Theoretical Study on Large Deformation of Fluid-Structure Interaction Problem on Elastic Plate

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Abstract. In this paper, a theoretical algorithm using united Lagrangian-Eulerian method was presented to study the fluid-structure interaction (FSI) problems. This method was used to solve the large deformation of the elastic plate in a continuous cross-flow of ideal fluid. In this approach, material was described by relatively form (e.g., Lagrangian for structures, Eulerian for fluids, Lagrangian and Eulerian for the interfaces of them). The coupling between the fluid and elastic plate domains were kinematic and dynamic conditions at the interface. For the elastic plate in a continuous flow of ideal fluid, the kinematic equation and dynamic equation of fluid- structure contact surfaces were established using united Lagrangian-Eulerian method. The knowledge of the large deformation is given by using the Fourier series expansions method. It is shown that the united Lagrangian - Eulerian method is an effectively method for the problem of elastic plate in an ideal cross-flow.

Introduction

The area of fluid-structure interaction(FSI) problems has received the greatest attention within aerospace, mechanical or biomedical applications, and thus has been studied by many authors over the past few years from different points of view (theoretical algorithm, numerical analysis and simulation)[1-3]. This paper deals with the mathematical analysis of problems dealing with steady fluid- structure interaction phenomena. To the authors knowledge, the situation has mainly been analysed the vibration and stability problems by numerical methods. On the contrary, the theoretical results of deformations and velocity are few. Examples of vibration are widespread [4, 5]. The different finite element methods such as Galerkin, Monolithical are described [6-9].

Typically, fluid and structure are given in different coordinate systems making a common solution. Fluid flows are given in Eulerian coordinates whereas the structure is treated in a Lagrangian framework. We use united Lagrangian-Eulerian method to present the large deformation of elastic plate in a continuous cross-flow of nonviscous, incompressible fluid. It is a new method of where fluid and structure equations are given in their preferred reference frames. The coupling between the fluid and structure domains are kinematic and dynamic conditions at the body surface. The effect of large deformation of the elastic plate is taken into account by united Lagrangian-Eulerian method.

A sketch of steady and irrotational flow of a nonviscous, incompressible fluid around an elastic plate is shown in figure 1. Thus the effect of viscosity, compression and fluid rotation are neglected. An orthogonal xyz coordinate system is used, and all investigated values of the plate and fluid are assumed to be functions independent of the coordinate z. The surfaces A and B of box plate are elastic, and the others are rigid. The surface A is very close to B. This article mainly studies the simply supported plate A. The pressure inside of the box plate is assumed to be constant. The physical parameters of the two-dimensional plate are: The length b, thickness h and bending stiffness $D = Eh^3/[12(1-\nu^2)]$, where E is Young's modulus and ν is the Poisson's ratio.

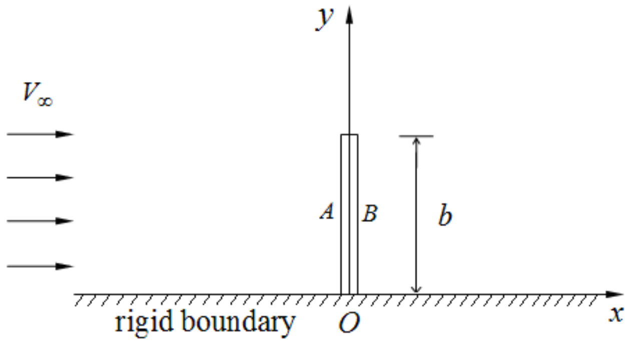

Fig. 1 Sketch of Flow around Elastic Plate

Steady State Equations

In this section the steady state governing equations for the fluid and structure are presented together with the interface coupling conditions.

Structure and Fluid Equations

The elastic plate equations of state is expressed using the Lagrangian formulation. The displacement field of the middle surface of the surface A of the plate is given by the following components: w, u; for x and y axes, respectively. The balanced equations are [10, 11]

$$\frac{\partial^4 w}{\partial y^4}+\frac{1}{2}\frac{\partial^2}{\partial y^2}\left[\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{1}{2}\left(\frac{\partial^2 w}{\partial y^2}\right)^3=\frac{1}{D}Z_1. \tag{1}$$

where $Z_i\,(i=1,2)$ are the projections of the external force vector.

The fluid state equations can be written in an Eulerian reference frame. The steady state equations for a potential flow can be described as

$$\nabla^2\varphi=0, \tag{2}$$

$$p=p_\infty+\frac{\rho_\infty}{2}\left[V_\infty^2-(\nabla\varphi)^2\right]. \tag{3}$$

in which ϕ is the velocity potential, p is the pressure, p_∞, ρ_∞ and V_∞ are the pressure, mass density, and velocity of the stationary flow on the infinite boundary, respectively. And ϕ satisfies the condition

$$\varphi=V_\infty x,\quad \left(x^2+y^2\to\infty\right). \tag{4}$$

Contact Conditions

The structure and fluid state equations can be written in Lagrangian and Eulerian reference frame, respectively. The coupling between the fluid and structure domains are kinematic and dynamic conditions at the interface. The kinematic condition is the noslip condition, i.e., continuity in velocity, and the dynamic condition is interface continuity in tractions. The kinematic and dynamic conditions can be written as [10, 11]

