Multi-dimensional Seismic Control with TLD in Vortex Effect

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Abstract. Seismic control of the horizontal and torsional response by cylindrical tuned liquid dampers is studied. The hydraulic pressure model of the cylindrical TLD including pulse pressure and convection pressure is established by membrane analogy method. The torsional model of the cylindrical TLD is obtained based on the quasi-uniform vortex theorem. The equation of motion for the control system considering eccentric torsion effect is derived and the solving method is presented. A 10-story eccentric structure with TLD is analyzed to verify the control performance for the horizontal and torsional coupled system under earthquake. The results show that the coupled response can be reduced effectively and the vortex torsional force by TLD cannot be ignored.

Introduction

In recent years, strong earthquakes occurred constantly and the dynamic action has brought great destruction to build structures, which make the importance of structural aseismic and disaster prevention and mitigation more appreciated. As for the important eccentric structure, only considering single dimensional earthquake will underestimate the actual dynamic response of the structure, therefore, the effect of the multi-dimensional seismic action on structure is necessary [1]. In addition to generating the translational vibration, the structure also produces torsional vibration which cannot be ignored. The cause of torsional vibration is as follows: there are rotational components exist in ground motion, or the ground motion at each point has a phase difference. On the other hand, the structure itself is eccentric, namely the center of mass and the center of stiffness are not coincident [2]. Earthquake damage shows that the torsional effect will aggravate the structural damage and in some cases it will become the main factor causing the structural failure.

In order to reduce the damage degree of structure under earthquake, the damping control technology emerges as the times require. At present, base isolation, energy dissipation, harmonic vibration, active control technology can play a role in damping control for the specific structure [3]. In recent years, as the tuned mass or tuned liquid control technology which develops fast does not need to take traditional measures to strengthen the structure, the damping effect is obvious and it’s easy to implement [4-7]. Tuning system is adding inertial mass to the top of the structure or adding the flowing liquid inside the subsidiary structure, using the second order system to absorb the vibrational energy of a major structure so as to control the vibration of the major structure. TLD has the characteristics of low cost, simple and practicable, low maintenance cost and good damping effect, so it has a good prospect of engineering application. But the tuned mass or tuned liquid damping system has the following problems: the analysis method of multi-dimensional damping still needs to be improved, and multi-dimensional damping effect should be enhanced etc. The seismic control of the horizontal and torsional response by cylindrical tuned liquid dampers is studied in paper [6] and the circulation torsion effect of the liquid is considered in the analysis. But anti twisted properties of the general cylindrical tuned liquid damper especially the anti-twisted ability formed by vortex inside the liquid was always ignored in previous studies. The mechanics principle and the damping effect of using cylindrical tuned liquid damper to realize the multi-dimensional coupled vibration control of the eccentric structure is studied in this paper.

The Hydraulic Pressure Model Established by Membrane Analogy Method

The principle of tuned liquid damper is using the internal inertia and viscosity of liquid in the container fixed on the structure to absorb and consume kinetic energy of the structure in order to achieve the purpose of reducing structure vibration. Many domestic and foreign researchers proposed different simplified dynamic model to reflect the energy dissipation characteristics of TLD. Housner has proposed equivalent mass method [8], this method classified the hydraulic pressure produced by liquid sloshing into two categories: Pulse pressure and Convection pressure, and simulated by vibration effect of two equivalent quality connected with the box body in different forms. Sayar etc. used the pendulum model to simulate the movement of containers partially filled with water and studied the movement of the container system and structure [9]. At present, the most widely used is the volatility model established by fluid wave dynamics theory, and it mainly includes the deep water model and the shallow water model [10]. Ju Rongchu proposed the model of equivalent quality established by membrane analogy method [7].

For shallow water container, the hydraulic pressure can be divided into pulse pressure and convection pressure, pulse pressure is associated with the inertia force caused by the movement of the pulse of the vessel wall, the amplitude is directly proportional to the vessel wall acceleration. The convection pressure is due to the hydraulic pressure generated by liquid sloshing. For the container which has vertical side wall and the horizontal plate, if apply a horizontal pulse acceleration in a certain direction of the side wall, and the acceleration which is orthogonal to the side wall can be ignored, equivalent to the liquid in the container is limited by the number of and the bottom plate and film force directions are orthogonal. It is equivalent to the liquid in the container which is limited by some membrane analogs that are orthogonal to the baseboard and the direction of force. Thus, the pulse pressure of liquid can be calculated by only considering the pulse pressure generated by the membrane analog. When the side wall of the container is subjected to the excitation, the liquid will produce hydraulic pressure in the vessel wall and floor. If only considering the first vibration mode of liquid, it can be regarded as the rigidity membrane analog which can rotate freely in the horizontal direction.

