Linguistic Truth-Valued Lattice Value Propositional Logic System ℓP(X)

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Abstract

In the present paper, as continuous work about linguistics truth-valued LIA and its properties (CESA2006), the lattice value propositional logic system whose valuation field look as linguistic truth value LIA (briefly, L-LIA) is focused. Firstly, some properties about linguistic truth value LIA are discussed. On the other hand, some concepts about linguistic truth value lattice-valued propositional logic system ℓP(X) is established, whose truth value domain is a linguistic truth-valued lattice implication algebra, and the semantic problems of ℓP(X) are investigated.

Keywords: Linguistic truth-valued LIA, Valuation, Valid formula, (α,β) -valid.

1. Introduction

L. A. Zadeh introduced and developed the theory of approximate reasoning based on the notions of linguistic variable and fuzzy logic [1-3], and distinguished the importance of fuzzy truth values as very true, quite true etc., that its are fuzzy subsets of the set of all truth degrees, i. e., its truth-valued are linguistic values of the linguistic truth variable, which are represented by fuzzy sets in the interval [0,1]. In 1987, G. Takeuti and S. Titani investigated so-called globalization which can be seen as an interpretation of connective "fully true" [4]. Nguyen Cat Ho and Wolfgang Wechler proposed an algebraic model of Hedge algebra for deal with linguistic information [5-6]. Since then, there existed some importance results on uncertainty information processing with linguistic terms. In 2000, P. Hájek and D. Harmancova adopted A. D. Yashin axioms of the "strong future tense operator" [8] in Gödel logic and obtained a complete axiomatization for logical connective "more or less" [7]. Since then, P. Hájek has discussed logic BL_{vt} which is a conservative extension of BL-logic including logical connective "very true", and semantics given by BL-algebras extended by a unary function V interpreting "very true" [9]. In 2006, Vilém Vychodil has introduced a complete axiomatization of unary connectives interpreted by monotone and super diagonal truth functions, so-called truth-depressing hedges [10]. These connectives formalize linguistic hedges likes "slightly true" and "more or less". Nevertheless, how far can even this sort of fuzzy logic be captured by standard methods of mathematical logic. Therefore, there some approach which use linguistic assessments take the place of numerical values by means of linguistic variables [11-13]. Moreover, variable values are not numbers but words or sentences in a natural or artificial language. In real uncertainty reasoning and approximate inference, there exist many situations in which the information can not be assessed precisely in a quantitative form but may be in a qualitative one that is description in natural language [14]. For example, when ones try to evaluate "Age", ones tend to apply natural language "slightly young, somewhat young, almost young and very young etc." description. We know these descriptions are generated from modifiers and meta truth values by various linguistic and connectives [15-18]. In these situations, a modifier and meta truth value application is efficient. Moreover, some linguistic modifiers seem difficult to distinguish their boundary sometimes, but their meaning of common using can be understood. According to the above viewpoints, a linguistic truth-valued lattice implication algebra for a valuation domain has been proposed in [19]. As a continuous work of [19, 15, 16, 17], this paper extends lattice-valued propositional logic system LP(X) [20, 21, 22, 23, 24] to the corresponding linguistic truth-valued lattice value propositional logic system ℓP(X).

2. Preliminaries

First of all, we recall some definitions and results which will be needed.

Definition 2.1 [25, 24] Let $(L,\vee,\wedge,')$ be a bounded lattice with an ordered-reversing involution $'$

and the universal bounds O, I, $\rightarrow: L\times L\rightarrow L$ be a mapping. $(L,\vee,\wedge,',\rightarrow)$ is called a lattice implication algebra if it satisfies the following axioms:

(L1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$,

(L2) $x\rightarrow x=I$,

(L3) $x\rightarrow y=y'\rightarrow x'$,

(L4) $x\rightarrow y=y\rightarrow x=I$ imply $x=y$,

(L5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$,

(L6) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$,

(L7) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$,

for all $x,y,z\in L$.

Definition 2.2 [24] Let L be a lattice implication algebra, for all $x,y,z\in L$, $J\subseteq L$ is said to be a filter of L, if it satisfies the following conditions:

(1) $I\in J$; (2) if $x\in J$ and $x\rightarrow y\in J$, then $y\in J$.

