Solution for Direct Kinematics of 3-PRS Parallel Manipulator Using Sylvester Dialytic Elimination Method

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Abstract. This paper presents the direct kinematics of a 3-PRS parallel manipulator. Through analysis position, the direct kinematics model is established, which is a nonlinear system of three equations in three unknown. The equations are derived using recursively the Sylvester dialytic elimination method. Finally, a numerical example is provided. Therefore, the direct kinematics is minimal..

Introduction

Parallel manipulator is a mechanism composed of a moving platform connected to a fixed base by means of at least two limbs [1]. Parallel manipulators have constituted a very active field of research over the last 20 years. Compared to serial manipulators, parallel manipulators essentially have two well-known advantages, namely greater precision in positioning and increased rigidity with respect to the relationship between size and workload limit. Many parallel manipulators with less than six Degrees Of Freedom (DOF) have been introduced, such as the famous DELTA robot with three translational DOF [2].

One of the challenges in studying parallel manipulators consists of the difficulty in solving their direct kinematics problems, which leads to systems of polynomial equations [3]. For direct kinematics, the input joint variables are given and all possible moving platform positions that would result from the given input values need to be found. Direct kinematic analysis is an essential component of the design, programming and control of any mechanism. Solution approaches for such a problem can be divided into two classes: numerical methods and analytic techniques [4]. To find all possible solutions of the direct kinematics problems of parallel manipulators, analytical techniques are found in the literature [5, 6]. Three common approaches to solving these systems of polynomial equations are the Dialytic Elimination, Polynomial Continuation and Grobner bases.

The purpose of this paper is to solve the direct kinematics problem of a 3-PRS parallel manipulator. Deriving the equations governing the problem leads to three couple equations. Then the equations are derived using recursively the Sylvester dialytic elimination method. Finally, a numerical example is provided.

Description of the 3-PRS Parallel Manipulator

The schematic of the 3-PRS parallel manipulator is shown in Fig. 1. The architecture of the manipulator is composed of a moving platform, a fixed base and three PRS type active limbs with the linear actuators fixed the base. Each PRS type active limb connects the moving platform to the base by a prismatic joint, P at B_i, a revolute joint at C_i, and a spherical joint at M_i. The prismatic joint is actuated and the other joints are passive. A reference frame O-xyz is attached to the base at point O, located at the center of the base. The z axes is perpendicular to the base. Point B_i is assumed to lie at a radial distance of r_{B} from the point O. B_iC_i is the direction of the prismatic joint movement, which is parallel to z axes. The axes of the revolute joint is perpendicular to the B_iO and B_iC_i. Point O_{M} is the center of the moving platform. And point M_i is the center of the spherical joint, which is assumed to lie at a radial distance of r_{M} from the point O_{M}. The distance from C_i to M_i is L. The angle φ_i is defined from B_iC_i to C_iM_i. And the angle $\theta_{\mathrm{B}i}$ is defined from x axes to OB_i. The drive parameters q_i is the distance from B_i to C_i.

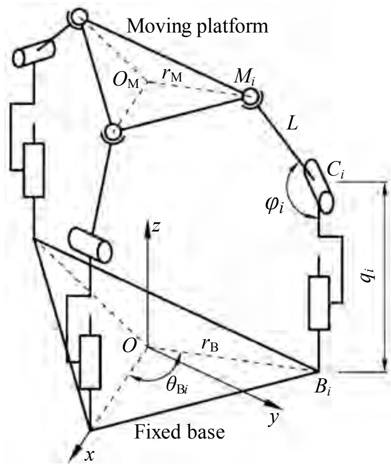

Fig. 1 Schematic of the 3-PRS parallel manipulator

Direct Kinematics

The position of M_i in the O-xyz coordinate frame for the i^{th} limb can be expressed as x_{Mi}, y_{Mi} and z_{Mi}

$$\begin{cases} x_{\mathrm{M}i} = \left(r_{\mathrm{B}} - L\sin\varphi_i\right)\cos\theta_{\mathrm{B}i} \\ y_{\mathrm{M}i} = \left(r_{\mathrm{B}} - L\sin\varphi_i\right)\sin\theta_{\mathrm{B}i} \quad \left(i = 1 \sim 3\right). \\ z_{\mathrm{M}i} = q_i - L\cos\varphi_i \end{cases} \tag{1}$$