For a cylindrical container, the equivalent mass of the hydraulic pressure of the side wall is $m_{c0} = m_l h / \sqrt{3} r \tanh(\sqrt{3} r / h)$. In this formula, m_l is the total liquid quality, h is the height of the liquid in the cylinder, r is the radius of the cylinder. The total liquid dynamic pulse pressure acting on the walls of the container is

$$F_{pp} = \sqrt{3} / 3 c_s \rho \pi r h^2 \tanh(\sqrt{3} r / h) \ddot{u} \quad (1)$$

Where, c_s is structure coupling coefficient, value of 0.3 to 0.4, ρ is the liquid density, $\ddot{u}$ is horizontal acceleration the container bear.

The first equivalent quality of the cylindrical container on side wall for convection pressure is $m_{c1} = 0.386 r m_l \tanh(1.837 r / a) / h$, the first equivalent quality corresponding stiffness is $k_c = 5.4 m_{c1}^2 g h / m r^2$, g is the acceleration of gravity. The total liquid convection pressure acting on the walls of the container is

$$F_{pf} = \frac{11}{48} c_s \rho \pi r^4 \omega^2 \theta_0 \sin \omega t \quad (2)$$

Where, ω is the circular frequency corresponding to the first equivalent quality of liquid, θ_0 is the free surface of the liquid at the corner. The maximum total liquid convection pressure is $F_{pf\max} = 0.831 \frac{11}{48} c_s \rho \pi r^4 \omega^2 \frac{\alpha_{c1}}{g}$, where α_{c1} is the earthquake influence coefficient liquid storage of the natural vibration period of liquid storage. The natural vibration period of the liquid storage in container is $T_{c1} = 2\pi / \left[(\sqrt{27/8} g / r) \tanh(\sqrt{27/8} h / r) \right]^{1/2}$.

Torsional Model of the Cylindrical TLD based on Quasi-uniform Vortex Theorem

Under the actual multi-dimensional earthquake, in addition to energy consumption and shock absorption in two horizontal directions, the liquid of the TLD installed in structure will produce vortex phenomenon which can suppress the structure torsional to a certain extent. But at present the mechanism of vortex and torsional effects is not involved in TLD dynamic model. This paper will study.

The theory of fluid mechanics has proved ellipsoidal cavity is the only geometry cavity which can achieve homogeneous vortex motion [12]. Pfeiffer proposed an approximate theoretical calculation of non ellipsoidal cavity fluid flow. The theory uses the vorticity of each point in the mean value to describe the actual flow vorticity approximately [13-14]. The motion that uses uniform vortex motion to describe the liquid moving under the sense of the average is called the quasi-uniform vortex motion. Under the actual earthquake, the rotary of liquid in a cylindrical TLD belongs to non homogeneous vortex, so this paper will establish the cylindrical TLD torsional model based on the quasi-uniform vortex theorem.

The motion of an ideal and not compressible liquid in any cavity, if it existence of the vortex motion which cannot be ignored, then introduce the mean vorticity $\boldsymbol{\Omega}_a$ into the cavity liquid, it is the average value of the vorticity in the flow field.

$$\boldsymbol{\Omega}_a = \frac{1}{V} \int_V \boldsymbol{\Omega}(t, x_1, x_2, x_3) d\tau \tag{3}$$

Where, V represents the volume of the fluid domain. Suppose the fluid motion close to uniform vortex motion, its velocity distribution is written as

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{\Omega}_a \times \boldsymbol{r} + \nabla \varphi \tag{4}$$

Where, v is the absolute velocity of the fluid at any point in the P, v0 is a liquid filled rigid coordinate velocity, r is the radius vector that fluid particle P relative to the origin of the system, $\nabla \varphi$ is the velocity potential function of the liquid. According to the theory of fluid mechanics, we can obtain

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{\omega} \times \boldsymbol{r} + \boldsymbol{u} \tag{5}$$

Where, ω is the rotation angular velocity of inertia coordinates system compared to the liquid filled rigid body fixed coordinate system. U is the relative velocity of the fluid particle compared to the motion of the cavity. The relationship between relative velocity and potential function is:

$$\boldsymbol{u} = (\boldsymbol{\Omega}_a - \omega) \times \boldsymbol{r} + \nabla \varphi \tag{6}$$