$J\subseteq L$ is said to be a implicative filter of L, if it satisfies the following conditions:

(1) $I\in J$; (2) if $x\rightarrow y\in J$ and $x\rightarrow(y\rightarrow z)\in J$, then $x\rightarrow z\in J$.

Definition 2.3 [18, 19] Denote MT={True (Tr for short), False (Fa for short)}, which is called as the set of meta truth values. The lattice implication algebra defined on the set of meta truth values is called a meta linguistic truth-valued lattice implication algebra, where Fa $\prec$ Tr, the operation " ' " is defined as: Tr'=Fa and Fa'=Tr, the operation " $\rightarrow$ " is defined as $\rightarrow: MT\times MT\rightarrow MT$, $x\rightarrow y=x'\vee y$.

Definition 2.4[19] Denote AD={Slightly (Sl for short), Somewhat(So), Rather(Ra), Almost (Al), Exactly(Ex), Quite(Qu), Very(Ve), Highly(Hi), Absolutely(Ab)}, which is called as the set of modifiers. The lattice implication algebra defined on the chain Sl $\prec$ So $\prec$ Ra $\prec$ Al $\prec$ Ex $\prec$ Qu $\prec$ Ve $\prec$ Hi $\prec$ Ab is called lattice implication algebra with modifiers if its implication is Łukasiewicz implication.

In the following, denote $L=AD\times MT$. Let

$L_9=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9\}$

$L_2=\{b_1,b_2\}$.

We can define two Łukasiewicz lattice implication algebras on them respectively, and still denote them as L_9, L_2;

$L_9: a_1\prec a_2\prec a_3\prec a_4\prec a_5\prec a_6\prec a_7\prec a_8\prec a_9$,

$a_i\rightarrow_{(L_9)} a_j=a_{(9-i+j)\wedge 9}$, $a^{'(L_9)}=a_i\rightarrow_{(L_9)} a_1$;

$L_2: b_1\prec b_2$, $b_1^{'(L_2)}=b_2$, $b_2^{'(L_2)}=b_1$,

$b_i\rightarrow_{(L_2)} b_j=\begin{cases} b_1\,(i=2,j=1)\\ b_2\,(otherwise)\end{cases}$.

Definition 2.5[19] The lattice implication algebra L defined above is called a linguistic truth-valued lattice implication algebra generated by AD and MT, denoted a L-LIA.

Theorem 2.6[19] The following conclusions hold for any $(x,y)\in L_{18}$.

(1) $(x,y)'=(x',y')$; (2) $(x',y')\geq(x',y)$ iff $y'\geq y$; (3) $(x',y')\geq(x,y)$ iff $x'\geq x$;

(4) The relation between (x',y) and (x,y') are shown in Table 1, where "$(x',y)//(x,y')$" means that they are incomparable.

Table 1 The relation between (x',y) and (x,y'):

	$y\succ y'$	$y=y'$	$y=y'$
$x\succ x'$	//	$\prec$	$\prec$
$x=x'$	$\succ$	=	$\prec$
$x\prec x'$	$\succ$	$\succ$	//

3. The main results of L-LIA

In what follows let L_{2N} denote a linguistic truth-valued lattice implication algebra unless otherwise specified.

Firstly, in a $L_{2N}=L_N\times L_2$, $I=(a_N,b_2)$ and $O=(a_1,b_1)$ the greatest and the smallest element of respectively, we define binary operations $\vee,\wedge,\otimes,\oplus$ as follows:

$(a_i,b_m)\vee_{(L_{2N})}(a_j,b_n)=(a_i\vee_{(L_N)}a_j,b_m\vee_{(L_2)}b_n)$;

$(a_i,b_m)\wedge_{(L_{2N})}(a_j,b_n)=(a_i\wedge_{(L_N)}a_j,b_m\wedge_{(L_2)}b_n)$;

$(a_i,b_m)\otimes_{(L_{2N})}(a_j,b_n)=((a_i,b_m)\rightarrow(a_j,b_n)')'$;

$(a_i,b_m)\oplus_{(L_{2N})}(a_j,b_n)=(a_i,b_m)'\rightarrow(a_j,b_n)$

for any $(a_i,b_m),(a_j,b_n)\in L_{2N}$.

