

# R/S Analysis of Regional Divergence Temporal Evolution of Urban Domestic Source Pollution in China

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**Abstract**—Research on domestic pollution sources is actively carried out to be studied as an international environmental issue after that on industrial pollution sources. Contrasted with industrial source, neither regularity nor mechanism are shown in the studies on domestic source. The domestic source of urban residents was taken as the investigated object in this study, where multisequentially social, economic and environmental statistics were collected systematically. Using fractal and fractal dimension theories and geo-statistic methods, the studies were carried out on the temporal and spatial evolution laws of China's urban domestic source pollution discharge by province. The achieved results proved that regional differences in China's urban domestic source revealed an expansion trend in the future with its spatial agglomeration shown as weak positive spatial autocorrelation, and the overall variation trend was transformed from agglomeration to discrete state. the in-depth studies on temporal and spatial evolution of China's urban domestic source pollution discharge have important theoretical and practical significances for promoting domestic source research.

**Keywords**—urban domestic source; regional divergence; temporal evolution; R/S analysis; fractal dimension;

configuration, they are significantly limited in the application of actual complex system. To describe some inherent commonalities of different time-series data within different time range, traditional methods of time series analysis fail. R/S analysis method (Rescaled Range Analysis Method) in Fractal Theory will be proposed in the paper to describe temporal evolution of provincial urban domestic discharge in China<sup>[2]</sup>.

## II. DATA SOURCE

Main factors of domestic discharge (domestic sewage, COD, NH<sub>3</sub>-N) are taken as statistical objects in this chapter, along with systematic collection of the information from China Environment Yearbook, Annual report on environment in China, China Statistical Yearbook as well as statistical yearbook in provinces in 1998-2008.

## I. INTRODUCTION

Time series refers to dynamic data set generated and arranged chronologically, features the dependency or correlation and integrates the characteristics and laws of sometime changes for some phenomena, processes or systems. Time series analysis traditional in autoregressive, moving average and other methods etc. may be used for the analysis and fitting of variation characteristics of certain variables within a specific time period, and to predict future changes using the extrapolation<sup>[1]</sup>. Due to great impact of specific time series for traditional analysis methods on model structure and fitting accuracy of the

## III. RESEARCH METHODS

R/S analysis is usually used to analyze fractal characteristics and long-term memory processes of time series, initially proposed by Hurst, a British hydrologist, in 1951 in the research of Nile dam project. Later the method was applied in the analysis of various time series. R/S analysis is used to study the laws of statistical characteristics changes based on Hurst index change in different time scale, through the change of time scale of

the object, and predict future change trend under known development conditions<sup>[4]-[11]</sup>.

$$X(t_i, n) = \sum_{i=1}^n \{\xi_i - \langle \xi \rangle_n\}$$

Where,  $X(t, n)$  is the cumulative departure at  $t_j$ ,  $\xi_i$  is time series at  $T_i$ , and  $\langle \xi \rangle_n$  is the average of time series.

$$R(t_n - t_1) = R(T) = \max_{t_1 \leq t \leq t_n} X(t, n) - \min_{t_1 \leq t \leq t_n} X(t, n)$$

Where,  $R(T)$  is the range,  $\max_{t_1 \leq t \leq t_n} X(t, n)$  is the max in  $n$  time series, and  $\min_{t_1 \leq t \leq t_n} X(t, n)$  is the min in  $n$  time series.

$$S = \left( \frac{1}{n} \sum_{i=1}^n \{\xi_i - \langle \xi \rangle_n\}^2 \right)^{1/2}$$

$$\frac{R}{S} = \frac{\max_{t_1 \leq t \leq t_n} X(t, n) - \min_{t_1 \leq t \leq t_n} X(t, n)}{\left( \frac{1}{n} \sum_{i=1}^n \{\xi_i - \langle \xi \rangle_n\}^2 \right)^{1/2}}$$

$$R/S = (T/2)^H$$

$H$  in the above formula stands for Hurst index, and it is different as time series is different, and varies as the nature of time series changes. The value of  $H$  is between 0-1, and in case of  $H = 0.5$ , it indicates that the time series is random and irrelevant, so the status quo does not have an impact on the development of future trend; in case of  $H > 0.5$ , it indicates the persistence of time series, and its future development maintains original trend, so the greater  $H$  is, the more obvious original trend is maintained; in case of  $0 < H < 0.5$ , it means that time series is persistent or ergodic in inverse state, and future development trend is opposite to current situation, so the smaller  $H$  is, the more obvious the regularity is. Thus, the laws can be deeply discussed, by calculating the variation of value  $H$  for regional divergence measurement index of urban domestic source pollution in China.