As the distance between the M_i are known constant, and it can be expressed as

$$\begin{aligned} f_{12} &: \left\| M_1 - M_2 \right\| = d_{12} \\ f_{13} &: \left\| M_1 - M_3 \right\| = d_{13} \\ f_{23} &: \left\| M_2 - M_3 \right\| = d_{23} \end{aligned} \quad . \tag{2}$$

where d_{ij} is the distance between point M_i to M_j. Substituting Eq. 1 into Eq. 2, yields

$$f_{ij} : u_{ij1}\cos\varphi_i\cos\varphi_i + u_{ij2}(\cos\varphi_i - \cos\varphi_j) + u_{ij3}\sin\varphi_i\sin\varphi_i + u_{ij4}(\sin\varphi_i + \sin\varphi_j) + u_{ij5} = 0 \tag{3}$$

where u_{ijk} can be expressed as

$$\begin{aligned} u_{ij1} &= -2L^2 \\ u_{ij2} &= -2L(q_i - q_j) \\ u_{ij3} &= -2L^2(\cos\theta_{\mathrm{B}i}\cos\theta_{\mathrm{B}j} + \sin\theta_{\mathrm{B}i}\sin\theta_{\mathrm{B}j}) \\ u_{ij4} &= 2Lr_{\mathrm{B}}(\cos\theta_{\mathrm{B}i}\cos\theta_{\mathrm{B}j} + \sin\theta_{\mathrm{B}i}\sin\theta_{\mathrm{B}j} - 1) \\ u_{ij5} &= -2r_{\mathrm{B}}^2(\cos\theta_{\mathrm{B}i}\cos\theta_{\mathrm{B}j} + \sin\theta_{\mathrm{B}i}\sin\theta_{\mathrm{B}j} - 1) + (q_i - q_j)^2 + 2L^2 - d_{ij}^2 \end{aligned} \quad . \tag{4}$$

The half-angel tangent relationships can be expressed as

$$\sin\varphi_i = \frac{2t_i}{1+t_i^2} \text{ and } \cos\varphi_i = \frac{1-t_i^2}{1+t_i^2} \quad . \tag{5}$$

Substituting Eq. 5 into Eq. 4, direct kinematics of the 3-PRS parallel manipulator can be shown as

$$F_{ij} : t_i^2 t_j^2 + v_{ij1}(t_i^2 t_j + t_i t_j^2 + t_i + t_j) + v_{ij2} t_i^2 + v_{ij3} t_j^2 + v_{ij4} t_i t_j + 1 = 0 \quad . \tag{6}$$

where v_{ijk} can be expressed as

$$
\begin{aligned}
v_{ij1} &= 2u_{ij4} / (u_{ij1} + u_{ij5}) \\
v_{ij2} &= (-u_{ij1} - 2u_{ij2} + u_{ij5}) / (u_{ij1} + u_{ij5}) \\
v_{ij3} &= (-u_{ij1} + 2u_{ij2} + u_{ij5}) / (u_{ij1} + u_{ij5}) \\
v_{ij4} &= 4u_{ij3} / (u_{ij1} + u_{ij5})
\end{aligned} \quad . \tag{7}
$$

Eq. 6 is a nonlinear system of three equation in three unknown t_1, t_2 and t_3.The Dialytic elimination method is applied to F_{23} and F_{13}, producing the following equation

$$
\begin{bmatrix}
t_1^2 + v_{13,1}t_1 + v_{13,3} & v_{13,1}t_1^2 + v_{13,4}t_1 + v_{13,1} & v_{13,2}t_1^2 + v_{13,1}t_1 + 1 & 0 \\
0 & t_1^2 + v_{13,1}t_1 + v_{13,3} & v_{13,1}t_1^2 + v_{13,4}t_1 + v_{13,1} & v_{13,2}t_1^2 + v_{13,1}t_1 + 1 \\
t_2^2 + v_{23,1}t_2 + v_{23,3} & v_{23,1}t_2^2 + v_{23,4}t_2 + v_{23,1} & v_{23,2}t_2^2 + v_{23,1}t_2 + 1 & 0 \\
0 & t_2^2 + v_{23,1}t_2 + v_{23,3} & v_{23,1}t_2^2 + v_{23,4}t_2 + v_{23,1} & v_{23,2}t_2^2 + v_{23,1}t_2 + 1
\end{bmatrix}
\begin{bmatrix} t_3^3 \\ t_3^2 \\ t_3 \\ 1 \end{bmatrix} = 0 \quad . \tag{8}
$$