We now obtain the following results:

Theorem 3.1 Let L_{2N} be a linguistic truth-valued lattice implication algebra. For any (a_i,b_m), (a_j,b_n), (a_p,b_h), $(a_q,b_k)\in L_{2N}$, then

$$((a_i,b_m)\rightarrow(a_j,b_n))'\leq((a_p,b_h)\rightarrow(a_i,b_m))\wedge((a_j,b_n)\rightarrow(a_q,b_k))$$

holds.

Proof. Since

$$\begin{aligned}
&((a_i,b_m)\rightarrow(a_j,b_n))'\rightarrow(((a_p,b_h)\\
&\rightarrow(a_i,b_m))\wedge((a_j,b_n)\rightarrow(a_q,b_k)))\\
&=(((a_i,b_m)\rightarrow(a_j,b_n))'\rightarrow((a_p,b_h)\rightarrow(a_i,b_m)))\\
&\wedge(((a_i,b_m)\rightarrow(a_j,b_n))'\rightarrow((a_j,b_n)\rightarrow(a_q,b_k)))\\
&=((a_p,b_h)\rightarrow(((a_i,b_m)\rightarrow(a_j,b_n))'\rightarrow(a_i,b_m)))\\
&\wedge((a_j,b_n)\rightarrow(((a_i,b_m)\rightarrow(a_j,b_n))'\rightarrow(a_q,b_k)))\\
&=((a_p,b_h)\rightarrow((a_i,b_m)'\rightarrow((a_i,b_m)\rightarrow(a_j,b_n))))\\
&\wedge((a_j,b_n)\rightarrow((a_q,b_k)'\rightarrow((a_i,b_m)\rightarrow(a_j,b_n))))
\end{aligned}$$

$= ((a_p,b_h) \to ((a_i,b_m)' \to ((a_j,b_n)' \to (a_i,b_m)')))$
$\wedge((a_q,b_k)' \to ((a_j,b_n) \to ((a_i,b_m) \to (a_j,b_n))))$
$= ((a_p,b_h) \to I) \wedge ((a_q,b_k)' \to I) = I$.

Moreover, we get

$((a_i,b_m) \to (a_j,b_n))' \le ((a_p,b_h) \to (a_i,b_m))$
$\wedge((a_j,b_n) \to (a_q,b_k))$.

This means the proposition holds.

Theorem 3.2 Let L_{2N} be a linguistic truth-valued lattice implication algebra.

For any (a_i,b_m), $(a_j,b_n) \in L_{2N}$,
then $((a_i,b_m) \vee (a_j,b_n)) = ((a_i,b_m) \to (a_j,b_n))$
$\to (a_j,b_n)$ holds.

Proof. It follows from the operations "$\vee$" and "$\to$", we can get

$(a_i,b_m) \vee (a_j,b_n) = (a_i \vee_{(L_N)} a_j, b_m \vee_{(L_2)} b_n)$,
$((a_i,b_m) \to (a_j,b_n)) \to (a_j,b_n)$
$= ((a_i \to_{(L_N)} a_j) \to_{(L_N)} a_j, (b_m \to_{(L_2)} b_n) \to_{(L_2)} b_n))$.

Since $a_i, a_j \in L_N$, $b_m, b_n \in L_2$, and L_N, L_2 are chains. Thus, we have

$a_i \vee_{(L_N)} a_j = (a_i \to_{(L_N)} a_j) \to_{(L_N)} a$ and
$b_m \vee_{(L_2)} b_n = (b_m \to_{(L_2)} b_n) \to_{(L_2)} b_n$,

it follows that

$((a_i,b_m) \vee (a_j,b_n)) = ((a_i,b_m) \to (a_j,b_n))$
$\to (a_j,b_n)$ holds.

Similarly, we can get

$(a_i,b_m) \wedge (a_j,b_n) = ((a_i,b_m)' \vee (a_j,b_n)')'$.

Theorem 3.3 Let L_{2N} be a linguistic truth-valued lattice implication algebra. If $a_i \le a_j$ or $b_m \le b_n$ for any (a_i,b_m), $(a_j,b_n) \in L_{2N}$, then

$(a_i,b_m) \le (a_i,b_n)$ and $(a_i,b_m) \le (a_j,b_m)$ hold.

