Vibration Analysis of Moderately Thick Coupled Rectangular Plates

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Keywords: Coupled Plates, Mindlin Theory, Improved Fourier Series.

Abstract. Taking into account the impact of the flexural and in-plane vibrations , an improve Fourier series method is employed to analyze the vibration of moderately thick coupled plates. The vibration displacement is sought as the linear combination of a double Fourier cosine series and auxiliary series functions. The use of these supplementary functions is to solve the discontinuity problems along the edges. Then Rayleigh-Ritz method can give the matrix eigenvalue equation which is equivalent to governing differential equations of the plate. Finally the numerical results are presented to validate the correct of the method.

Introduction

Coupled structures are commonly used structural components in many branches, and their vibrational behavior is very important to the performance of the equipments, so many scholars studied the vibration characteristics of coupled plates and proposed various modeling methods. Cuschieri used mobility power flow analysis approach to compute the transmitted vibrational power of two plates joined at a right angle along a common edge [1]. Other researchers used wave propagation and modal analysis method [2], dynamic stiffness method [3] to research the coupled plates. These investigations mentioned above considered only the bending waves and ignored the in-plane waves, but the exclusion of in-plane waves may lead to errors at high frequencies. In order to establishment more correct vibrational model of coupled plates, some researchers studied the vibration characteristic by using the bending and in-plane vibration. [4, 5, 6].

These researches of the coupled plates above are based on the classical Kirchhoff hypothesis, and this theory neglects the effect of shear deformation which results in the over-estimation of vibration frequencies. The more correct model can be established by Mindlin hypothesis, so McCollum et al used this theory to study the vibration characteristic of coupled plates [7]. But the researches based on Mindlin theory are rarely, and the boundary conditions of coupled plates which have been researched are classic conditions, the coupling angle is right, the coupling stiffness is infinite.

To solve the limitions of the analysis method to the coupled plates with boundary supports and coupling conditions, taking into account the impact of the flexural and in-plane vibrations and based on Mindlin theory, the vibration model of coupled plates is established. To simulate arbitrary coupled conditions and boundary conditions, six types of springs along the coupling edges and five kinds of springs along the edges are attached. The displacement functions of the flexural and in-plane vibration are described by the improve Fourier series method. Using Rayleigh-Ritz method, the model of coupled plates with arbitrary boundary conditions is established. Finally the numerical results are presented to validate the correct of the method.

Structural Model of Coupled Plates

In order to establish a general model, the structural model of coupled plate is shown in Fig. 1. Coupled plate consists of plate i (i=1, 2,…, n), and the coupling angle of these two plates is θ_i. The length, width and thickness of plate i is a_i, b_i and h_i. Suppose that the second plate is connected to plate 1 at the common edge $x_1=a_1$ (or $x_2=0$). Oxyz coordinate system is the overall coordinate system of the coupled plated, Ox_iy_i is the local coordinate system of i-th rectangular plate structure. The boundary conditions and coupling conditions of coupled plates are simulated by setting restraining springs along edges and coupling edge. To simulate the boundary conditions of coupled plates, five types of linear spring (translational, rotational, torsional, normal and tangential springs) are needed. The six types of spring are three types of displacement restraining spring and three types of rotational restraining spring to simulate the arbitary coupling conditions. All classical homogeneous boundary conditions and the rigid coupling condition can be easily derived by simply setting each of the spring constants to be infinite or zero.

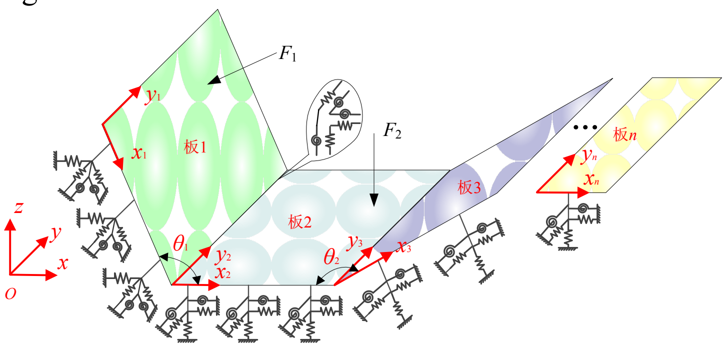


Fig. 1 A Couple Plate Structure with General Elastic Boundary Support and Coupling Conditions

