A Modified CamClay Constitutive Model Based on Thermodynamic Approach

Xiao-Xia GUO[1,2]

[1]State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian, Liaoning, China

[2]Department of Engineering Mechanics, Dalian University of Technology, Dalian, Liaoning, China

whanyuer@163.com

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Abstract. Mechanical behavior of granular media is important in many fields of studies such as the deformation of soil, slope stability, or the bearing capacity and so on. The mechanical behavior of granular media has been studied by borrowing the stress-strain models, such as elastic, elasto-plastic, or plastic models, developed for continuum materials. The aim of this research is to develop a constitutive equation taking into account the fabric tensors and their evolution modes. A modified Camclay constitutive model is effectively used, and link modern ideas of thermomechanics to void fabric tensors. After discussing the relationship between essential independent variables in constitutive equations and the state of granular materials from a viewpoint of the evolution mode of the void fabric tensors, the results show that granular materials can show less symmetry (more anisotropic). Moreover, the stress-strain relationship predicted by modified model is compared with the drained triaxial tests under the different consolidation pressures in order to assess the predictive capability. In each case good agreement with experiment is obtained.

Introduction

Granular materials are quite common in nature, as well as in industry, so that their related problems have been widely discussed in various engineering fields. In the geo-mechanics field, engineers must always deal with granular materials such as sand, gravel, seriously damaged risk and so on, and their efforts are often directed toward the establishment of a new field of mechanics.

As the experimental techniques on the deformation behavior of granular materials like sand have advanced, many complex deformation features have become to be known. In order to give a mathematically consistent fabric measure, the second order tensor characterizing the spatial distribution of microscopic quantities in a granular mass, which is called the fabric tensor, was proposed and discussed [1]. It can, however, be said that the constitutive formulation of granular materials based on thermodynamic approach taking the fabric tensor into account has yet been proposed. The formulation of constitutive equations including the fabric tensor is not simple to accomplish in an arbitrary manner. Even if one succeeds in developing a constitutive equation with more feasible features in an arbitrary manner, the resultant constitutive equation is likely to suffer from the lack of fundamental requirements for the non-linear constitutive equation. In the development of the specific constitutive model, physical considerations based on experimental observations and thermodynamic approaches become essential.

Fabric Measure Based on Void Space

Granular media, like cohesionless soil, consist of non-spherical particles with random, but still characteristic, statistical arrangement. The technical term, fabric, has been used to denote the spatial arrangement of particles and associated voids. This section gives a quantitative definition of the fabric, as generally as possible, through which the mechanical behavior can be discussed by taking into account the micro-structure. There is no doubt that the shape and the distribution of voids play an important role in defining the mechanical properties of a granular mass. Porosity or void ratio is often used to characterize the state of packing [2-4].

The analysis of the porosity distribution can also be extended to model the void ratio distribution as:

$$e(v)=e_m\left(1+\Omega_{ij}v_iv_j\right) \tag{1}$$

Where e_m is the mean void ratio of the soil.

Thermomechanical Formulation of Constitutive Laws for Elastic/Plastic Model

An overview [5-6] of the structure of this theory, indicating the steps needed to deduce the form of the yield function, flow incremental form etc. from the free energy and dissipation functions, is given in Fig. 1.

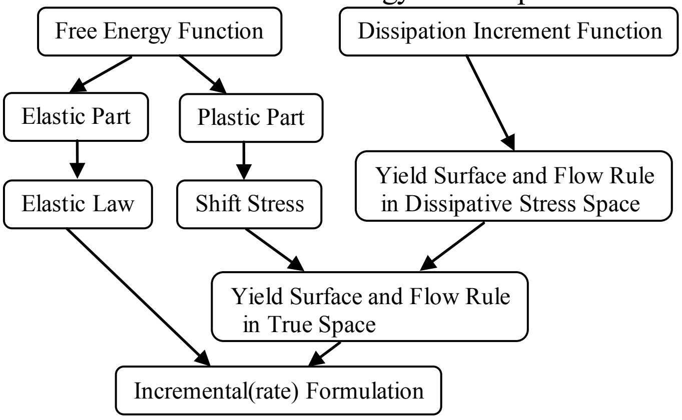


Fig. 1. Flow Chart Illustrating the Steps in Constructing the Elastic/Plastic Constitutive Model for Soils by Use of Thermodynamics

Elastic Potential Function for Soils

The total elastic energy can be divided two parts: one is the contribution of elastic volume modulus; the other is the contribution of elastic shear modulus. That is to say, Gibbs free energy can be written as

$$g_e=g_{e1}+g_{e2} \tag{2}$$

Where g_{e1} is the contribution of elastic volume modulus; g_{e2} is the contribution of elastic shear modulus.