Theorem 3.4 Let L_{2N} be a linguistic truth-valued lattice implication algebra, for any $(a_i,b_m), (a_j,b_n), (a_p,b_h) \in L_{2N}$. Then

(1) $(a_i,b_m) \to (a_j,b_n) \ge (a_i,b_m)' \vee (a_j,b_n)$;

(2) $(a_j,b_n) \to (a_i,b_m) = (a_p,b_h) \to (a_i,b_m)$ if and only if $a_j = a_p$ and $b_n = b_h$ if and only if $(a_i,b_m) \to (a_j,b_n) = (a_i,b_m) \to (a_p,b_h)$.

Theorem 3.5 Let L_{2N} be a linguistic truth-valued LIA, for any $(a_i,b_m), (a_j,b_n) \in L_{2N}$. Then

$((a_i,b_m) \to (a_j,b_n)) \vee ((a_j,b_n) \to (a_i,b_m)) = I$.

Proof.Since

$((a_i,b_m) \to (a_j,b_n)) \vee ((a_j,b_n) \to (a_i,b_m))$
$= (a_i \to_{L_N} a_j, b_m \to_{L_2} b_n)$
$\vee(a_j \to_{L_N} a_i, b_n \to_{L_2} b_m)$
$= (a_{(N-i+j)\wedge N}, b_{(2-m+n)\wedge 2})$
$\vee(a_{(N-j+i)\wedge N}, b_{(2-n+m)\wedge 2})$
$= (a_{(N-i+j)\wedge N} \vee a_{(N-j+i)\wedge},$
$b_{(2-m+n)\wedge 2} \vee b_{(2-n+m)\wedge 2})$
$= (a_{((N-i+j)\wedge N)\vee((N-j+i)\wedge N)},$
$b_{((2-m+n)\wedge 2)\vee((2-n+m)\wedge 2)})$
$= (a_N, b_2) = I$.

Hence the proposition is proved.

Theorem 3.6 Let L_{2N} be a linguistic truth-valued LIA, for any (a_i,b_m), (a_j,b_n), $(a_k,b_h) \in L_{2N}$. Then

$((a_i,b_m) \to (a_k,b_h)) \to ((a_j,b_n) \to (a_k,b_h))$
$= (a_j,b_n) \to ((a_i,b_m) \vee (a_k,b_h))$.

Proof.Since

$((a_i,b_m) \to (a_k,b_h)) \to ((a_j,b_n) \to (a_k,b_h))$
$= (a_j,b_n) \to (((a_i,b_m) \to (a_k,b_h)) \to (a_k,b_h))$
$= (a_j,b_n) \to ((a_i \to_{L_N} a_k) \to_{L_N} a_k,$
$(b_m \to_{L_2} b_h) \to_{L_2} b_h))$
$= (a_j,b_n) \to (a_i \vee_{L_N} a_k, b_m \vee_{L_2} b_h)$
$= (a_j,b_n) \to ((a_i,b_m) \vee (a_k,b_h))$

Therefore, we have

$((a_i,b_m) \to (a_k,b_h)) \to ((a_j,b_n) \to (a_k,b_h))$
$= (a_j,b_n) \to ((a_i,b_m) \vee (a_k,b_h))$.

We can get the following Theorem 3.7 and Theorem 3.8 by the properties of LIA and operators.

Theorem 3.7 Let L_{2N} be a linguistic truth-valued LIA, for any (a_i,b_m), $(a_j,b_n) \in L_{2N}$. Then the following statements hold:

(1) $(a_i,b_m) \vee (a_j,b_n) \ge (a_i,b_m)$ or $(a_i,b_m) \vee (a_j,b_n) \ge (a_j,b_n)$;

(2) $(a_i,b_m) \wedge (a_j,b_n) \le (a_i,b_m)$ or $(a_i,b_m) \wedge (a_j,b_n) \le (a_j,b_n)$.

