Completeness of (μ,ν)-Resolution Principle of Intuitionistic Operator Fuzzy Logic

Hongliang Zheng[1] **Benqiang Xu**[1] **Li Zou**[1,2]

[1]School of Computer and Information Technology, Liaoning Normal University, Dalian 116029, P.R.China
[2] Intelligent Control Development Center, Southwest Jiaotong University, Chengdu 610031, P.R.China

Abstract

In this paper, a primary interpretation for intuitionistic operator fuzzy logic is presented. The concepts of (μ,ν)-complementary literal and (μ,ν)-similar literal about complex literals are proposed. Then the properties of (μ,ν)-false and the (μ,ν)-resolution method of the complex literals are discussed. Based on the concepts of (μ,ν)-weak implication and (μ,ν)-strong implication, the completeness of (μ,ν)-resolution of intuitionistic operator fuzzy logic hold. An example is given to show that the proposed (μ,ν)-resolution method is a layered resolution method.

Keywords: (μ,ν)-resolution, the completeness of (μ,ν)-resolution, intuitionistic operator fuzzy logic.

1. Introduction

Since the fuzzy logic was proposed by Zadeh [1], many researches have studied reasoning about fuzzy logic. The resolution principle introduced by Robinson [5] is a fundamental technique for mechanical reasoning or question-answering system [6]. The resolution principle of fuzzy logic was discussed by Liu on the lattice [7,8]. From the view of intuitionistic fuzzy logic which presented by K.Atanassov [2,3], the truth value of fuzzy proposition can be described by two real number (μ,ν) on the closed interval [0, 1], which represents its truth degree and its false degree. Some resolution methods of many-valued logic were discussed using operator[11-13]. In paper [9] the intuitionistic fuzzy degree was expressed by operator which lies on the left of fuzzy proposition atom and intuitionistic operator fuzzy logic (*IOFL*) was proposed on the operator lattice $L=\{(\mu,\nu)|\mu,\nu\in[0,1],\ \mu+\nu\leq 1\}$. The (μ,ν)-resolution principle was discussed, but it wasn't completeness. We will discuss the completeness of (μ,ν)-resolution principle in this paper.

The paper is organized as follows. In Section 2, we give primary interpretation for intuitionistic operator fuzzy logic, the (μ,ν)-weak implication and (μ,ν)-strong implication are proposed. In Section 3, the concepts of (μ,ν)-complementary literal and (μ,ν)-similar literal about complex literals are presented and the completeness of (μ,ν)-resolution of intuitionistic operator fuzzy logic hold. An example to apply the resolution method shows in Section 4. Finally, some conclusions are given in Section 5.

2. (μ,ν)-weak implication and (μ,ν)-strong implication

Definition 2.1 Assume $(\mu,\nu)P$ is an atom of *IOFL*,

$$\mathrm{V}_{\mathrm{I}}(\mu,\nu)P=\begin{cases}\mu, & when\ P\ is\ appointed\ T\ by\ I\\ \nu, & when\ P\ is\ appointed\ F\ by\ I\end{cases}$$

The interpretation of P in this definition is two kinds: true or false [7-9]. The world described in this system is : any proposition P is certain, crisply, true or false. Because of different understanding degree, different person gives different intuitionistic fuzzy proposition. This degree is represented by operator (μ,ν). We can interpret the operator (μ,ν): the certainty and uncertainty of P, or the obverse demonstration and inverse demonstration of P and so on.

Definition 2.2 Let $L=\{(\mu,\nu)|\mu,\nu\in[0,1],\ \mu+\nu\leqslant 1\}$, the operation "∘" of the operator on $(L, *, \oplus, ')$ is defined as follows: for any (μ_1,ν_1) , $(\mu_2,\nu_2)\in L$,

$$(\mu_1,\nu_1)\circ(\mu_1,\nu_2)=((\mu_1+\mu_2)/2,(\nu_1+\nu_2)/2)$$

where

$$(\mu_1,\nu_1)*(\mu_1,\nu_2)=(\min(\mu_1,\mu_2),\max(\nu_1,\nu_2))$$
$$(\mu_1,\nu_1)\oplus(\mu_1,\nu_2)=(\max(\mu_1,\mu_2),\min(\nu_1,\nu_2))$$
$$(\mu_1,\nu_1)'=(\nu_1,\mu_1)$$

Hence L is an operator lattice.

The operator "∘" can be regarded the evidence of existing P. The different operation can get different interpretation. We can define and take the operator according to our need.