In Duncan-Chang E-B model, Unloading and reloading modulus and bulk modulus can be expressed as

$$E_{ur}=K_{ur}p_a\left(\frac{p}{p_a}\right)^n;\ K_t=K_bp_a\left(\frac{p}{p_a}\right)^m \tag{3}$$

Where p_a is atmospheric pressure and K_{ur}, n, K_b, m are material constants respectively.

Double-integrating by the corresponding stresses in compliance matrix and considering the initial conditions: when $p=p_0$ and $q=q_0$, so $\varepsilon_v=0$ and $\varepsilon_s=0$, we can get the elasticity component of the Gibbs free energy function as follows.

$$-g_e=\frac{p_a^{m-1}}{K_b}\frac{1}{(-m+1)}\left[\frac{1}{(-m+2)}p^{-m+2}-p_0^{-m+1}p\right]$$
$$-\frac{1}{18K_bp_a^{1-m}}\left[q^2p^{-m}+mq_0^2p_0^{-m-1}p-2q_0p_0^{-m}q\right]+\frac{1}{2K_{ur}p_a^{1-n}}\left[q^2p^{-n}+nq_0^2p_0^{-n-1}p-2q_0p_0^{-n}q\right] \tag{4}$$

Starting from elastic component of the Gibbs free energy function and double-differentiating by the stresses, incremental stress-strain matrix may be given

$$\begin{pmatrix} dp \\ dq \end{pmatrix} = \begin{bmatrix} K^* & J^* \\ J^* & 3G^* \end{bmatrix} \begin{pmatrix} d\varepsilon_v \\ d\varepsilon_s \end{pmatrix} = \begin{bmatrix} D_{11}/A & D_{12}/A \\ D_{21}/A & D_{22}/A \end{bmatrix} \begin{pmatrix} d\varepsilon_v \\ d\varepsilon_s \end{pmatrix} \tag{5}$$

Where $D_{11} = -\frac{1}{9E_1} + \frac{1}{E_2}$; $D_{12} = D_{21} = -\frac{m\omega}{9E_1} + \frac{n\omega}{E_2}$; $D_{22} = \frac{1}{E_1} - \frac{\bar{m}\omega^2}{18E_1} + \frac{\bar{n}\omega^2}{2E_2}$; $\bar{m} = m(m+1)$; $\bar{n} = n(n+1)$; $\omega = p/p_a$

Considering the void fabric tensor

$$d\varepsilon_v^e = (d\varepsilon_v)_m^e (1+\Omega_{ij}v_iv_j) + \frac{n_m}{(1-n_m)}\beta d\varepsilon_q^e \tag{6}$$

Dissipation Function for Modified Camclay Model

As shown by Houlsby[7] and further discussed by Collins & Houlsby[8], true work equation associated with the modified Cam clay model is

$$\hat{W}^p = p\dot{\varepsilon}_v^p + q\dot{\varepsilon}_q^p = \frac{1}{2}p_c\dot{\varepsilon}_v^p + \frac{1}{2}p_c\sqrt{\dot{\varepsilon}_v^{p^2} + M^2\dot{\varepsilon}_q^{p^2}} \tag{7}$$

Starting from the dissipation function proposed above, obtain yield function expressions in the dissipative stress space by use of Ziegler's orthogonality principle[5]. Moreover, obtain yield function in the true stress space through adding the shift stress to the dissipative stress. The dissipative stress can be obtained as follows

The dissipative principle stress

$$\chi_p = \frac{\partial d}{\partial \dot{\varepsilon}_v^p} = \frac{1}{2}p_x\frac{1}{2}\left\{\left[\dot{\varepsilon}_v^p(1+\Omega_{ij}v_iv_j) + \left(\frac{n_m}{1-n_m}\right)\beta\dot{\varepsilon}_v^p\right]^2 + M^2\dot{\varepsilon}_q^{p^2}\right\}^{-1/2} 2\left[\dot{\varepsilon}_v^p(1+\Omega_{ij}v_iv_j) + \left(\frac{n_m}{1-n_m}\right)\beta\dot{\varepsilon}_q^p\right](1+\Omega_{ij}v_iv_j) \tag{8}$$

