lyapunov index greater than zero, the simulation conditions for [0100] programming with matlabe is this period Fig .4 as follows.

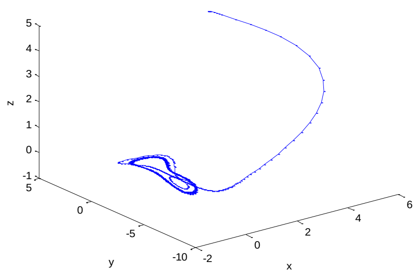

Figure 4. The simulation time is [0100] phase diagram

At this time, there were chaotic attractors in the system, the system with the change of time is chaotic, that is to say, when the system from these values, the phenomenon of arbitrage in the financial system, so there is a certain financial risk, it is necessary to decision-makers quick reaction, some powerful measures, control the financial market, to achieve stability state, reduce the economic risk. Change the delay will bring uncertain systems.

V. ENDING

Get some inspiration from the previous analysis:

First, combination of each parameter improper system is caused by the economic system appear chaos roots, it is possible to make the system tends to chaos and out of control, there may be a system of stagnation rigid state, therefore, whether in the period of inflation is serious, or in the period of economic recession, the conversion mechanism, structural adjustment is an important task for finance system.

Second, lack of elastic variables will cause the delay information feedback, only error signal given the decision-making body of the distorted, adverse consequences arising therefrom is one can imagine, what is more serious is that these adverse consequences will be to the vicious spiral form continues to spread. Therefore, properly increasing the variable elastic will help to stabilize the economy, the normal operation of the financial system.

Third, the amount of storage system variables must be maintained at an appropriate level. The results of the study show that, smaller, the larger fluctuation, too hours can lead to chaos of the situation; however, it can not be too large, otherwise, will make the economy less energy.

Fourth, different combinations of different parameters in the system can lead to the bifurcation phenomenon, bifurcation represents the transition between economic growth during different growth pattern.

Fifth, if the selection of appropriate control strategy and direction, a good grasp of financial control, power system can be stabilized.

Need to explain is: Although nonlinear science have made great progress in all areas, but compared to natural science, its research results achieved in the field of economics, finance limited. The nonlinear dynamics method applied to economics, a master key is not the solution to all problems. In the field of finance, equity and social economy due to the interaction of nonlinear factors, economic problems become more and more complicated and from low - to high dimensional evolution process, the internal structure of system is also showing a diversity and complexity, therefore, there are still a lot of work to do.

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