Definition 2.3 Let G is a formula of *IOFL*, $\mu,\nu\in L$, assume $V_I(G)=(\mu_G,\nu_G)$,the formula G is called (μ,ν)-true if $\mu_G\geqslant\mu$ and $\nu_G\leqslant\nu$ for an arbitrary interpretation I. Whereas, the formula G is called (μ,ν)-false if $\mu_G\leq\mu$ and $\nu_G\geq\nu$.

Definition 2.4 Let G an H are formulas of *IOFL*, $\mu,\nu\in$L, G is called (μ,ν)-weak implicate H (or H is a weak-logical result of G) if $(G\rightarrow H)$ is (μ,ν)-true, denoted by $G\Rightarrow H$.

Theorem 2.1 Assume $V_I(G)=(\mu_G, \nu_G)$ and $V_I(H)=(\mu_H, \nu_H)$, $(G\rightarrow H)$ is (μ,ν)-true iff if $\mu_G>\nu$ and $\nu_G<\mu$ then $\mu_H\geq\mu$ and $\nu_H\leq\nu$ for arbitrary interpretation I.

Proof （<=）

$\mu_{(G\rightarrow H)}=\mu_{(\sim G\vee H)}=\max\{\mu_{\sim G},\mu_H\}=\max\{\nu_G,\mu_H\}=\mu_H\geq\mu$ and $\nu_{(G\rightarrow H)}=\nu_{(\sim G\vee H)}=\min\{\nu_{\sim G}, \nu_H\}= \nu_H\leq\nu$, hence $(G\rightarrow H)$ is (μ,ν)-true.

(=>) For an arbitrary interpretation I, $\mu_{(G\rightarrow H)}\geq\mu$ and $\nu_{(G\rightarrow H)}\leq\nu$, if $\mu_G>\nu$ and $\nu_G<\mu$, then

$\mu_{(G\rightarrow H)}=\mu_{(\sim G\vee H)}=\max\{\mu_{\sim G,}\mu_H\}=\max\{\nu_G, \mu_H\}\geq\mu$ since $\nu_G<\mu$, we have $\mu_H\geq\mu$,

$\nu_{(G\rightarrow H)}=\nu_{(\sim G\vee H)}=\min\{\nu_{\sim G,} \nu_H\}= \min\{\mu_{G,}\nu_H\}\leq\nu$ and $\mu_G>\nu$, therefore $\nu_H\leq\nu$.

Definition 2.5 Assume S is a set of clause, $S_{PR}^{(\mu,\nu)}$ is called (μ,ν)-primary reduced set of S, $(\mu,\nu)\in L$,, $S_{PR}^{(\mu,\nu)}$ is obtained by the method as follows: for arbitrary $(\mu^*,\nu^*)P\in S$,

(1)while $\mu\geq0.5,\nu\leq0.5$, if $\nu\leq\mu^*\leq\mu$ or $\nu\leq\nu^*\leq\mu$, delete $(\mu^*,\nu^*)P$ from S.

(2)while $\mu<0.5$, $\nu>0.5$, if $\mu\leq\mu^*\leq\nu$ or $\mu\leq\nu^*\leq\nu$, delete $(\mu^*, \nu^*)P$ from S.

Theorem 2.2 Let C_1 and C_2 are two clauses , $\mu\geq0.5$, $\nu\leq0.5$, $C_{1PR}^{(\mu,\nu)}$ and $C_{2PR}^{(\mu,\nu)}$ are (μ,ν)-primary reduced clause of C_1， C_2, and then

$(C_{1PR}^{(\mu,\nu)}\wedge C_{2PR}^{(\mu,\nu)})=>R_{(\mu,\nu)}(C_{1PR}^{(\mu,\nu)}, C_{2PR}^{(\mu,\nu)})$

Proof. We can obtain it from definition 2.1, definition 2.2 and theorem 2.1.(omitted)

Theorem 2.3 Let C_1 and C_2 are two clauses , assume $(\mu, \nu)=(0.5,0.5)$, and then $C_1\wedge C_2\Rightarrow R_{(\mu,\nu)}(C_1,C_2)$.

Proof .While $(\mu, \nu) = (0.5, 0.5)$, there is $C_1=C_{1PR}^{(\mu,\nu)}$ and $C_2=C_{2PR}^{(\mu,\nu)}$;

From theorem 2.1 we can get $C_1\wedge C_2\Rightarrow R_{(\mu,\nu)}(C_1,C_2)$.