$$\frac{\partial w}{\partial y}\left(\frac{\partial\varphi}{\partial y}+u\frac{\partial^2\varphi}{\partial y^2}+w\frac{\partial^2\varphi}{\partial y\partial x}\right)-\left(1+\frac{\partial u}{\partial y}\right)\left(\frac{\partial\varphi}{\partial x}+w\frac{\partial^2\varphi}{\partial x^2}+u\frac{\partial^2\varphi}{\partial x\partial y}\right)-\frac{w^2}{2}\frac{\partial\varphi^3}{\partial x^3}=0, \tag{5}$$

$$Z_2 = 0, \quad Z_1 = p + u\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial x} + \frac{w^2}{2}\frac{\partial^2 p}{\partial x^2}. \tag{6}$$

Thus the balanced equation of the plate is expressed as

$$\frac{\partial^4 w}{\partial y^4} + \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{1}{2}\left(\frac{\partial^2 w}{\partial y^2}\right)^3 = \frac{1}{D}\left(p + u\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial x} + \frac{w^2}{2}\frac{\partial^2 p}{\partial x^2}\right). \tag{7}$$

Theoretical Solutions

Let us consider the general forms of the displacement and velocity potential

$$w = \sum_{n=1,2,\cdots} W_n \sin\frac{n\pi y}{b}, \tag{8}$$

$$\varphi = V_\infty x + B\cos\frac{\pi y}{b} e^{-\frac{\pi x}{b}}. \tag{9}$$

where W_n and B are coefficients to be determined. We obtain

$$p = p_\infty + \frac{1}{2}\rho_\infty B\frac{\pi}{b}\left[2V_\infty \cos\frac{\pi y}{b} e^{-\frac{\pi x}{b}} - B\frac{\pi}{b} e^{-\frac{2\pi x}{b}}\right]. \tag{10}$$

For the displacement w and u, the following equation can be written

$$\frac{\partial u}{\partial x} = -\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left[1 + \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^2\right]. \tag{11}$$

Substituting (8), (9), (10) and (11) into (5) and (7), we obtain the following expressions by using the Fourier series expansions method

$$A_{11} - \frac{1}{2}\left(\frac{\pi}{b}\right)^2 A_{21} + \frac{1}{2}A_{31} - \frac{1}{D}\left(\frac{4M}{\pi} + \frac{4K}{\pi} + A_{51}\right) = 0, \tag{12}$$

$$A_{12} - \frac{1}{2}\left(\frac{2\pi}{b}\right)^2 A_{22} + \frac{1}{2}A_{32} - \frac{1}{D}\left(A_{62} + \frac{4K}{3\pi} + A_{52}\right) = 0, \tag{13}$$

$$\left(1 + A_{40} + A_{41} + A_{42}\right)\left(V_\infty - B\frac{\pi}{b}\right) = 0. \tag{14}$$

where

$$A_{11} = \left(\frac{\pi}{b}\right)^4 W_1, \quad A_{12} = \left(\frac{2\pi}{b}\right)^4 W_2,$$

$$A_{21} = -\frac{1}{4}\left(\frac{\pi}{b}\right)^3 W_1^3 - 2\left(\frac{\pi}{b}\right)^3 W_1 W_2^2, \quad A_{22} = -\left(\frac{\pi}{b}\right)^4 W_1^2 W_2 + 2\left(\frac{\pi}{b}\right)^3 W_2^3 - 8\left(\frac{\pi}{b}\right)^4 W_2^3,$$

$$A_{31} = -\frac{3}{4}\left(\frac{\pi}{b}\right)^6 W_1^{\ 3} - 24\left(\frac{\pi}{b}\right)^6 W_1 W_2^{\ 2}, \qquad A_{32} = -6\left(\frac{\pi}{b}\right)^6 W_1^{\ 2} W_2 - 48\left(\frac{\pi}{b}\right)^6 W_2^{\ 3}.$$

$$A_{40} = -\frac{1}{4}\left(\frac{\pi}{b}\right)^2 W_1^{\ 2} - \frac{1}{4}\left(\frac{2\pi}{b}\right)^2 W_2^{\ 2} - \frac{1}{32}\left(\frac{\pi}{b}\right)^4 W_1^{\ 4} - \frac{6}{32}\left(\frac{\pi}{b}\right)^2\left(\frac{2\pi}{b}\right)^2 W_1^{\ 2} W_2^{\ 2} - \frac{1}{32}\left(\frac{2\pi}{b}\right)^4 W_2^{\ 4},$$

$$A_{41} = -\left(\frac{\pi}{b}\right)^2 W_1 W_2 - \frac{1}{4}\left(\frac{\pi}{b}\right)^4 W_1^{\ 3} W_2 - \frac{1}{8}\frac{\pi}{b}\left(\frac{2\pi}{b}\right)^3 W_1 W_2^{\ 3},$$