The Displacement Functions of Coupled Plates

According to the bending and in-plane vibration theory, the transverse displacement of the plate median surface and the rotations of the cross-section, respectively, along the x direction and the y direction, the in-plane longitudinal and shear displacements are utilized. In this study, these quantities are expressed in form of improved Fourier series expansions:

$$w_i(x_i,y_i)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{imn}\cos(\lambda_{im}x_i)\cos(\lambda_{in}y_i) + \sum_{s=1}^{2}\sum_{m=0}^{\infty}d_{sm}^{1,i}\xi_{sb}(y_i)\cos(\lambda_{im}x_i)+\sum_{t=1}^{2}\sum_{n=0}^{\infty}f_{tn}^{1,i}\xi_{ta_i}(x_i)\cos(\lambda_{in}y_i) \tag{1}$$

$$\psi_{ix}(x_i,y_i)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}B_{imn}\cos(\lambda_{im}x_i)\cos(\lambda_{in}y_i) + \sum_{s=1}^{2}\sum_{m=0}^{\infty}d_{sm}^{2,i}\xi_{sb}(y_i)\cos(\lambda_{im}x_i)+\sum_{t=1}^{2}\sum_{n=0}^{\infty}f_{tn}^{2,i}\xi_{ta_i}(x_i)\cos(\lambda_{in}y_i) \tag{2}$$

$$\psi_{iy}(x_i,y_i)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{imn}\cos(\lambda_{im}x_i)\cos(\lambda_{in}y_i) + \sum_{s=1}^{2}\sum_{m=0}^{\infty}d_{sm}^{3,i}\xi_{sb}(y_i)\cos(\lambda_{im}x_i)+\sum_{t=1}^{2}\sum_{n=0}^{\infty}f_{tn}^{3,i}\xi_{ta_i}(x_i)\cos(\lambda_{in}y_i) \tag{3}$$

$$u_i(x_i,y_i)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}D_{imn}\cos(\lambda_{im}x_i)\cos(\lambda_{in}y_i) + \sum_{s=1}^{2}\sum_{m=0}^{\infty}d_{sm}^{4,i}\xi_{sb}(y_i)\cos(\lambda_{im}x_i)+\sum_{t=1}^{2}\sum_{n=0}^{\infty}f_{tn}^{4,i}\xi_{ta_i}(x_i)\cos(\lambda_{in}y_i) \tag{4}$$

$$v_i(x_i,y_i)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}E_{imn}\cos(\lambda_{im}x_i)\cos(\lambda_{in}y_i) + \sum_{s=1}^{2}\sum_{m=0}^{\infty}d_{sm}^{5,i}\xi_{sb}(y_i)\cos(\lambda_{im}x_i)+\sum_{t=1}^{2}\sum_{n=0}^{\infty}f_{tn}^{5,i}\xi_{ta_i}(x_i)\cos(\lambda_{in}y_i) \tag{5}$$

Where l=1, 2, s=1,2,⋯,5, $\lambda_m=m\pi/a$, $\lambda_n=n\pi/b$, A_{imn}, B_{imn}, C_{imn}, D_{imn}, E_{imn}, $d_{lm}^{s,j}$ and $f_{ln}^{s,j}$ are the expansion coefficients of i-th rectangular plate, and

$$\xi_{1a_i}(x_i)=\frac{a_i}{2\pi}\sin\frac{\pi x_i}{2a_i}+\frac{a_i}{2\pi}\sin\frac{3\pi x_i}{2a_i},\quad \xi_{2a_i}(x_i)=-\frac{a_i}{2\pi}\cos\frac{\pi x_i}{2a_i}+\frac{a_i}{2\pi}\cos\frac{3\pi x_i}{2a_i} \tag{6}$$

Theoretically, there are an infinite number of these supplementary functions. However, one needs to ensure that the selected functions will not nullify any of the boundary conditions. It is easy to verify that $\xi_{1a}(0)=\xi_{1a}(0)=\xi_{1a}'(a)=0$, $\xi_{1a}'(0)=1$, $\xi_{2a}(0)=\xi_{2a}(0)=\xi_{2a}'(0)=0$, $\xi_{2a}'(a)=1$, similar conditions exist for the supplementary function in y-direction. Though these conditions aren't necessary, they can simplify the subsequent mathematical expressions and the corresponding solution procedures.

One shall notice from Eqs. (1), (2), (3), (4) and (5) that beside the standard double Fourier series; four single Fourier series are also included. The potential discontinuity associated with the x-derivative and y-derivative of the original function along the four edges can be transferred onto these auxiliary series functions. Then, the Fourier series would be smooth enough in the whole solving domain. Therefore, not only is this Fourier series representation of solution applicable to any boundary conditions, but also the convergence of the series expansion can be improved.

Energy Model of Coupled Plates

The Rayleigh-Ritz method will be used to find the solution, specifically the Fourier expansion coefficients in eqs. (1) – (5). The Lagrangian's function L for the coupled plate system can be generally defined as

$$L=V-T \tag{7}$$

In the above equation, the total potential energy V can be expressed as

$$V=\sum_{i=1}^{n}\left(V_{\text{bend}}^{i}+V_{\text{inplane}}^{i}+V_{\text{spring}}^{i}+V_{\text{couple}}\right) \tag{8}$$

And the total kinetic energy T as

$$T=\sum_{i=1}^{n}\left(T_{\text{bend}}^{i}+T_{\text{inplane}}^{i}\right) \tag{9}$$

Where V_{bend}^{i} and V_{inplane}^{i} represent the strain energy associated with the bending and in-plane vibration of the i-th plate; V_{spring}^{i} indicates the potential energy stored in the bending-related and in-plane-related boundary springs of the i-th plate; T_{bend}^{i} and T_{inplane}^{i} are the kinetic energies of the i-th plate; V_{couple}^{i} denotes the potential energy associated with the coupling springs.