#### IV. TREND OF REGIONAL DIVERGENCE VARIATION OF URBAN DOMESTIC SOURCE POLLUTION IN CHINA

During research time series (1998-2008), in the moment of rapid economic development and urbanization in China, environmental quality also will be changed dramatically. From main index of domestic discharge, it includes waste (sewage) water, COD and  $\text{NH}_3\text{-N}$ , in different trends (Fig. 1).

From time series of waste water discharge time in the research, total waste increases by annual 3.77% on average, while domestic sewage rapidly grows by 5.43%. Since

1999, the structure of sewage discharge structure has changed, and domestic sources are beyond industrial sources; especially after 2005, industrial wastewater has been effectively controlled, showing negative growth trend year by year, while domestic sewage waste has become a major driving force of waste water growth.

From COD discharge time series, it shows that COD emission has been effectively controlled, slower in overall growth and with a diminishing trend after 2006; domestic COD emission shows annually increased 3.12% on average, while that in industrial sources declines 4.44%, so it is apparent that domestic COD control is key currently.

After  $\text{NH}_3\text{-N}$  is included in total control index since 2002, upon the peak of its total discharge in 2006, it shows the "first increased and then decreased" situation, and domestic and industrial sources emerge in the same development trend, declining year by year during the of life and industrial sources is the same trends in diminishing during the Eleventh Five-Year Plan.

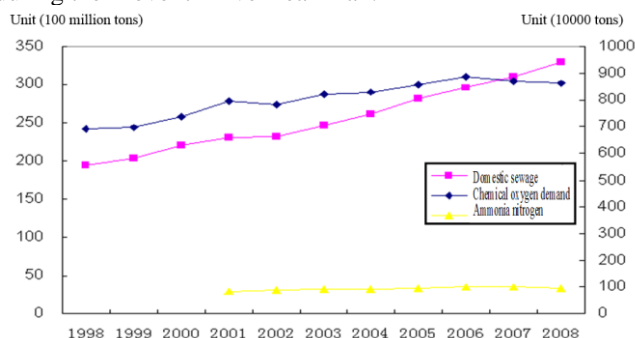


Figure 1 Statistics of domestic discharge (1998-2008)

#### V. R/S ANALYSIS OF REGIONAL DIVERGENCE TEMPORAL EVOLUTION OF URBAN DOMESTIC SOURCE POLLUTION IN CHINA

##### A. Divergence measurement

In order to analyze variation trend of domestic source pollution, weighted variation coefficient ( $V_w$ ) and Theil coefficient ( $L$ ) are selected in the paper as divergence measurement indicators to measure and analyze regional divergence of three domestic source factors including domestic sewage, COD and  $\text{NH}_3\text{-N}$  during time series research (Table 1, Fig. 2).

TABLE I WEIGHTED COEFFICIENT AND THEIL INDEX OF DOMESTIC SOURCE POLLUTANT FROM 1998-2008

Indicator	Domestic sewage		COD		$\text{NH}_3\text{-N}$	
	$V_w$	$L$	$V_w$	$L$	$V_w$	$L$
Year						
1998	0.723	0.083	0.383	0.031	/	/
1999	0.672	0.074	0.388	0.032	/	/
2000	0.617	0.064	0.361	0.028	/	/
2001	0.684	0.076	0.351	0.027	0.390	0.033
2002	0.623	0.066	0.313	0.022	0.378	0.033
2003	0.633	0.068	0.325	0.030	0.377	0.031
2004	0.580	0.058	0.310	0.027	0.373	0.029
2005	0.556	0.060	0.312	0.027	0.371	0.035
2006	0.577	0.062	0.303	0.026	0.360	0.034
2007	0.563	0.060	0.383	0.031	0.364	0.035
2008	0.535	0.055	0.388	0.032	0.370	0.034