Theorem 3.8 Let L_{2N} be a linguistic truth-valued LIA, for any $(a_i,b_m), (a_j,b_n), (a_q,b_k)$, $(a_p,b_h) \in L_{2N}$. Then the following statements hold:

(1) $(a_i,b_m) \otimes (a_j,b_n) = (a_j,b_n) \otimes (a_i,b_m)$,
$(a_i,b_m) \oplus (a_j,b_n) = (a_j,b_n) \oplus (a_i,b_m)$;

(2) $(a_i,b_m) \le (a_j,b_n)$ and
$(a_p,b_h) \le (a_q,b_k)$ imply
$(a_i,b_m) \otimes (a_p,b_h) \le (a_j,b_n) \otimes (a_q,b_k)$;

(3) $(a_i,b_m) \le (a_j,b_n)$ implies
$(a_i,b_m) \otimes (a_p,b_h) \le (a_j,b_n)$;

(4) $(a_i,b_m) \otimes (a_1,b_1) = (a_1,b_1)$,
$(a_i,b_m) \oplus (a_1,b_1) = (a_i,b_m)$;

(5) $(a_i,b_m) \otimes (a_N,b_2) = (a_i,b_m)$,
$(a_i,b_m) \oplus (a_N,b_2) = (a_N,b_2) = I$;

(6) $((a_i,b_m) \otimes (a_j,b_n))' = (a_i,b_m)' \oplus (a_j,b_n)'$,
$((a_i,b_m) \oplus (a_j,b_n))' = (a_i,b_m)' \otimes (a_j,b_n)'$.

4. The semantic of linguistic truth-valued lattice values propositional logic system ℓP(X)

Language Let $L^* = (L, \vee, \wedge, ', \to)$ be a Linguistic Truth-Valued LIA, $L = L_N \times L_2$.

The symbols in $\ell P(X)$ are

(1) the set of propositional variable: $X = \{p, q, r, ...\}$;

(2) the set of constants: L;

(3) logical connectives: $\to$, $'$;

(4) auxiliary symbols:) , (.

The set F of formula of ℓP(X) is the least set Y satisfying the following conditions:

(1) $X \subseteq Y$; (2) $L \subseteq Y$;

(3) if $(a_i, b_m), (a_j, b_n) \in Y$, then $'(a_i, b_m), \to (a_i, b_m), (a_j, b_n) \in Y$.

In the following, we denote $'(a_i, b_m)$ as $(a_i, b_m)'$ and $\to (a_i, b_m), (a_j, b_n)$ as $(a_i, b_m) \to (a_j, b_n)$.

Definition 4.1 The free T^* algebra of the set X of the propositional variable is said to be the propositional algebra ℓP(X) of the linguistic truth-valued lattice value propositional calculus system and denote by $\ell P(X)$ if it satisfies the following conditions: (1) $T^* = L^* \cup \{', \to\}$ be a type; (2) for any $(\alpha, \beta) \in L$, $ar(\alpha, \beta) = 0$, $ar(') = 1$, $ar(\to) = 2$.

Definition 4.2 A mapping φ: ℓP(X) →L is called a valuation of ℓP(X), if it is a T-homomorphism.

Since ℓP(X) is a free T-algebra on X, there exist only one valuation ψ such that $\varphi|_X = \psi$ foe any mappin-g $\psi : X \to L^*$.

Let L^* be a complete lattice, $F_{L^*}(Y)$ is the set of all L^*-type fuzzy sets in Y.

Corollary 4.3 Let $f : \ell P(X) \to L^*$ be a mapping, then f is a valuation of ℓP(X) if and only if it satisfies:

(1) $f(\alpha, \beta) = (\alpha, \beta)$ for any $(\alpha, \beta) \in L^*$;

(2) $f((a_i, b_m) \to (a_j, b_n))$
$= f(a_i, b_m) \to f(a_j, b_n)$ for any formula $p = (a_i, b_m)$ and $q = (a_j, b_n)$;

(3) $f((a_i, b_m)') = (f(a_i, b_m))'$ for any formula $p = (a_i, b_m)$.

Proof. "⇒": Suppose f is a valuation of ℓP(X), then (1), (2) and (3) are hold by Definition4.3.

"⇐": Since $(a_i, b_m) \vee (a_j, b_n)$
$= ((a_i, b_m) \to (a_j, b_n)) \to (a_j, b_n)$ and $f((a_i, b_m) \to (a_j, b_n))$
$= f(a_i, b_m) \to f(a_j, b_n)$.