The dissipative shear stress

$$\chi_q = \frac{\partial d}{\partial \dot{\varepsilon}_q^p} = \frac{1}{2}p_x\frac{1}{2}\left\{\left[\dot{\varepsilon}_v^p(1+\Omega_{ij}v_iv_j) + \left(\frac{n_m}{1-n_m}\right)\beta\dot{\varepsilon}_q^p\right]^2 + M^2\dot{\varepsilon}_q^{p^2}\right\}^{-1/2}$$
$$\left\{2\left[\dot{\varepsilon}_v^p(1+\Omega_{ij}v_iv_j) + \left(\frac{n_m}{1-n_m}\right)\beta\dot{\varepsilon}_q^p\right]\left(\frac{n_m}{1-n_m}\right)\beta + 2M^2\dot{\varepsilon}_q^p\right\} \tag{9}$$

The yield surface in the true stress space is shown as follows

$$p'^2 + \left(q' - \frac{n_m\beta}{(1-n_mM)}p'\right)^2 = 1 \tag{10}$$

Where $\frac{(p-1/2\,p_x)^2}{\frac{1}{2}p_x(1+\Omega_{ij}v_iv_j)} = p'$, $\frac{q}{\frac{1}{2}Mp_x} = q'$.

Dilatancy Relationship in the Dissipative Stress Space and in the True Stress Space

In the dissipative stress space, strain increment vector direction is orthogonal to the elliptical yield locus. Dissipative yield function is obtained. So the volumetric and the plastic shear strain increment are respectively

$$d\varepsilon_v^p = d\lambda\left[\frac{8\chi_p}{p_x^2\left(1+\Omega_{ij}v_iv_j\right)^2} - \frac{8\chi_q}{Mp_x}\frac{1}{p_x\left(1+\Omega_{ij}v_iv_j\right)}\frac{\left(\frac{n_m}{1-n_m}\right)\beta}{M} + \frac{8\chi_p}{p_x^2\left(1+\Omega_{ij}v_iv_j\right)^2}\frac{\left(\frac{n_m}{1-n_m}\right)^2\beta^2}{M^2}\right] \tag{11}$$

$$d\varepsilon_s^p = d\lambda\left[\frac{8\chi_q}{M^2p_x^2} - \frac{8}{Mp_x}\frac{\chi_p}{p_x\left(1+\Omega_{ij}v_iv_j\right)}\frac{\left(\frac{n_m}{1-n_m}\right)\beta}{M}\right] \tag{12}$$

In which $d\lambda$ is a non-negative proportional coefficient.

Furthermore, dilatancy relationship in the dissipative stress space can be obtained. Generally, in the traditional plastic mechanics, the incremental plastic strain can be determine by derivative to the plastic potential function. But in the thermodynamic theory, only to obtain flow rule in the dissipative force, it is orthogonal to dissipative yield function with associated flow rule, and then transfer them to the true stress space through the back stress. In this process, does not need to consider plastic potential surface in the true stress space.

Yield Surface Characteristics in the Dissipative Stress and in the True Stress Space

For the same β and with the different Ω, the yield loci in the dissipative stress space and the true stress space are shown in Fig. 2 and Fig. 3.

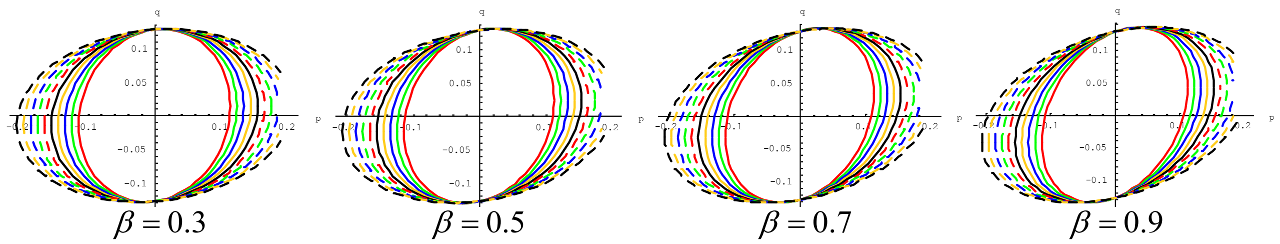


Fig. 2. The Change of the Yield Surface in the Dissipative Stress Space with Different Ω