Definition 2.6 Assume G and H are two formulas of IOFL, $(\mu, \nu)\in L$, for arbitrary interpretation I, if $\mu_G\geq\mu$ and $\nu_G\leq\nu$ there must be $\mu_H\geq\mu$ and $\nu_H\leq\nu$, G is called (μ,ν)-strong implication H or H is a logical result of G, denoted $G\equiv>H$.

The following propositions are obviously.

Proposition 2.1 When $\mu>0.5$ and $\nu<0.5$, if $G=>H$ then $G\equiv>H$; When $\mu=0.5$ and $\nu=0.5$, if $G\equiv>H$ then $G=>H$.

Proposition 2.2 Let G is a formula,

(1)When $\mu\leq0.5$ and $\nu\geq0.5$, $A=>A$;

(2)$A\equiv>A$.

Proposition 2.3 Let A, B, C are the formulas of *IOFL* respectively,

(1)When $\mu>0.5$ and $\nu<0.5$, if $A=>B$, $B=>C$ then $A=>C$;

(2)If $A\equiv>B$ and $B\equiv>C$ then $A\equiv> C$.

Proposition 2.4 Let A, B, C are formulas of *IOFL*

(1) If $A=>B$ and $A=>C$ then $A=>(B\wedge C)$;

(2) If $A\equiv>B$ and $A\equiv>C$ then $A\equiv>(B\wedge C)$.

Theorem 2.4 Let C_1 and C_2 are two clauses, $\mu\geq0.5$ and $\nu\leq0.5$, and then

$C_1\wedge C_2\equiv>R_{(\mu,\nu)}(C_1,C_2)$.

Corollary Let C_1 and C_2 are two clauses, $\mu\geq0.5$ and $\nu\leq0.5$, for arbitrary interpretation I, if

$\mu_{(C1\wedge C2)}>\mu$, $\nu_{(C1\wedge C2)}<\nu$

Then $\mu_{R_{(\mu,\nu)}(C_1,C_2)}>\mu$, $\nu_{R_{(\mu,\nu)}(C_1,C_2)}<\nu$

3. Completeness of (μ,ν)-resolution principle

Definition 3.1 Let $(\mu_{11},\nu_{11})\ldots(\mu_{1n},\nu_{1n})P$ and $(\mu_{21},\nu_{21})\ldots(\mu_{2n},\nu_{2n})P$ are two literals, $(\mu,\nu)\in L$. Assume $V_I(\mu_{11}, \nu_{11})\ldots(\mu_{1n}, \nu_{1n})P=(\mu_1, \nu_1)$, $V_I(\mu_{21}, \nu_{21})\ldots(\mu_{2n}, \nu_{2n})P=(\mu_2, \nu_2)$,

When P is appointed T, $\mu_1>\max(\mu,\nu)$, $\nu_1<\min(\mu,\nu)$ and $\mu_2<\min(\mu,\nu)$, $\nu_2>\max(\mu,\nu)$;

When P is appointed F, $\mu_1<\min(\mu,\nu)$, $\nu_1>\max(\mu,\nu)$ and $\mu_2>\max(\mu,\nu)$, $\nu_2<\min(\mu, \nu)$.

These two literals are called (μ, ν)-complementary literals each other.

Definition 3.2 Let $(\mu_{11}, \nu_{11})\ldots(\mu_{1n},\nu_{1n})P$ and $(\mu_{21},\nu_{21})\ldots(\mu_{2n},\nu_{2n})P$ are two lilterals, $(\mu,\nu)\in L$.

Assume $V_I(\mu_{11}, \nu_{11})\ldots(\mu_{1n}, \nu_{1n})P=(\mu_1, \nu_1)$, $V_I(\mu_{21}, \nu_{21})\ldots(\mu_{2n}, \nu_{2n})P=(\mu_2, \nu_2)$

While P is appointed T, $\mu_1>\max(\mu,\nu)$, $\nu_1<\min(\mu,\nu)$ and $\mu_2>\max(\mu,\nu)$, $\nu_2<\min(\mu, \nu)$

While P is appointed F, $\mu_1<\min(\mu,\nu)$, $\nu_1>\max(\mu,\nu)$ and $\mu_2<\min(\mu,\nu)$, $\nu_2>\max(\mu, \nu)$

These two literals are called (μ, ν)-similar literals.