By substituting the displacement functions (1) - (5) into the Lagrangian (8) and minimizing the result against all unknown Fourier coefficients, one is able to obtain a final system of linear equations as

$$(\mathbf{K}-\rho h\omega^2\mathbf{M})\mathbf{G}=\mathbf{0} \tag{10}$$

Where **K** and **M** are the stiffness and mass matrices, and **G** is a vector of the entire unknown Fourier expansion coefficient. The natural frequencies and eigenvectors of coupled plates can be obtained through solving Eqs. (10).

Result and Discussion

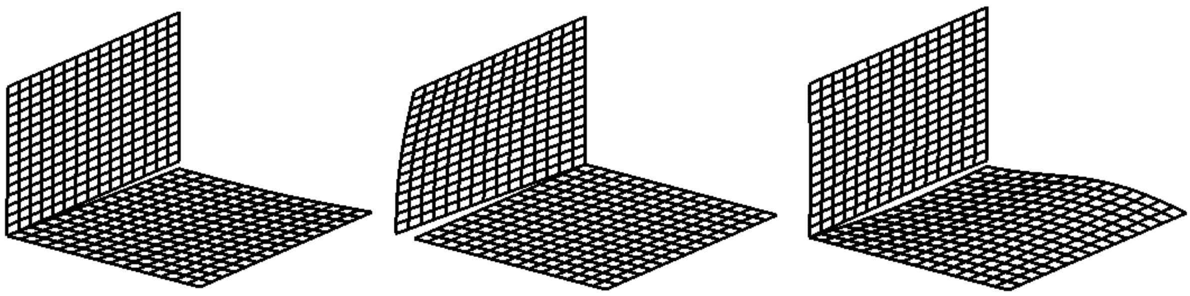
Several examples involving various boundary conditions will be discussed in this section. To avoid any comparison of the round off results which might be unrealistic, the non-dimensional frequency is used. For the analysis, and Poisson's ratio μ=0.3, and shear correction factor k=5/6 are used. The lengths for plate 1 and 2 are a_1=1m and a_2=1.4m, respectively, and the widths are b_1=b_2=1.2m. In identifying the boundary conditions, letters F, S and C have been used to indicate the free, simply and clamped boundary conditions along an edge, respectively.

In previous studies, the coupling angle is mostly assumed to be 90°. The coupling angle is not always 90° in engineering practice, so it's also significance to establish the vibration model of coupled plates with arbitrary angle, this method can obtain the desired model through setting the coupling angle θ. The non-dimensional frequencies $\Omega=\omega(\rho_1 h_1/D_1)^{1/2}$ of coupled plates with different angle are shown in Table 1. The thicknesses of two plates are h_1=h_2=0.1m, the coupling stiffness is rigid, the boundary condition of bending and in-plane vibration all are free. At the same time, the solutions with FEA are also presented.

Tab. 1 The First Seven Frequency Parameters for Coupled Plates with Different Coupling Angle

θ	method	the non-dimensional frequency $\Omega=\omega(\rho_1 h_1/D_1)^{1/2}$						
		1	2	3	4	5	6	Difference
90°	IFSM	3.0193	4.6708	6.9868	9.6565	15.2791	16.8031	0.79%
	FEA	3.0124	4.6337	6.9562	9.6673	15.246	16.8130	
120°	IFSM	3.3268	4.5372	7.7774	9.7742	15.5197	16.7193	0.68%
	FEA	3.3199	4.5063	7.7479	9.7857	15.4950	16.7300	
150°	IFSM	3.6271	4.4907	9.0659	10.4740	16.1683	16.3911	0.26%
	FEA	3.6199	4.4849	9.0420	10.4580	16.1710	16.4060	

Traditionally, the coupling conditions and boundary conditions are limit to rigid coupling and classic boundary. In this paper, the coupling and boundary conditions are simulated by setting the restrain springs along edges, and the vibration model of coupled plates with arbitrary elastic coupling and boundary conditions. The first three modes are shown in Fig.2. The coupling spring stiffness K_{c1}=K_{c2}=K_{c3}=k_{c1}=k_{c2}=k_{c3}=10×D_1, and the boundary condition of bending and in-plane vibration all are CFCCCF. It's seen that the weak coupling actually allows the plates to move almost independently.



(a) The first mode (b) The second mode (c) The third mode

Fig. 2 The first Three Mode Shapes of Coupled Plates with Elastic Coupling

Conclusion

An improved Fourier series method is proposed to analyze the free vibration of coupled plates with general elastic boundary supports and arbitrary coupling conditions. The unknown expansion coefficients can be solved through using Rayleigh-Ritz method. In comparison with most existing techniques, the current one does not need any formulation or implementation modifications to accommodate different boundary and coupling conditions. Finally, the numerical results are presented to validate the accuracy and convergence of the method.

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