

# Study on the Mechanical Essence of Gyro Characteristics

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**Keyword:** Gyro Rotor, Simplified Model, Nutation, Precession, Mechanical Essence.

**Abstract.** Simplify the gyro rotor into a hollow disk, each particle in the disk circular motion around the z axis with High speed. At same time, it around y axis rotation follows the disc. That is, the disk does not rotation, but each particle in the rotor circular motion relative to the disk, at the same time, it rotate with the angular velocity  $\omega$  follows the disc. On the basis of the simplified model, apply the knowledge on the synthetic movement of theoretical mechanics; research each particle acceleration in the gyro rotor. Then according to the Newton's law to analysis the force situation of each particle in the rotor, to reveal the mechanical essence of nutation and precession characteristics of the Gyro rotor. Then, according to the situation of the applied forces on each particle in the rotor, reveal mechanics essence of nutation and precession characteristics of the gyro.

## Characteristics of the Gyro

After a high-speed free rotating gyroscope transient is disturbed, its axis deviation. But after the deviation from the axis of rotation for a period of time, the rotation shaft gradually returned to the pre disturbance location. If applying a couple in a free rotating gyro rotor, not as people imagine, the rotor will produce rotation around perpendicular vectors applied couple axis. Above two phenomena known as the nutation and precession are strange. However, how it happened? What is the essence of its mechanics? The many relevant present information state it from the Euler equations, little involve mechanical essence. It result the people on the properties of gyroscope some doubts. In this paper, from the angle of inertial coordinate system. First, apply synthetic motion knowledge to analysis the each particle acceleration of gyro rotor. Then, according to the stress situation of each particle of the rotor on the Newton's law, reveal the mechanical essence of gyro nutation and precession characteristics, and combined with concrete examples to illustrate, Is easy to understand and trust.

## The Simplified Model and the Motion Analysis of the Particles of a Free Gyro Rotor

### The Simplified Model and Object of Study

The gyro rotor with the high speed free rotate (three degree of freedom gyro), its initial state see Figure 1.

The rotor rotates around the axis z with high-speed rotation angular velocity  $\Omega$ . After being disturbed, it produced a rotation angular velocity  $\omega$  around the vertical axis y. And  $\omega \perp \Omega$ ,  $\omega$  is far less than  $\Omega$ .

See Figure 1, o-xyz is the inertial coordinate system (the earth can be regarded as the inertial coordinate system). At some moment, assume the direction of angular velocity  $\Omega$  of the disk coincides with coordinate axis z, and the direction of disturbed angular velocity  $\omega$  coincide with the coordinate axis y.

The each particle of rotor motion with high-speed in a circular around the axis z, at same time follow the rotor to rotate around the coordinate axis y. The gyro rotor can be simplified as homogeneous disc, assume that the disk without rotation, and each particle relative to the disc in a circular motion. At same time each particle motion in a circle of the disc with angular velocity rotation around y axis. The disc is the object of our study.

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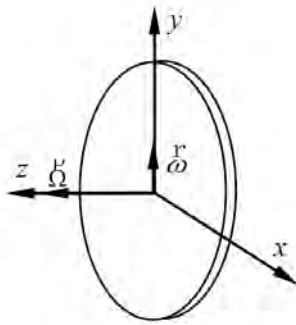


Fig. 1 The Object of Study

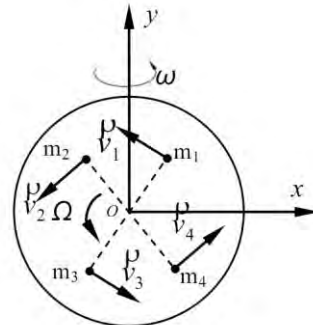


Fig.2 The Relative Velocity of Particle

### Analysis of the Motion of Each Particle on the Gyro Rotor

By the knowledge of theory mechanics can know, the each particle of the disk is at synthetic motion. We selected the particle as a motion point, motion coordinate system fixed on the disc with angular velocity  $\omega$  around axis  $y$  rotation, fixed coordinate system is  $o$ -xyz. Then the circular motion around the axis  $z$  of a particle is the relative motion, the particle rotate with the disk around  $y$  axis is implicated movement. In order to describe simple and clear, selected 4 particles, its mass is  $m_1$ ,  $m_2$ ,  $m_3$ , and  $m_4$ , its position is symmetrical, represent all the particles on the disc. See figure 2 Next we will analysis the acceleration of the 4 particles. The velocity  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  is the relative velocity of 4 particle (That is the circular motion velocity of particle). Its ranges and the disc are in a same plane. According to the composition theorem of acceleration while Implicate motion as rotational motion, Absolute acceleration of the points equal to the vector sum of its relative acceleration, transport acceleration and Coriolis acceleration.