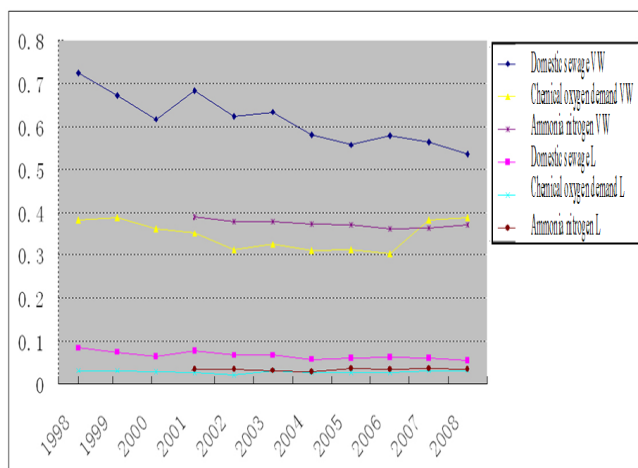


Figure 2 Variation graph of regional divergence measurement index of urban domestic source pollution in China from 1998-2008

Seen from Fig .2, the variation of regional divergence of urban domestic source pollution in China shows the following trends:

1) Weighted variation coefficient of domestic sewage shows overall downward trend. In 1998-2000, regional divergence of urban domestic sewage in China became smaller; in 2000-2001, the expansion showed again; in 2001-2008, a progressively smaller trend was basically emerged. Theil coefficient of domestic sewage has the same time evolution characteristics, but the variation is less.

2) Weighted variation coefficient of COD presents the "keystone" trend. In 1998-2002, regional divergence became smaller; in 2002 – 2006, it was relatively stable; in 2006-2008, a progressively larger trend showed. For COD, its Theil coefficient shares basically the same variation trend with its weighted variation coefficient.

3) Weighted variation coefficient of  $\text{NH}_3\text{-N}$  is relatively stable. It slowly decreased in 2001-2006, while it showed a slight increase in 2006-2008. For  $\text{NH}_3\text{-N}$ , its Theil coefficient shares basically the same variation trend with its weighted variation coefficient.

*B.R/S analysis of divergence measurement index* to further discuss the characteristics of regional divergence temporal evolution of urban domestic source pollution in China, an R/S analysis method is used in this section, to analyze development trend of domestic source pollution which is extremely complex and highly uncertain.

In the event of R/S analysis of divergence measurement index for domestic sewage and COD, taking into account the changes in social development in China, it is the year 2002 as a cut-off point (In 2002, it is China's accession to WTO and the implementation of national strategies including key shift of regional development strategies like Western Development that inevitably bring about certain impact on our economic development pattern and people living); due to the shortage of statistical data about  $\text{NH}_3\text{-N}$ , it is the year 2006 as a cut-off point (*National Economic and Social Development in Eleventh Five-Year Plan and 2020 Vision Outline* promulgated in 2006 proposed that the shift from regional non-balanced development strategy to regional

coordinated development strategy will also have a profound impact on social and economic development <sup>[19])</sup>

### 1) Variation characteristics of value H

In R/S analysis of regional divergence measurement index for domestic source factors (Table 2 - 4), the results show that all H values are downward, specifically as follows:

a) Domestic sewage in time series research and the value of  $H_{\text{weighted}}$  and  $H_{\text{Theil}}$  for  $\text{NH}_3\text{-N}$  between (0.5, 1). From the values H for weighted variation coefficient and Theil coefficient, weighted variation is positively correlated as the time goes, and the self-similarity for value H shows continuous decrease of regional divergence of domestic source pollution in China, but the change of value H from big to small indicates the sustainability of declining trend becomes weaker.

b) In 1998-2008, domestic COD shows different development trend with the other two factors. From the change of value H of weighted variation, it shows an overall trend from big to small over time. In various decomposition in 1998-2008,  $H_{\text{weighted}}$  (1998-2006) is between (0.5, 1), consistent with development trend of the other two factors, namely, regional divergence of domestic COD is gradually reduced, but its trend is becoming weaker. In the event of  $H_{\text{weighted}}$  between (0, 0.5) after 2006, weighted variation is negatively correlated significantly over time, so it is the year 2006 that becomes a turning point in two different development trends, namely, regional divergence for COD before 2006 is gradually reduced, but it changes in 2006, showing a slowly increasing trend. Theil coefficient basically shares the same variation trend with weighted coefficient.

### 2) Characteristics of fractal dimension D variation

The size of fractal dimension D reflects the irregular and chaotic extent of fractional Brownian motion in time series, and when D is 1.5, it indicates that it is in random Brownian motion in time series change, and out of such 1.5, time-series change will gradually be stabilized. Based on spatial divergence temporal evolution of urban domestic source pollution in China, that fractal dimension D change is conducive to further analyze the regularity of its evolution will be discussed in this section.