$$A_{42} = -\frac{1}{4}\left(\frac{\pi}{b}\right)^2 W_1^{\ 2} - \frac{1}{8}\left(\frac{\pi}{b}\right)^4 W_1^{\ 4} - \frac{3}{8}\left(\frac{\pi}{b}\right)^2\left(\frac{2\pi}{b}\right)^2 W_1^{\ 2} W_2^{\ 2},$$

$$A_{51} = \rho_\infty B\left(\frac{\pi}{b}\right)^2\left(B\frac{\pi}{b}W_1 - \frac{1}{2}V_\infty W_2\right), \qquad A_{52} = \rho_\infty B\left(\frac{\pi}{b}\right)^2\left(B\frac{\pi}{b}W_2 - \frac{1}{2}V_\infty W_1\right),$$

$$A_{62} = \frac{4M}{3\pi} + \rho_\infty B\frac{\pi}{b}V_\infty\frac{8}{3\pi},$$

$$K = -\frac{1}{2}\rho_\infty B\left(\frac{\pi}{b}\right)^2 V_\infty\left(A_{40}\frac{2b}{\pi} + A_{41}\frac{b}{\pi}\right), \qquad M = -\frac{1}{2}\rho_\infty B^2\left(\frac{\pi}{b}\right)^2.$$

The coefficients W_1, W_2 and B are reduced from (12)-(14). Then the deformation w, velocity $V_x = \frac{\partial\phi}{\partial x}$ and $V_y = \frac{\partial\phi}{\partial y}$ are observed.

Numerical Example

In this section, numerical examples are presented. The test elastic plate and fluid flow have the following characteristics: plate thickness $h = 1\times10^{-3}$ m, Young's modulus $E = 200\times10^9$ N/m2, Poisson's ratio $\nu = 0.3$, flow velocity $V_\infty = 0.12$m/s, mass density $\rho_\infty = 1000$ kg/m3. The results are shown in figure 2-4.

Figure 2 shows the deformation w by varying the length of plate. The deformation w is the maximum value near the middle of the plate, closer to $y = 0$. And w is zero at $y = 0, b$, respectively.

Figure 3 illustrates the fluid velocity from united Lagrangian-Eulerian method. Bernoulli's equation predicts a maximum velocity would exist near the plate tips.

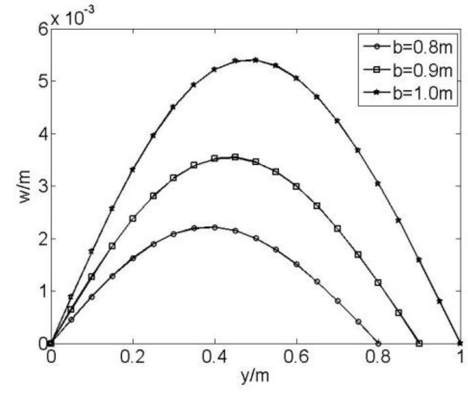

Fig. 2 Deformations of Plate for b=0.8m, 0.9m and 1.0m

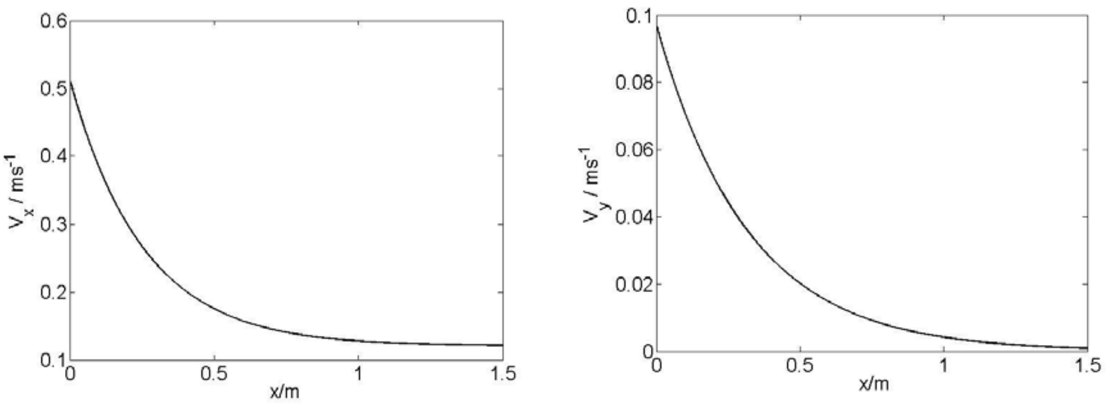

Fig. 3 $V_x - x$ and $V_y - x$ Curves when b=1m and $y = b$

Conclusions

To united Lagrangian-Eulerian (ULE) method, the structure and fluid state equations can be written in Lagrangian and Eulerian reference frame, respectively. The coupling between the fluid and plate domains are kinematic and dynamic conditions at the interface. The theoretical large deformation of elastic plate around by ideal fluid has been derived by ULE method. For FSI problem, Lagrangian and Eulerian coordinates for the interfaces of structure and fluid is easier than only Lagrangian or Eulerian coordinates. Numerical results reveal the streamlines with very high velocities near the plate tips. It is shown that the united Lagrangian - Eulerian method is an effectively methods for the problem of elastic structure acted by ideal fluid flow.

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