Definition 3.3 For $(\mu,\nu)\in L$, (μ^*, ν^*)P is an arbitrary word of a clause which satisfied with

$\nu\leq\mu^*\leq\mu$ or $\nu\leq\nu^*\leq\mu$.

This clause is called (μ,ν)-null clause, denoted by (μ,ν)-□.

Theorem 3.1 Let $\mu\geq0.5$ and $\nu\leq0.5$ if a deduction that (μ,ν)-□ can be deduced from S with (μ,ν)-resolution method exists, then S is (μ,ν)-false.

Proof. If otherwise, there will be an interpretation I, cause $\mu_S>\mu$ and $\nu_S<\nu$,
from theorem 2.4 there is $C_1\wedge C_2\equiv> R_{(\mu,\nu)}(C_1,C_2)$

From proposition 2.3 and the corollary of theorem 2.4 there is

$$\mu_{(\mu,\nu)\text{-}\square}>\mu,\ \nu_{(\mu,\nu)\text{-}\square}<\nu,$$

It is a contradiction for definition 3.1.

Theorem 3.2[9] For $(\mu, \nu)\in L$, if the clause set S is (μ, ν)-false, there must be a (μ,ν)- resolution deduction which can deduce (μ,ν)-□ from S.

From theorem 3.1 and theorem 3.2 we hold as follow:

Theorem 3.3 (Completeness Theorem) Assume $\mu\geq0.5$ and $\nu\leq0.5$, S is a clause set, then S is (μ,ν)-false iff there is a (μ,ν)-resolution deduction which can deduce (μ,ν)-□ from S.

From above, in order to keep the intuitionistic property of two clauses, $(\mu, v)=(0.5,0.5)$ should be taken in (μ, v)-weak implication; While $\mu \geq 0.5$ and $v \leq 0.5$ should be taken in (μ,v)-strong implication, that can make the (μ,v)-resolution formula of two clause is logical result of their parent clause.

While $\mu+v=1$, it can be obtained λ-weak implication and λ-strong implication of operator fuzzy logic which defined in paper [4].

Theorem 3.4 If the (μ,v)-resolution deduction which can deduce (μ,v)-□ beginning with the clause set S, then S is both (μ, v)-false and (v,μ)-false.

Proof. If $(\mu,v)=(0.5, 0.5)$, it can prove easily.

If $\mu \geq 0.5$ and $v \leq 0.5$, the null clause can be obtained by (μ,v)-resolution, then S is (μ,v)-false. At last we can obtained the (μ,v)-resolution formula of two (μ,v)-complementary literals.

Assume (μ,v)-complementary literals are $(\mu_1,v_1)P_1$ and $(\mu_2,v_2)P_2$,$\mu_1 > \mu$ and $v_1 < v$, $\mu_2 < v$ and $v_2 > \mu$, therefore $S \equiv > (\mu_1, v_1)P_1^{\sigma}$, $S \equiv > (\mu_2,v_2)P_2^{\sigma}$.

Hence $S \equiv > (\mu_1, v_1) P_1^{\sigma} \wedge (\mu_2, v_2) P_2^{\sigma}$, in which $P_1^{\sigma} = P_2^{\sigma}$.

Hence $\mu_S < v$ and $v_S > \mu$. Otherwise, if $\mu_S > v$ then $(\mu_1, v_1) P_1^{\sigma} \wedge (\mu_2, v_2) P_2^{\sigma} > v$,but this is impossible .

From above S is (v,μ)-false.

4. Application

For instance a production rule

if A then B (μ^*, v^*)

is described by formula in *IOFL* as follows:

$$(\mu^*, v^*)(A \rightarrow B) \text{ or } (\mu^*, v^*)(\sim A \vee B)$$

in which (μ^*, v^*) is the intuitionistic fuzzy degree of this rule.

A group of production rule A and a group of fact B are known:

$$A\begin{cases} If \quad E_1 \quad then \quad E_2 \quad (0.7,0.2) \\ If \quad E_2 \quad then \quad E_2 \quad (0.9,0.1) \\ If \quad E_4 \quad then \quad E_5 \quad (0.6,0.2) \end{cases}$$

$$B\begin{cases} (1,0)E_4 \\ (0.8,0.1)E_5 \end{cases}$$

We can prove that（0.8, 0.1）H will be deduced from A and B.

Use (μ,v)-resolution method, we can prove $A \wedge B \rightarrow (0.8,0.1)H$ is (μ,v)-false.