For a uniform vortex motion, the Helmholtz equation is

$$\frac{d\boldsymbol{\Omega}}{dt} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\Omega} + \omega \times \boldsymbol{\Omega} = (\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{v} \tag{7}$$

For an arbitrary axisymmetric liquid in a cavity, its mean vorticity $\boldsymbol{\Omega}_a$ satisfies the averaged Helmholtz equation:

$$\frac{d\boldsymbol{\Omega}_a}{dt} = \frac{1}{V}\int_V (\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{v}d\tau \tag{8}$$

Through the continuity equation $\nabla\cdot\boldsymbol{u}=0$ and the condition that liquid cannot penetrable in the cavity wall $\boldsymbol{u}\cdot\boldsymbol{n}=0$, equations and boundary conditions are derived to satisfy the potentials

$$\nabla^2\varphi = 0 \qquad P\in V \tag{9}$$

$$\frac{\partial\varphi}{\partial n} = (\omega-\boldsymbol{\Omega}_a)\cdot\boldsymbol{r}\times\boldsymbol{n} \qquad P\in\partial V \tag{10}$$

Leading into the Stokes-Rukovskii vector potential $\boldsymbol{\Psi}$ to meet the needs

$$\varphi = \boldsymbol{\Psi}\cdot(\omega-\boldsymbol{\Omega}_a) \tag{11}$$

Thus, quasi-uniform velocity formula of vortex motion of liquid (6) can be rewritten as

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{\Omega}_a\times\boldsymbol{r} + \nabla\boldsymbol{\Psi}\cdot(\omega-\boldsymbol{\Omega}_a) \tag{12}$$

Averaging the Helmholtz equation in the cavity space, and put (12) into (8), consolidate it can be obtained

$$\frac{d\boldsymbol{\Omega}_a}{dt} = \frac{1}{V}\int_V (\boldsymbol{\Omega}\cdot\nabla)\nabla\boldsymbol{\Psi}d\tau\cdot(\omega-\boldsymbol{\Omega}_a) \tag{13}$$

For the quasi-homogeneous vortex motion of rotationally symmetric in the cavity liquid, leading into the Pfeiffer coefficient

$$\Gamma = \frac{1}{V}\int_{\partial S} n_z \frac{\partial(\boldsymbol{r}\boldsymbol{\Psi})}{\partial\boldsymbol{r}}ds \tag{14}$$

The average Helmholtz equation only retains the linear becomes

$$\dot{\boldsymbol{\Omega}}_a + (1-\Gamma)\omega_0\boldsymbol{e}_3\times(\boldsymbol{\Omega}_a-\omega) = 0 \tag{15}$$

For the cylindrical cavity that radius is r and the height is h, the Pfeiffer coefficient is expressed as

$$\Gamma_c = 1-\frac{8r}{h}\sum_{n=1}^{\infty}\frac{1}{\zeta_n(\zeta_n^2-1)}\tanh(\frac{\zeta_n h}{2r}) \tag{16}$$

ζ_n is the root that derivative of the variables in the first class of Bessel functions. Using the method of separation of variables to solve the partial differential equation (8), the moment of inertia of the cylindrical cavity can be obtained with its geometric center as a vertical spindle:

$$J_c = m_l r^2\gamma = m_l(\frac{r^2}{4}+\frac{h^2}{12}) - m_l r^2\left[1-\frac{16r}{h}\sum_{n=1}^{\infty}\frac{1}{\zeta_n^3(\zeta_n^2-1)}\tanh(\frac{\zeta_n h}{2r})\right] \tag{17}$$

The moment of inertia considered the internal liquid vortex effect of cylindrical TLD from the formula can be obtained.