For a nontrivial solution exist for Eq.8, the determinant of the square matrix must equal 0. This is developed as

$$
F_{(13)(23)} : n_1 t_2^4 + n_2 t_2^3 + n_3 t_2^2 + n_4 t_2 + n_5 = 0 \quad . \tag{9}
$$

where n_i can be expressed as

$$
\begin{aligned}
n_1 &= o_{11}t_1^4 + o_{12}t_1^3 + o_{13}t_1^2 + o_{14}t_1 + o_{15} \\
n_2 &= o_{21}t_1^4 + o_{22}t_1^3 + o_{23}t_1^2 + o_{24}t_1 + o_{25} \\
n_3 &= o_{31}t_1^4 + o_{32}t_1^3 + o_{33}t_1^2 + o_{34}t_1 + o_{35} \\
n_4 &= o_{41}t_1^4 + o_{42}t_1^3 + o_{43}t_1^2 + o_{44}t_1 + o_{45} \\
n_5 &= o_{51}t_1^4 + o_{52}t_1^3 + o_{53}t_1^2 + o_{54}t_1 + o_{55}
\end{aligned} \quad . \tag{10}
$$

F_{12} is developed as

$$
F_{12} : m_1 t_2^2 + m_2 t_2 + m_3 = 0 \quad . \tag{11}
$$

where m_i can be expressed as

$$
\begin{aligned}
m_1 &= t_1^2 + v_{12,1}t_1 + v_{12,3} \\
m_2 &= v_{12,1}t_1^2 + v_{12,4}t_1 + v_{12,1} \\
m_3 &= v_{11,2}t_1^2 + v_{11,1}t_1 + 1
\end{aligned} \quad . \tag{12}
$$

The Dialytic elimination method is applied to $F_{(13)(23)}$ and F_{12}, producing the following equation

$$
\begin{bmatrix}
m_1 & m_2 & m_3 & 0 & 0 & 0 \\
0 & m_1 & m_2 & m_3 & 0 & 0 \\
0 & 0 & m_1 & m_2 & m_3 & 0 \\
0 & 0 & 0 & m_1 & m_2 & m_3 \\
n_1 & n_2 & n_3 & n_4 & n_5 & 0 \\
0 & n_1 & n_2 & n_3 & n_4 & n_5
\end{bmatrix}
\begin{bmatrix} t_2^5 \\ t_2^4 \\ t_2^3 \\ t_2^2 \\ t_2 \\ 1 \end{bmatrix} = 0 \quad . \tag{13}
$$

Eq. 13 produces a 16^{nd} degree polynomial in t_1. This equation can be solved for t_1, and then the values of t_2 and t_3 are determined by back-substitution.

Numerical Example

As an example of the direct kinematics solution for a general case mechanism, let the parameters be as: r_B=0.30, θ_{B1}=0, θ_{B2}=2π/3, θ_{B3}=4π/3, r_M=0.18, $d_{ij}=\sqrt{3}r_M$, L=0.22, q_1=0.16, q_1=0.22, q_3=0.26. The roots for t_i are given in Table 1. There are 16 solutions, and in which there are 8 real roots. So, for the given drive parameters q_i, there are 8 possible poses for the 3-PRS parallel mechanism. Those real roots, which are depicted in Fig. 2, correspond to the assembly modes of the manipulator. Therefore, the direct kinematics is minimal.