Thus, we get

$f((a_i, b_m) \vee (a_j, b_n))$
$= f(((a_i, b_m) \to (a_j, b_n)) \to (a_j, b_n))$
$= (f(a_i, b_m) \to f(a_j, b_n)) \to f(a_j, b_n)$
$= f(a_i, b_m) \vee f(a_j, b_n)$.

Similarly, we can get

$f((a_i, b_m) \wedge (a_j, b_n)) = f(a_i, b_m) \wedge f(a_j, b_n)$.

Hence f is a T-homomorphism, i.e., f is a valuation of ℓP(X). This completes the proof.

Definition 4.4 Let $A \in F_{L^*}(F)$, φ is a valuation of $\ell P(X)$. It is called that φ satisfies A if $A(a_i, b_m) \le \varphi(a_i, b_m)$ for any $(a_i, b_m) \in F$. A is called satisfiable if there exists a valuation φ, which satisfies A.

Definition 4.5 Let $A \in F_{L^*}(F)$, $(a_i, b_m) \in F$, $(\alpha, \beta) \in L^*$. (a_i, b_m) is called semantically implied from A with truth value level (α, β) if $\varphi(a_i, b_m) \ge (\alpha, \beta)$ for any valuation φ, which satisfies A, of ℓP(X), and denote as $A \models_\alpha (a_i, b_m)$. (a_i, b_m) is called valid with truth value level (α, β) and denoted by $\models_{(\alpha,\beta)} (a_i, b_m)$, if $\varphi(a_i, b_m) \ge (\alpha, \beta)$ for any φ valuation of $\ell P(X)$. (a_i, b_m) is called a valid formula if $\models_{(a_N, b_2)}$. (a_i, b_m) is called (α, β)-valid in A if $(\alpha, \beta) = \wedge\{\varphi(a_i, b_m) | \varphi$ is a valuation and satisfies A$\}$.

Definition 4.6 Let (a_i, b_m) and $(a_j, b_n) \in \ell P(X)$.
(a_i, b_m), (a_j, b_n) are called equivalent if $\varphi(a_i, b_m)$ $= \varphi(a_j, b_n)$ for any valuation φ of ℓP(X), deno-ted by $(a_i, b_m) = (a_j, b_n)$.

Definition 4.7 $A \in F_{L^*}(\ell P(X))$ is said to be closed if it satisfies the following conditions: (1) $A((a_i, b_m) \to (a_j, b_n)) \otimes A(a_i, b_m) \le A(a_j, b_n)$;
(2) $(\alpha, \beta) \to A(a_i, b_m) \le A((\alpha, \beta) \to (a_i, b_m))$ for any $(a_i, b_m), (a_j, b_n) \in \ell P(X), (\alpha, \beta) \in L^*$.

Definition 4.8 Let $X_n = \{(a_1, b_1), (a_2, b_2), ..., (a_n, b_n)\}$ and $\omega((a_1, b_1), (a_2, b_2), ..., (a_n, b_n)) \in \ell P(X)$, for any element $\varpi((a_1, b_1), (a_2, b_2), ..., (a_n, b_n))$ of $\ell P(X)$ and $\omega((a_1, b_1), (a_2, b_2), ..., (a_n, b_n))$
$= \varpi((a_1, b_1), (a_2, b_2), ..., (a_n, b_n))$.

Define a mapping $f_\varpi : L^{*^n} \to L^*$, f_ϖ is called a truth value function of L^{*^n} if it satisfies for any

$((\alpha_1,\beta_1),(\alpha_2,\beta_2),...,(\alpha_n,\beta_n)) \in L^{*^n}$, there exists a valuation φ of ℓP(X) such that $\varphi(a_i,b_m)=(\alpha_i,\beta_m)\ (1\le i\le N, 1\le m\le 2)$,

$f_{\varpi}((\alpha_1,\beta_1),(\alpha_2,\beta_2),...,(\alpha_n,\beta_n))$
$=\varphi(\omega((a_1,b_1),(a_2,b_2),...,(a_n,b_n)))$

We can easy to verify the following results by definitions and properties of LIA.