Multidimensional Dynamic Model of Eccentric Structure with TLD

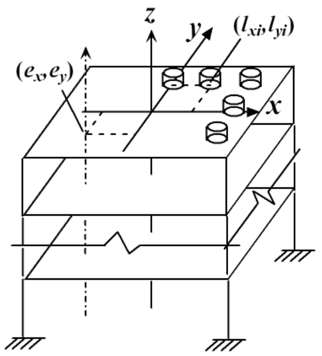

Fig. 1 Eccentric Structure Model with TLD

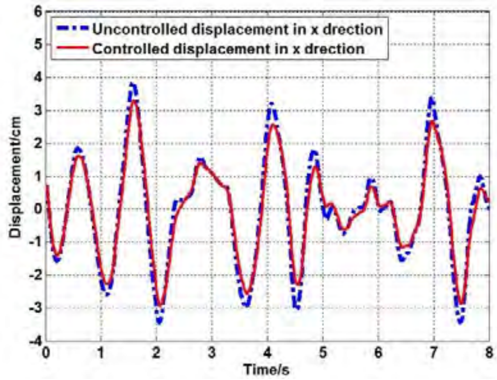

Fig. 2 Displacement History Comparison of Top Story

On top of the eccentric structure which has n layers set s TLDs along the longitudinal and transverse, as is shown in Fig.1, in the two horizontals and around the vertical shaft torsional, each floor is equivalent to a particle of single degree of freedom respectively, thus the structure is simplified to be a layer model. The equation of motion under multiple earthquake excitations is

$$\boldsymbol{M\ddot{U}} + \boldsymbol{C\dot{U}} + \boldsymbol{KU} = -\boldsymbol{M\ddot{U}}_g(t) + \boldsymbol{F}(t) \tag{18}$$

Where, M, C and K are 3n ×3n mass matrix, damping matrix and stiffness matrix of the structure. $\boldsymbol{U} = [\boldsymbol{U}_x, \boldsymbol{U}_y, \boldsymbol{U}_\theta]^T = \{u_{x1}, \cdots, u_{xn}, u_{y1}, \cdots, u_{yn}, u_{\theta 1}, \cdots, u_{\theta n}\}^T$ is the displacement vector of three dimensional centroid. $\ddot{\boldsymbol{U}}_g(t) = [\ddot{\boldsymbol{U}}_{xg}(t), \ddot{\boldsymbol{U}}_{yg}(t), \ddot{\boldsymbol{\Phi}}_{\theta g}(t)]^T$ is the earthquake input. $\boldsymbol{F}(t) = \{0, \cdots, F_x(t), 0, \cdots, F_y(t), 0, \cdots, F_\theta(t)\}^T$ is the control force vector of three dimensional. Where the expressions of Fx(t), Fy(t) and Fθ(t) are

$$\boldsymbol{F}_x(t) = -\sum_{i=1}^{s} m_{Ti}\ddot{u}_{axn} + \sum_{i=1}^{s} m_{Ti} l_{yi}\ddot{u}_{a\theta n} - \boldsymbol{F}_{px} \tag{19}$$

$$\boldsymbol{F}_y(t) = -\sum_{i=1}^{s} m_{Ti}\ddot{u}_{ayn} - \sum_{i=1}^{s} m_{Ti} l_{xi}\ddot{u}_{a\theta n} - \boldsymbol{F}_{py} \tag{20}$$

$$\boldsymbol{F}_\theta(t) = \sum_{i=1}^{s} m_{Ti} l_{yi}\ddot{u}_{axn} - \sum_{i=1}^{s} m_{Ti} l_{xi}\ddot{u}_{ayn} - \sum_{i=1}^{s} m_{Ti} R_i^2 \ddot{u}_{a\theta n} - \boldsymbol{F}_{p\theta} \tag{21}$$

In the formula, mTi represents the quality of liquid in the first i TLD. $\ddot{u}_{axn} = \ddot{u}_{xn} + \ddot{u}_{xg}$, $\ddot{u}_{ayn} = \ddot{u}_{yn} + \ddot{u}_{yg}$, $\ddot{u}_{a\theta n} = \ddot{u}_{\theta n} + \ddot{u}_{\theta g}$, $R_i^2 = l_{xi}^2 + l_{yi}^2$. $\boldsymbol{F}_{px}$ and $\boldsymbol{F}_{py}$ represent the hydraulic pressure of X and Y direction of TLD, respectively. Combined formula (1) and (2), there is

$$\boldsymbol{F}_{px} = \sum_{i=1}^{s} c_{si}[\frac{\sqrt{3}}{3}\rho\pi r_i h_i^2 \tanh(\sqrt{3}r_i / h_i) + \frac{11}{48}\rho\pi r_i^4 \omega_i^2 \theta_0 \sin \omega_i t]\ddot{u}_{axn} \tag{22}$$

$\boldsymbol{F}_{py}$ has the similar expressions, $\boldsymbol{F}_{p\theta}$ represents the anti-torsion vortex effect considering TLD. Combined Eq.(17), then

$$\boldsymbol{F}_{p\theta} = \sum_{i=1}^{s} J_{ci}\ddot{u}_{a\theta n} \tag{23}$$

