

# Research advances of heterogeneity representation methods for rocks

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## Abstract

Heterogeneity of rocks has a great influence on macroscopic mechanical behaviors. So in rock mechanics research, the heterogeneity should be described properly. This paper presents Weibull distribution, which can describe the microstructure parameters heterogeneity, and sums up heterogeneous parameter's influences on the failure process, macro-strength and rock burst. Then random parameters assignment method of rock mineral cell unit is presented, and the influences of heterogeneity on macroscopic mechanical behaviors are showed. Combing digital image processing with numerical simulation software, the new method of building actual microstructures model of rocks is presented.

**Keywords:** heterogeneity, Weibull distribution, random parameters assignment method, digital image processing

## 1. Introduction

Rocks are heterogeneous materials, consisting of mineral particles of various sizes, shapes and properties, and adhering together by cementing materials [1-2]. Its heterogeneous micro-characteristics affect both the physically and mechanically macroscopic properties of rocks [3-5]. Thus studying the effect of rock heterogeneity on mechanically macroscopic behaviors contributes to the further research of rock burst tendency, failure laws, and other hot issues. It has great academic value and engineering significance [6-7]. This

paper summarizes the present commonly used rock heterogeneous representation methods, such as Weibull distribution, random parameters assignment method of rock mineral cell unit and rock microscopic structure representation method based on digital image processing, etc. Research results about different heterogeneous influences on rock failure patterns, macroscopic strength characteristics, etc. are also summarized.

## 2. Heterogeneity of micro-parameters

### 2.1. Weibull distribution

Weibull distribution is a widely used method for micro-parameters setting at present stage of heterogeneity research [8-10], and its probability density function  $f(x)$  is shown in Equation (1).

$$f(x) = \frac{m}{x_0} \left( \frac{x}{x_0} \right)^{m-1} e^{-(x/x_0)^m} \quad (1)$$

where  $x_0$  is expectation value, and  $m$  is heterogeneous parameter.

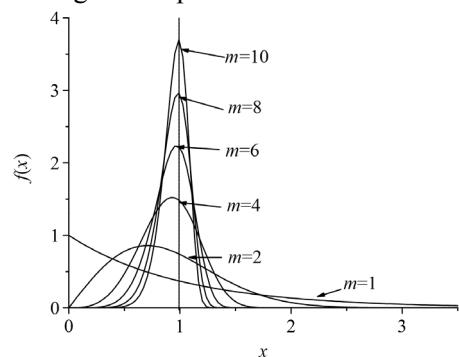


Fig. 1. Curves of probability density of Weibull distribution

The probability density function curves of different values of  $m$  are shown in Fig. 1. These curves show that the greater heterogeneous parameter  $m$  is, the lesser discrete of micro-parameters and the more uniform of whole properties.

## 2.2. Impact of heterogeneity on failure modes

Rock heterogeneity is one of essential factors that cause the morphological diversity of rock damages. Tang [11-12], Liang [13-14], Wang [15], etc. researched the failure process of heterogeneous rocks systematically by Weibull distribution.

When rock heterogeneous parameter  $m$  is low, unit content of low-intensity microstructure is more, rock deformation of linear features is weak, while nonlinear characteristics are strong; under lower external load, structural unit of rocks starts to break, acoustic emission events occur early and continuously, failed elements before the main damage occur are in large numbers, and the distribution is scattered, as shown in Fig. 2(a).

When heterogeneous parameter  $m$  is high, micro-structural unit's strength is higher and its distribution is uniform, linear features of rock deformation are enhanced; when external load is high, structural units begin to destroy, and because of little differences between the intensity of units, structural destruction will have a chain reaction that rocks become break in a very short period, damage is focused in one region, and the number of damage zones is small, as shown in Fig. 2(b).

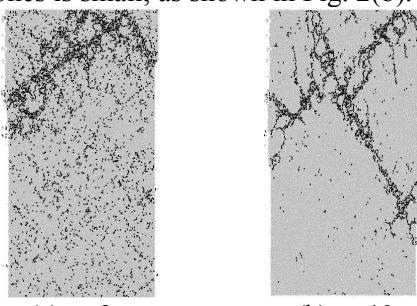
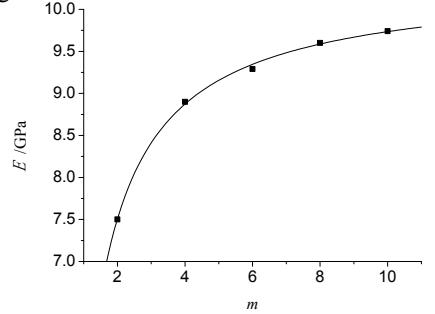


Fig. 2. Failure modes with heterogeneous parameters

## 2.3. Effect of heterogeneity on macro-strength

When heterogeneous parameter  $m$  is small, rock compressive strength and elasticity modulus are low; with the increase of heterogeneous parameters, elasticity modulus and compressive strength increase, but the increase gradient decreases, eventually comes close to a stable value [16], as shown in Fig. 3.