Tab.1 Solutions of t_i for the Direct Kinematics

No.	t_1	t_2	t_3	No.	t_1	t_2	t_3
1	0.3552134	0.2521578	0.8071060	9	-1.1643614 -0.4539659I	0.4648174 +0.9031792I	0.4506974 +0.8752161I
2	0.3257314	0.2829163	0.3251804	10	0.4679279 +0.9105487I	0.4462254 +0.8689908I	-0.7681163 -0.2933693I
3	1.5397644	0.3319772	0.2676018	11	0.4697463 +0.8870099I	0.4542512 +0.8910920I	0.4638804 +0.8834293I
4	4.3773386	0.8363213	2.4761929	12	0.4724809 -0.9259680I	-0.9157076 +0.3263262I	0.4371669 -0.8569080I
5	0.4047012	1.1001490	0.2276883	13	0.4724809 +0.9259680I	-0.9157076 -0.3263262I	0.4371669 +0.8569080I
6	3.9293282	2.8349249	0.6712913	14	0.4697463 -0.8870099I	0.4542512 -0.8910920I	0.4638804 -0.8834293I
7	2.9185724	3.5289475	3.1583887	15	0.4679279 -0.9105487I	0.4462254 -0.8689908I	-0.7681163 +0.2933693I
8	1.3284484	1.3284484	2.9984559	16	-1.1643614 +0.4539659I	0.4648174 -0.9031792I	0.4506974 -0.8752161I

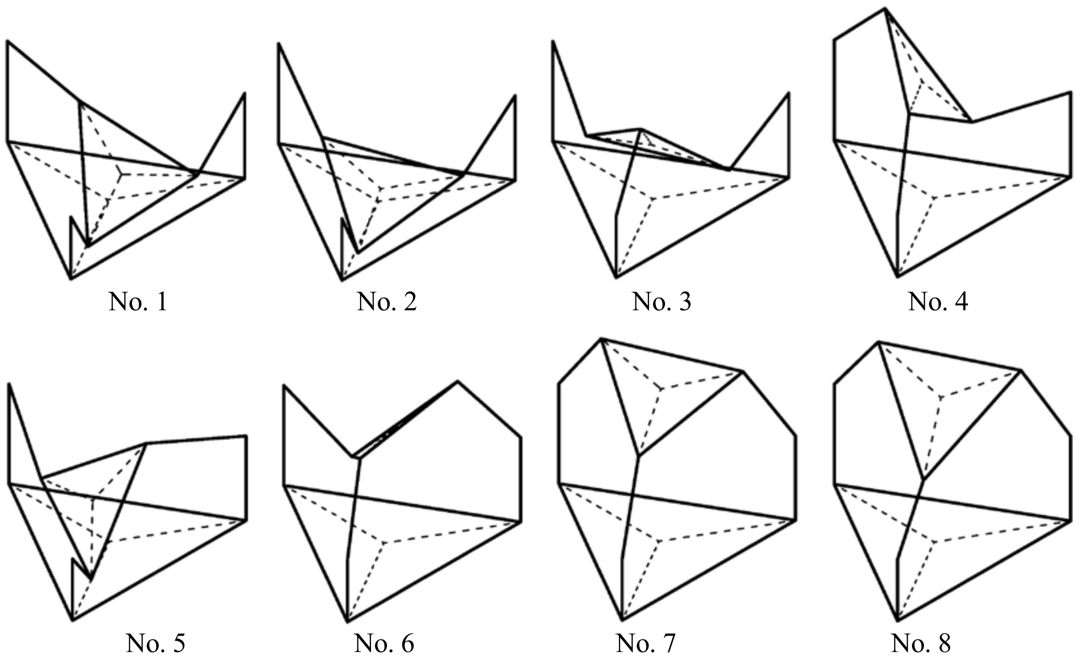

Fig. 2 The Eight Assembly Model of the Example

Conclusion

In this paper, the direct kinematics of 3-PRS parallel manipulation has been successfully approached. Firstly, the position analysis was carried out, and direct kinematics of the 3-PRS parallel manipulator was established. The direct kinematics, which was nonlinear system, was carried out using Sylvester dialytic elimination method. Then, a 16^{nd} degree polynomial in one unknown was produced. The results imply that, for a set of three given drive parameters, the foregoing polynomial admitted up to sixteen solutions. A numerical example having eight real solutions was included. Therefore, the polynomial is minimal.

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