Since the liquid damping matrix is introduced in the structure and the coupling system with TLD, The coupling damping matrix is not orthogonal, belongs to non- classical damping, so cannot use real modal transform to analyze decoupling and time history of Eq. (18). In order to solve the problem, we can use the state space method, by introducing the state vector to transform the higher order ordinary differential equation of the system into the first order differential equation consisting of state variables. First of all, define the equation $\ddot{U}_{ge}(t) = (I - \frac{F(t)}{M\ddot{U}_g(t)})\ddot{U}_g(t)$, where I is the unit matrix. Then the Eq. (18) can be rewritten as

$$M\ddot{U} + C\dot{U} + KU = -M\ddot{U}_{ge}(t) \tag{24}$$

Define $X = \{U\ \dot{U}\}^T$, $\{Y\} = \{U\ \ddot{U}\}^T$ as the state vector of the system, then Eq.(2) can be expressed as the form of state equation

$$\dot{X} = AX + B\ddot{U}_{ge}(t) \quad Y = CX + D\ddot{U}_{ge}(t) \tag{25}$$

Where, $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$, $C = \begin{bmatrix} I & 0 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$, $B = D = \begin{bmatrix} 0 \\ -I \end{bmatrix}$. Thus, according to the structural acceleration excitation vector, by solving the system state space equation can calculate the coupling dynamic structural response with TLD.

Calculation Examples

A 10-story eccentric structure with TLD is analyzed to verify the control performance for the horizontal and torsional coupled system under earthquake. The structure plane size is 36m × 16.5m, each layer is 3m. Column section size is 600mm × 600mm, the reinforcement ratio is 2%. Beam section size is 300mm×650mm, the reinforcement ratio is 1.5%. Slab thickness is 120mm, Concrete strength grade is C40, structure eccentric in the X direction and Y direction are respectively 6m and 6m.As is shown in Figure 1, on the top of the building along two horizontal eccentric skew symmetric install 10 cylindrical TLDs, each TLD's ratio is 2m,the liquid height is 1m. All TLD accounted for the total mass of the structure is 7.31%.

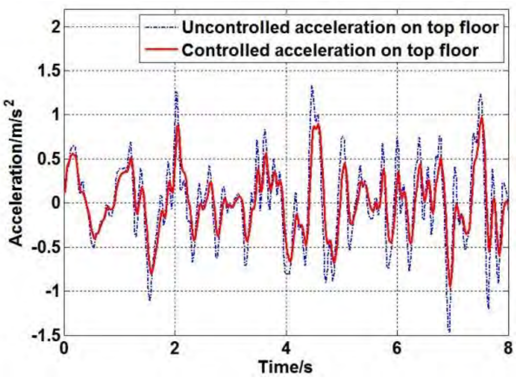

Fig. 3 Acceleration History Comparison of Top Story

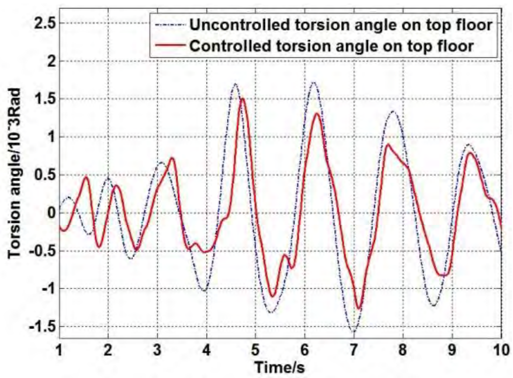

Fig. 4 Torsion Angle History Comparison of Top Story

El Centro wave and any other seismic waves are used as ground motion after establishing a layer model, calculation the multi-dimensional seismic response and seismic mitigation effect. In the presence of El Centro wave, the dynamic response time history curve whether to install the TLD structure shows from Fig. 2 to figure 4,we can see that after the installation of TLD, the structural response including the displacement, acceleration and angle of twist, have a certain degree of inhibition. The damping effect of TLD is ideal. Fig. 5 and Figure 6 represent the displacement of the structure and the torsional damping effect, the results show that TLD has the effect of damping vibration on the other layer, but the damping rate gradually weakened with the number of floors decreases.