(a) elasticity modulus

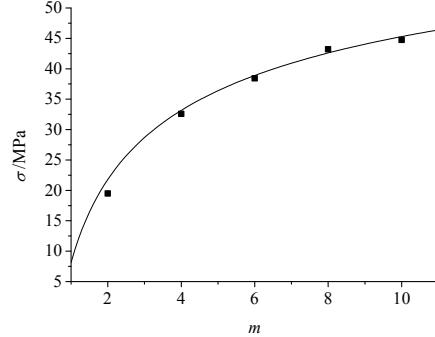


Fig. 3. Curves of heterogeneity and macro-strength

## 2.4. Impact of heterogeneity on burst tendency

Feng studied the relationship between rock burst tendency indicator  $E/\lambda$  and heterogeneous parameter  $m$ , and found that they had a negative exponential relationship [17].

Elasticity modulus and compressive strength are two parameters commonly used in rock mechanics, when considering non-uniformity of one parameter, its effect on burst tendency is different [16]. With the increase of elasticity modulus or

bonding strength, rock burst energy index is increasing rapidly; bonding strength has larger effect on rock burst tendency than elasticity modulus.

### 3. Heterogeneity of spatial anisotropy

From the view of mechanics, rock heterogeneity is not only reflected in microscopic parameters of the non-uniform, but also showed in spatial variability of mechanical parameters, this variation is random and a structural duality.

#### 3.1. Random parameters assignment method of rock mineral cell unit

According to the random of types and structures of rocks' composition, Luo put forward a new parameter assignment method to describe the heterogeneity of rocks—random parameter assignment method of rock mineral cell unit [18-19]. The assignment process includes: mesh generation, definition of mineral cell element categories, judge of cell element categories and parameters assignment of rock mineral cell unit. A three minerals model obtained by random parameters assignment method of rock mineral cell unit is shown in Fig. 4.

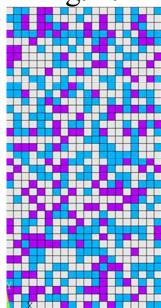


Fig. 4. Model of random parameters assignment of rock mineral cell unit<sup>[18]</sup>

#### 3.2. The influence of heterogeneous parameters on macro mechanical behaviors

Heterogeneity of rocks was defined by the parameter variation coefficient by Luo [19], the formula is:

$$C_v = \frac{\sigma(E_i)}{E(E_i)} \quad (2)$$

where  $\sigma(E_i)$  and  $E(E_i)$  are standard

deviation and expectations of mineral cell element elasticity modulus  $E_i$ .

Analyzing and comparing the effect of heterogeneity based on the new method of heterogeneity expression on macroscopic mechanical behavior under the two new methods, and discovering that:

(1) In the Weibull distribution, the compressive strength of rocks is nonlinear with the shape parameter  $m$ , but shows a simple linear relationship with the heterogeneity degree  $C_v$  (variation coefficient), which means  $C_v$  is more simple and convenient than  $m$  in describing the macroscopic mechanical properties of rocks affected by heterogeneous parameters.

(2) With the increase of heterogeneous parameters, the compressive strength of rocks decreases; for the two kinds of heterogeneous assignment method, macro mechanical properties of rocks affected by the laws of heterogeneous parameters are consistent, as shown in Fig. 5.

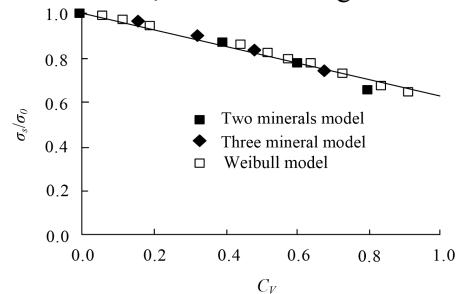


Fig. 5. Realtion of non-dimensional strength and heterogeneity degrees<sup>[19]</sup>

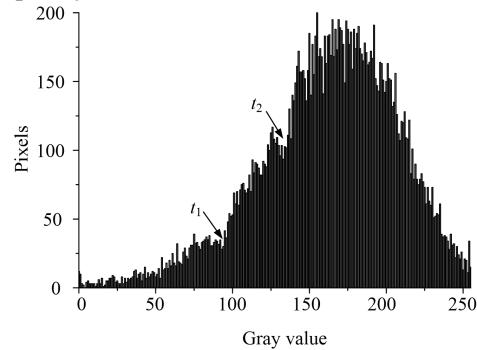
#### 4. Microscopic structure representation of rocks based on digital image processing

Rock heterogeneous models, which are created by Weibull distribution and random parameters assignment method, have a big difference with the real microscopic structure of rocks. Advanced digital image processing technology is introduced into rock mechanics research in order to establish models that can reflect the real microscopic structure of rocks.