From value D respectively for domestic sewage and  $\text{NH}_3\text{-N}$ , the value is between [1, 1.5], indicating that a stronger regularity emerges in index changes of spatial divergence of domestic source pollution in China, and positive correlation from strong gradually to weak is shown overall, namely, as time series increases, the regularity where regional divergence of domestic source pollution becomes reduced continuously becomes unstable.

From value D for COD, the value it approaching 1.5 before 2006, but it is away from 1.5 after 2006, showing the evolution of COD index coefficients is obviously staged, with positive correlation gradually weaker before 2006 and the same evolution between domestic sewage and  $\text{NH}_3\text{-N}$ . And negative correlation is gradually stronger after 2006, namely it will become stable as time series increases.

TABLE II R/S ANALYSIS RESULTS OF DOMESTIC WASTEWATER DIVERGENCE MEASUREMENT

Period	Time lag	H <sub>weighted</sub>	D <sub>weighted</sub>	R <sup>2</sup> <sub>weighted</sub>	F <sub>weighted</sub>	H <sub>Theil</sub>	D <sub>Theil</sub>	R <sup>2</sup> <sub>Theil</sub>	F <sub>Theil</sub>
1998-2002	4	0.697	1.303	0.987	75.812	0.697	1.303	0.999	3156.153
1998-2003	5	0.667	1.333	0.991	208.401	0.608	1.392	0.977	86.591
1998-2004	6	0.768	1.232	0.968	89.850	0.750	1.25	0.935	43.458
1998-2005	7	0.633	1.367	0.878	28.878	0.624	1.376	0.861	24.725
1998-2006	8	0.567	1.433	0.856	29.827	0.563	1.437	0.846	27.561
1998-2007	9	0.531	1.469	0.858	36.189	0.546	1.454	0.866	38.791
1998-2008	10	0.512	1.488	0.869	46.506	0.536	1.464	0.884	53.243

Note: H is Hurst exponent, D is fractal dimension, R<sup>2</sup> is multiple correlation coefficient, and F is F test value, and the following is similar.

TABLE III R/S ANALYSIS RESULTS OF COD DIVERGENCE MEASUREMENT

Period	Time lag	H <sub>weighted</sub>	D <sub>weighted</sub>	R <sup>2</sup> <sub>weighted</sub>	F <sub>weighted</sub>	H <sub>Theil</sub>	D <sub>Theil</sub>	R <sup>2</sup> <sub>Theil</sub>	F <sub>Theil</sub>
1998-2002	4	0.792	1.208	0.997	335.888	0.793	1.207	0.998	404.007
1998-2003	5	0.620	1.38	0.921	23.393	0.625	1.375	0.927	25.299
1998-2004	6	0.583	1.417	0.937	44.696	0.596	1.404	0.945	51.452
1998-2005	7	0.557	1.443	0.947	71.552	0.519	1.481	0.906	38.725
1998-2006	8	0.492	1.508	0.906	48.228	0.461	1.539	0.876	35.163
1998-2007	9	0.457	1.543	0.896	51.784	0.431	1.569	0.873	41.210
1998-2008	10	0.440	1.56	0.903	65.070	0.420	1.58	0.888	55.413

TABLE VI R/S ANALYSIS RESULTS OF NH<sub>3</sub>-N DIVERGENCE MEASUREMENT

Period	Time lag	H <sub>weighted</sub>	D <sub>weighted</sub>	R <sup>2</sup> <sub>weighted</sub>	F <sub>weighted</sub>	H <sub>Theil</sub>	D <sub>Theil</sub>	R <sup>2</sup> <sub>Theil</sub>	F <sub>Theil</sub>
2001-2006	5	0.712	1.288	0.970	65.565	0.717	1.283	0.998	1730.686
2001-2007	6	0.673	1.327	0.804	12.296	0.637	1.363	0.974	113.131
2001-2008	7	0.685	1.315	0.859	24.349	0.634	1.366	0.981	214.498

## VI. CONCLUSION

Fractal Theory is applied to further explore the rules of time evolution of urban domestic source pollution in China, and the results from R/S analysis on various indicators show that provincial divergence of urban domestic source pollution in China complies with Hurst law, with time self-similarity; regional divergence for domestic sewage and NH<sub>3</sub>-N discharge in research time series maintains a gradually smaller trend, but its sustainability is much weaker; regional divergence of COD emissions shows a gradually smaller trend in 1998-2005, but is has been reversed after 2006 and shows a trend to further expand in the future.

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