$A \wedge B \wedge \sim (0.8,0.1)H$ can decompose the set of clause:

(1)$(0.7, 0.2)((0, 1)E_1 \vee (1,0)E_2)$
(2)$(0.9, 0.1)((0,)E_2 \vee (1,0)H)$
(3)$(0.6, 0.2)((0, 1)E_4 \vee (0,1)E_5 \vee (1,0)E_1)$
(4)$(1, 0)E_4$
(5)$(0.8, 0.1)E_5$
(6)$(0.1, 0.8)H$

Take $(\mu,v)=(0.6,0.2)$, resolute with (0.6,0.2), then

(7)$(0.6, 0.2)((1, 0) E_1)$ from (3), (4), (5)
(8)$(0.7, 0.2)((1, 0) E_2)$ from (1), (7)
(9)$(0.9, 0.1)((1, 0) H)$ from (2), (8)
(10)□ from (6), (9)

Therefore, because the conclusion $(0.8, 0.1)H$ can be inferred from A and B, this theorem is (0.2,0.6)-true. From the last it can infer □ but not (μ,v)-□, this theorem is also (0.6,0.2)-true. Because the intuitionistic fuzzy degree of the conclusion H which inferred from A and B is (0.8, 0.1), this inference isn't credible completely. It is likely 0.6-true, but likely 0.2-false. The intuitionisdtic fuzzy degree in this inference process is taken (0.6, 0.2).

5. Conclusions

In this paper, following the operator intuitionistic fuzzy logic [6] and its (μ,v)-resolution principle we give the concepts of (μ,v)-complementary literal and (μ,v)-similar literal about complex literals and their properties. Based on (μ,v)-weak implication and (μ,v)-strong implication, the completeness of (μ,v)-resolution of intuitionistic operator fuzzy logic hold. How to find an intuitionistic fuzzy degree using more simple and convenient algorithm is an opening problem. It provides a new kind of the uncertain reasoning. The further work is to how to use it in the expert system or decision –making.

References

[1] L.A. Zadeh, The Concept of Linguistic Variable and its Application to Approximate Reasoning (I), *Information Sciences*, 8:199-249, 1975.

[2] K.Atanassov. Intuitionistic Fuzzy Set. *Fuzzy sets and System*. 20:87-96, 1986.

[3] K.Atanassov. Elements of intuitionistic fuzzy logic. Part I. *Fuzzy Set and Systems*, 95:39-52, 1998.

[4] T.Y. Lin, Neighborhood Systems: A Qualitative Theory for Fuzzy and Rough Sets, *Advances in Machine Intelligence and Soft Computing*, Volume Ⅳ, Ed.Paul Wang, pp.132-155, 1997.

[5] T.Y.Lin, and S. Tsumoto, Qualitative Fuzzy Sets Revisited: Granulation on the Space of Membership Functions, *The 19th International Meeting of the North American Fuzzy Information Processing Society*, pp.331-337, 2000.

[6] J.P.Robinson.，A Machine-oriented logic based on the resolution principle, *J.ACM,* 12:23-41, 1965.

[7] X.H. Liu, Automatic Reasoning Based on Resolution Method, *Science Publishing House*, Beijing, China, pp.347~360, 1994.

[8] X. Liu, L. Zou, Qualitative Fuzzy Logic System and Its Resolution Method, *Proceedings of International Conference on Machine Learning and Cybernetics*, pp. 2000-2004, 2004.

[9] T. Y. Chen, L. Zou, Intuitionistic Fuzzy Logic on Operator Lattice, *BUSEFAL*, 69:107-110, 1997.

[10] L. Zou, P. Das, Y. Xu and D. Meng, Soft-resolutiion Method of Linguistic Hedges Lattice-valued First-order Logic. *Proceedings of 8th International Joint Conference on Information Science*, pp.372-376, 2005.

[11] L. Zou, X. Liu, Y. Xu, Resolution Method of Linguistic Truth-valued Propositional Logic, *International Conference on Neural Networks and Brain*, pp.1996-2000, 2005.

[12] L. Zou, J. Ma, Y. Xu, A Framework of Linguistic Truth-valued Propositional Logic Based on Lattice Implication Algebra, *Proceedings of 2006 IEEE International Conference on Granular Computing*, pp. 574-577, 2006.

[13] L. Zou, B.H. Li, W. Wang and Y. Xu, Weighting Qualitative Fuzzy First-Order Logic and its Resolution Method, *Applied Artificial intelligenc*, pp. 103-111, 2006.