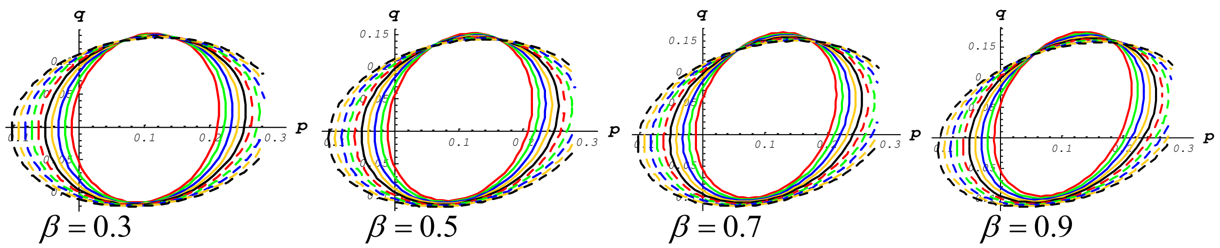
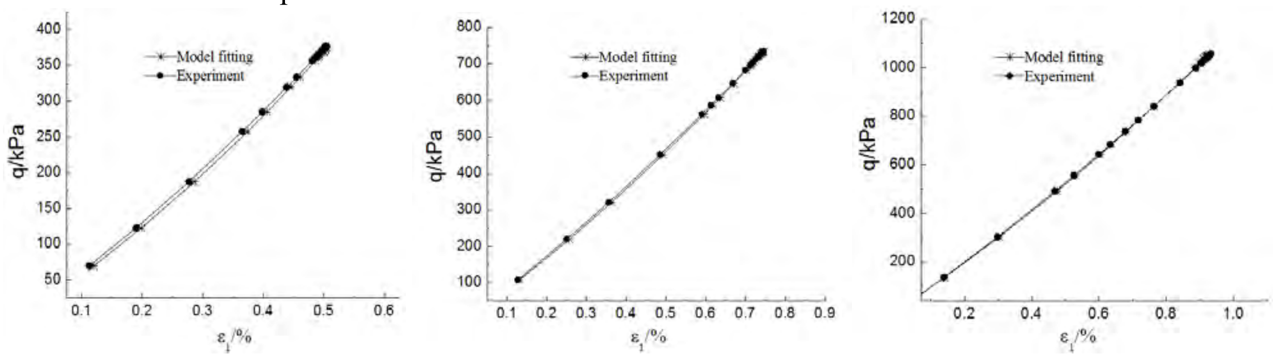


Fig. 3. The Change of the Yield Surface in the True Stress Space with Different Ω

From Fig. 2 and Fig. 3, we can see that under the different Ω, the intersection points of the different ellipse at the dissipative principle stress axis are the same point, and the different ellipses will deflect with the change of Ω. In the true stress space, the center of the ellipse is not at the origin, and it is not tangent to q axis. It indicates that fabric not only affects the deflection of the yield surface, but also affects the hardening rule.

Model Evaluation for Triaxial Tests

For consolidated drained triaxial tests, three different consolidation pressures are respectively considered for comparison. The deviatoric stress-deviatoric strain curves are shown in Fig. 4. Comparing to the calculation results of the different consolidation pressures under the different experiment conditions, we can note that the predicted responses by Modified model for all three pressures are nearly identical to stress-strain relationship curve of triaxial test.



(a) 100kPa (b) 300kPa (c) 500kPa

Fig. 4. Deviatoric Stress-deviatoric Strain Curves

Summary

Although modern developments in thermomechanics have had a large influence on many branches of mechanics, this is not yet true of geomechanics. In an effort to reach to the constitutive model taking into account more feasible features of granular materials, the thermodynamic formulation including the fabric tensors was examined. In this research, we have been mainly concerned with the representation of the void fabric tensors. It seems appropriate to indicate that we can apply the theorem to another important problems in the constitutive formulations based on the thermodynamic approaches. The theorem can be used to get anisotropic features, e. g., the anisotropic yield condition, which are of primary importance in formulating the equations. One another important application is to link the void fabric and the thermodynamic approach. Since we have less knowledge about the evolution modes of the internal variables than the stress strain relationships, as a consequence of the void fabric being used to satisfy the principle of material objectivity, seems to be more helpful in the thermodynamic formulation.

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