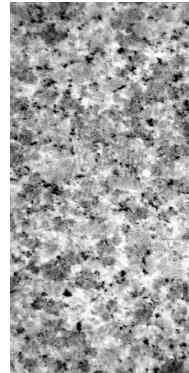
The digital image processing technology is a method, which can obtain real microscopic structures of rock surface effectively, and its basic theory is based on the different colors or brightness of mineral grains on the rock surface, dividing and recombining images [20]. The key of rock digital image processing is dividing and recombining different mineral grains, and the common method is histogram threshold method. In digital images, colors of same mineral grains are similar, while colors of different mineral grains have obvious differences. In the histogram, similarities and differences are showed as peak and valley of curves. A gray level histogram is shown in Fig. 6(a), and its actual image was Fig. 6(b). In the histogram, gray value between 0 and  $t_1$  is mica, between  $t_1$  and  $t_2$  is quartz, and greater than  $t_2$  is feldspar. The representation image after image processing is shown in Fig. 6(c), which is consistent with the microscopic structure distribution of Fig. 6(b). When the rock microscopic structure representation image is obtained, we can import the information of pixel into numerical simulation software to rebuild the real rock structures.

Many scholars rebuilt simulation models of real rock microscopic structures by digital image processing technology [21-23]. According to rock surface's variations of brightness, Chen obtained granite microscopic structure representation by using digital image processing technology, and established a heterogeneous model to carry out both uniaxial compression and Brazilian splitting tests [24]. Yu proposed a method, obtaining granite microscopic structure according to three kinds of color varieties of rock image RGB, and this method decreases information loss than using brightness or color separately [25]. Image processing methods above are all based on the histogram threshold method

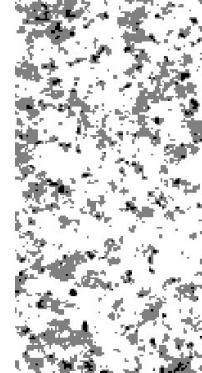
to divide and recombine images, and it will lead to microscopic structure information errors due to personal factors. In order to avoid personal factors, Zhu brought mean fuzzy clustering method into rock digital image processing [26]. Combining with the background of image filling technology, Feng achieved granite three-phase segmentation based on variational level setting method and this method can obtain rock microscopic structure and quantitative images of high quality [27].



(a) gray histogram



(b) granite image



(c) representation image  
Fig. 6. Gray level histogram and image of granite

## 5. Conclusions

This paper summarizes several commonly used methods of heterogeneous parameters representation, such as Weibull distribution, random parameters assignment method and heterogeneous model establishment method based on digital image processing method. The adoption of these methods makes a

significant contribution to rock mechanics research. Rocks contain not only heterogeneous mineral grains, but also cracks and joints, etc. Influences of these structures on rock heterogeneous description and mechanic behaviors are research priorities in the further work.