Theorem 4.9 Let L^* be a linguistic truth value LIA, for any (a_i,b_m), (a_j,b_n), $(a_p,b_k)\in L^*$, then $((a_i,b_m)\to(a_j,b_n))\to((a_i,b_m)\to(a_k,b_p))$ $=(a_i,b_m)\wedge(a_j,b_n)\to(a_k,b_p)$.

Theorem 4.10 Let L^* be a linguistic truth value LIA, for any (a_i,b_m), (a_j,b_n), $(a_p,b_k)\in L^*$, then $((a_i,b_m)\to(a_j,b_n))\to((a_i,b_m)\to(a_k,b_p))$ $\ge(a_j,b_n)\to(a_k,b_p)$ holds.

Theorem 4.11 Let L^* be a linguistic truth value LIA, for any (a_i,b_m), (a_j,b_n), $(a_p,b_k)\in \ell P(X)$, $(a_i,b_m)^2=(a_i,b_m)\otimes(a_i,b_m)$, $(a_i,b_m)^{t+1}=(a_i,b_m)^t\otimes(a_i,b_m)$.
Then $((a_i,b_m)^t\to((a_j,b_n)$ $\to(a_k,b_p)))\to((a_i,b_m)^s\to(a_k,b_p))$ $\to((a_i,b_m)^{t+s}\to(a_k,b_p))$ is a valid formula, for any $t,s\in N^+$.

Theorem 4.12 The valuation of ℓP(X) is closed.

Corollary 4.13 The valuation of ℓ_{18}P(X) is closed.

Let $\overline{F}=F/\equiv=\{\overline{(a_i,b_m)} \mid (a_i,b_m)\in F\}$, where $\overline{(a_i,b_m)}=\{(a_j,b_n)\mid(a_j,b_n)\in F,$ $(a_i,b_m)=(a_j,b_n)\}$. It can be proved that = is a congruence relation on $\overline{F}$. Hence, define operations $\vee$, $\wedge$, $'$, $\to$ on $\overline{F}$ as follows:
$(\overline{(a_i,b_m)})'=\overline{(a_i,b_m)'}$,
$\overline{(a_i,b_m)}\to\overline{(a_j,b_n)}=\overline{(a_i,b_m)\to(a_j,b_n)}$
$\overline{(a_i,b_m)}\vee\overline{(a_j,b_n)}=\overline{(a_i,b_m)\vee(a_j,b_n)}$,
$\overline{(a_i,b_m)}\wedge\overline{(a_j,b_n)}=\overline{(a_i,b_m)\wedge(a_j,b_n)}$.

Let $\ell P(X)=(\overline{F},\vee,\wedge,',\to)$, it is easy proved the following theorem 4.14, Theorem 4.15 and Theorem 4.16.

Theorem 4.14 Let $\overline{\ell P(X)}=(\overline{F},\vee,\wedge,',\to)$ is lattice implication algebra.

Theorem 4.15 (1) If φ is a valuation, $\overline{\varphi}:\overline{\ell P(X)}\to L^*$ satisfies $\overline{\varphi}(\overline{(a_i,b_m)})=\varphi(a_i,b_m)$ for any $(a_i,b_m)\in \ell P(X)$, then is a lattice implication epimorphism;

(2) Let $\varphi:\overline{\ell P(X)}\to L^*$ is a lattice implication homomorphism and satisfies $\varphi(\overline{(\alpha,\beta)})=(\alpha,\beta)$ for any $(\alpha,\beta)\in L^*$. If $\overline{\varphi_1}:\ell P(X)\to L^*$ satisfies $\varphi_1(a_i,b_m)=\varphi(\overline{(a_i,b_m)})$ for any $(a_i,b_m)\in L^*$. Then φ_1 is a valuation of ℓP(X).

Theorem 4.16 For any $\omega,\omega_1\in \ell P(X)$, if $f_{\overline{\omega}}=f_{\overline{\omega_1}}$ then $\omega=\omega_1$.

5. Further research

Future research will focus on the structure of linguistic truth-valued LIA and their resolution procedures based on linguistic truth-valued LIA and the construction method of reasonable linguistic truth-valued LIA.

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