Fig.7 is the curve of inertia force and hydraulic pressure that TLD applied to the top of structure in the x direction, combined Eq.(22) we can see that eccentric inertia force generated by TLD will produce stable and relatively obvious damping effects, also pulse pressure and convection pressure provided by TLD can play an important role in structure damping. Figure 8 is the curve of twisting force time history applied to the top by TLD, combined Eq.(22) we can see that the torsional force provided to structure by TLD is mainly obtained by relying on the vortex torsional obtained by arranged on the opposite direction of eccentric structure, the torsional force provided by the TLD vortex effect also accounted for a certain proportion, its role is bigger than the twisting force added by the horizontal inertial force by TLD, and cannot be ignored for serious eccentric structure.

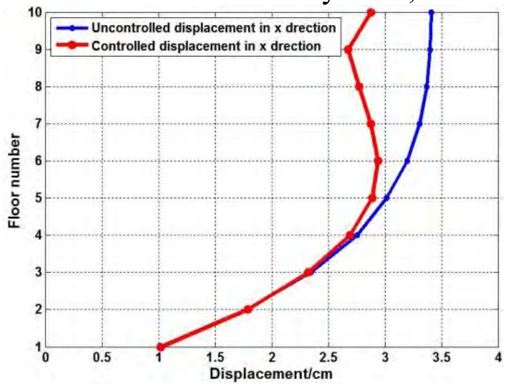

Fig. 5 Displacement Reduction of All the Stories

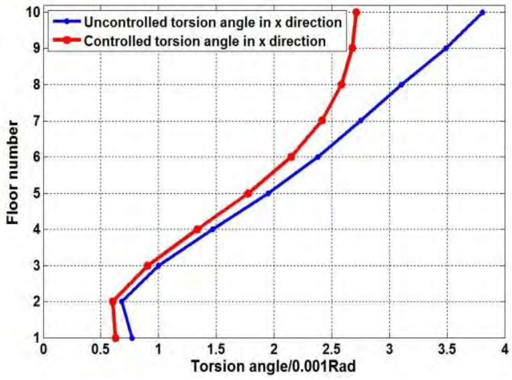

Fig. 6 Torsion Reduction of All the Stories

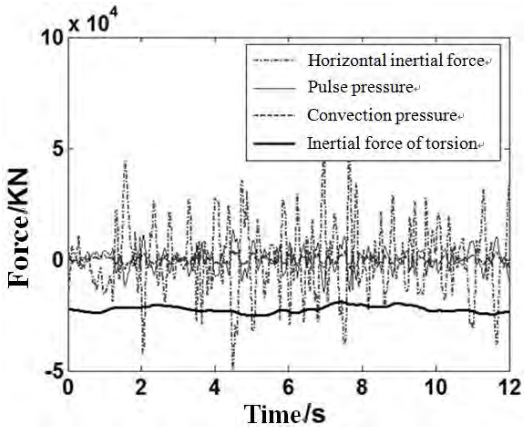

Fig. 7 Inertia Force and Hydraulic Pressure Curve of TLD

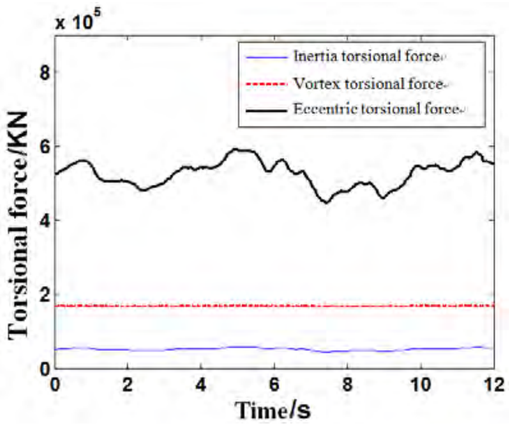

Fig. 8 Torsion Force Provided by TLD

Summary

The hydraulic pressure model of the cylindrical TLD including pulse pressure and convection pressure is established by membrane analogy method. This model can be coupled with dynamic equation directly, and it is more suitable for structure design. The torsional model of the cylindrical TLD is obtained based on the quasi-uniform vortex theory, in order to consider the respective vortex effect of tuned liquid. The equation of motion of the control system for considering eccentric torsion effect is derived and the solving method is presented. The analyzed results show that pulse pressure and convection pressure provided by TLD can play an important role in structure damping. The torsional force provided to structure by TLD is mainly obtained by relying on the vortex torsional obtained by arranged on the opposite direction of eccentric structure. The torsional force provided by the TLD vortex effect also accounted for a certain proportion, its role is bigger than the twisting force added by the horizontal inertial force, and cannot be ignored for serious eccentric structure.

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