## 6. Acknowledgments

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## 7. References

- [1] Tang C A, “Rock failure process of numerical experiments”. Beijing: science press, 2003.
- [2] ZHANG C H, “Seepage-stress coupled model of heterogeneous and random fractured rockmass”, Journal of China Coal Society(in Chinese), pp1460-1464, 2009.
- [3] Tan Y L, Yin Y C, Huang D M, “Inhomogeneous micro-Structure influence on macro-crack of sandstone”, Journal of Testing and Evaluation, pp1024-1031, 2013.
- [4] Yun S C, Harrison J P, “A review of the state of the art in modeling progressive mechanical breakdown and associated fluid flow in intact heterogeneous rocks”, International Journal of Rock Mechanics and Mining Sciences,pp 1001-1022, 2006.
- [5] SU H, DANG C H, LI Y J, “Study of numerical simulation of acoustic emission in rock of inhomogeneity”, Rock and Soil Mechanics(in Chinese), pp 1886-1890, 2011.
- [6] LV Z X, FENG Z C, ZHAO Y S , “Influence of rock inhomogeneity on strength-size effect of rock materials”, Journal of China Coal Society (in Chinese), pp 917-920, 2007.
- [7] ZHAO T B, TAN Y L, ZHANG Z, “Geomechanical mechanism of rock burst in deep level of Datai Mine”, Journal of China Coal Society(in Chinese), pp 2039-2044, 2010.
- [8] GAO F, XIE H P, “Fractal statistics strength theory of fragile material”, Acta Mechanica Solida Sinica(in Chinese), pp 239-244, 1996.
- [9] Xie H P, Gao F, “The mechanics of cracks and a statistical strength theory for rocks”, International Journal of Rock Mechanics and Mining Sciences, pp 477-488, 2000.
- [10] Tang C A, Fu Y F, Kou S Q, et al, “Numerical simulation of loading inhomogeneous rocks”, International Journal of Rock Mechanics and Mining Sciences, pp 1001-1007, 1998.
- [11] TANG C A, LIIU H Y, QIN S Q, et al, “Influence of the heterogeneity on crack propagation modes in brittle rock”, Chinese Journal of Geophysics(in Chinese), pp 116-121, 2000.
- [12] Tang C A, Liu H, Lee P K, et al, “Numerical studies of the influence of microstructure on rock failure in uniaxial compression—part I : effect of heterogeneity”, International Journal of Rock Mechanics and Mining Sciences, pp 555-569, 2000.
- [13] LIANG Z Z, YANG T H, TANG C A, et al, “Three-dimensional damage soften model for failure process of heterogeneous rocks and associated numerical simulation”, Chinese Journal of Geotechnical Engineering(in Chinese), pp 1447-1452, 2005.
- [14] Liang Z Z, Tang C A, Li H X, et al, “Numerical simulation of the 3-D failure process in heterogeneous

- rocks”, International Journal of Rock Mechanics and Mining Sciences, pp323-328, 2004.
- [15] WANG S M, ZHU H H, FENG X T, et al, “Influence of heterogeneity on macroscopical crack form of the brittle rock,” Rock and Soil Mechanics(in Chinese), pp 224-227, 2006.
- [16] ZHAO T B, YIN Y C, TAN Y L, et al, “Bursting liability of coal research of heterogeneous coal based on particle flow microscopic test”, Journal of China Coal Society(in Chinese), pp 280-285, 2014.
- [17] FENG Z C, ZHAO Y S, “Correlativity of rock inhomogeneity and rock burst trend”, Chinese Journal of Rock Mechanics and Engineering(in Chinese), pp 1863-1865, 2003.
- [18] LUO R, ZENG Y W, “Research on random parameter assignment method of rock mineral cell unit,” Rock and Soil Mechanics(in Chinese), pp2221-2228, 2012.
- [19] LUO R, ZENG Y W, CAO Y, et al, “Research on influence of inhomogeneity degree on mechanical parameters of inhomogeneous rock”, Rock and Soil Mechanics(in Chinese), pp3788-3794, 2012.
- [20] REID T R, HARRISON J P, “ A semi-automated methodology for discontinuity trace detection in digital images of rock mass exposures”, International Journal of Rock Mechanics and Mining Science, pp 1073-1089, 2000.
- [21] XU W J, HU R L, YUE Z Q, et al, “Mesostructural character and numerical simulation of mechanical properties of soil-rock mixtures”, Chinese Journal of Rock Mechanics and Engineering(in Chinese), pp 300-311, 2007.
- [22] XU W J, HU R L, WANG Y P, “PFC2D model for mesostructure of inhomogeneous geomaterial based on digital image processing”, Journal of China Coal Society(in Chinese), pp 358-362, 2007.
- [23] CHEN S, YUE Z Q, THAM L G, “Digital image-based numerical modeling method for prediction of inhomogeneous rock failure”, International Journal of Rock Mechanics and Mining Science, pp 939-957, 2004.
- [24] CHEN S, YUE Z Q, TAN G H, “Digital image based numerical modeling method for heterogeneous geomaterials”, Chinese Journal of Geotechnical Engineering(in Chinese), pp956-964, 2005.
- [25] YU Q L, YANG T H, LIIU H L, et al, “Numerical simulation on granite failure mechanism at meso-level under moderate or low confining pressure ”, Journal of Northeastern University (Natural Science), pp 1026-1029, 2009.
- [26] ZHU Z Q, XIAO P W, SHENG Q, et al, “Numerical simulation of fracture propagation of heterogeneous rock material based on digital image processing” ,Rock and Soil Mechanics (in Chinese), pp 3780-3786, 2011.
- [27] FENG B C, CAO J F, GAO H X, et al, “Obtaining rock’s mesostructure based on variational level set method”, Rock and Soil Mechanics (in Chinese), pp3592-3597, 2012.