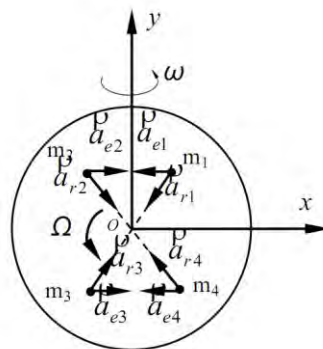


Fig.3 Each Article Acceleration

See figure 3.  $r_i$  is the distance from each particle to the rotation  $z$  axis,  $\theta$  is the angle between  $r_i$  and  $x$  axis. The particle  $m_i$  relative acceleration  $a_{ri} = r_i \Omega^2$ , Pointing to the origin  $O$ ; The particle  $m_i$  transport acceleration  $a_{ei} = r_i \omega^2 \cos \theta$ , its direction and perpendicular to the  $y$  axis; According to the

synthetic motion knowledge of theory mechanics, by each particle relative velocity  $v_1, v_2, v_3$  and  $v_4$  and the implicated angular velocity  $\omega$ , the coriolis acceleration  $a_{ci}$  of each particle can be learned.

The particle  $m_i$  coriolis acceleration  $a_{ci}=2\vec{\omega} \times \vec{v}_{ri}$ , that is the size of the particle  $m_i$  coriolis acceleration of the disc is  $a_{ci}=2r_i\omega\sin\theta$ ; According to the right-hand rule to determine the direct of it: The coriolis acceleration direction of particle  $m_1$  and  $m_2$ , is the same with the  $z$  axis (Figure 3, its direction is from the paper inside Point to the outside).

The coriolis acceleration direction of particle  $m_3$  and  $m_4$ , is the opposite with the  $z$  axis (Figure 3, its direction is from the paper outside Point to the inside).

We have analyzed the relative acceleration, transport acceleration and coriolis acceleration of the representative 4 particles, equals have analyses the absolute acceleration of all the particles on the rotor.

### Gyro Rotor Stress Analysis and Conclusion

According to Newton's second law  $F = m a$ , the relative acceleration, transport acceleration and transport acceleration of a particle to obtain on the rotor, so each particle is influenced by the corresponding force. Because of the rotation of the rotor is free, so the Particle stress can only come from a nearby other particles. That is can only exert a force to the particle by its adjacent particle. Because a particle only is correlating with its adjacent particles, by Newton's third law (Law of action and reaction), at this time there will be the force acting on a adjacent particle of the particle.

That is to say, the rotor rotation is around two axes at the same time, the relative acceleration, transport acceleration and coriolis acceleration of the rotor is objective existence. So the reaction from its adjacent particle of the particle is objective existence. Taking into account all the particles are in the rotor, so all the neighboring particle is affected by a reaction force is acting on the rotor. Here we study the situation of the adjacent particle has been reacted (actually that is the force situation of the rotor has been reacted).

See Figure 2. Because of the relative acceleration of particle point origin  $O$ , considering the symmetrical distribution of all particle of the rotor, can know their relative acceleration of every particle, and know the reaction on the adjacent particle not only are acting on the rotor, but also know the resultant force is zero.

See Figure 3. The transport acceleration of each particle  $a_{ei}=r_i\omega^2\cos\theta$  point to the  $y$  axis, Similar to 3.1, can know their transport acceleration of every particle, and know the reaction on the adjacent particle not only are acting on the rotor, but also know the resultant force is zero.

By the coriolis acceleration direction of  $m_1, m_2, m_3$ , and  $m_4$ , can know coriolis acceleration direction of each particle above the  $x$  axis are the same with the positive  $z$  axis. (Figure 3, its direction is from the paper inside point to the outside). The coriolis acceleration direction of each particle under the  $x$  axis is the opposite with the positive  $z$  axis. (Figure 3, its direction is from the paper outside point to the inside).

So can infer, the reaction to its adjacent particles is mutually parallel and pointing in the opposite of each particle by coriolis acceleration above or under the  $x$  axis. That is to say, the reaction by all adjacent particles, can be synthesized a couple of force to  $x$  axis. It is because of this couple, the rotor rotating around  $x$  axis trend, and with the passage of time, this trend makes the rotor formed angular velocity  $\omega'$  around  $x$  axis. It should be noted that, the vector direction of the angular velocity  $\omega'$  are vertical with  $\omega$  and  $\Omega$ .

### The Formation of Nutation and Precession

The above for the same reason, after the gyro rotor with angular velocity  $\omega'$  around  $x$  axis, will cause the reaction torque make the rotor to rotate around the  $y$  axis, and gradually the original around the  $Y$  axis rotation angular velocity  $\omega$  decreases, until the zero. The rotor is rotating at high-speed around the rotation axis at the same time, is rotating at around the  $x$  axis slowly. So the

cycle continues, is gyro nutation.

The situation of precession is different in form than nutation, the precession phenomenon of high speed rotating gyro disk happen under the external torque or have been implicated in the angular velocity. See Figure 2, assuming the rotor rotation rate  $\Omega$ , if applying a moment of couple  $M_y$  to the

rotor around the y axis, first the angular acceleration  $\alpha_y = \frac{M_y}{J}$  is generated, then the angular velocity  $\omega$  is generated, inevitably produce the reaction torque around x axis, and then form a rotation angular velocity  $\omega_x$  around x axis. Because of the existence of the angular  $\omega_x$ , will form a moment of the reaction force around y axis, this rotation direction of the couple moment is reverse of the applied torque, gradually offset the moment  $M_y$ . When this moment of the reaction force is equal to the moment  $M_y$ , angular velocity  $\omega_x$  reaches the maximum value. In this process, because there is always  $M_y$ , so the angular velocity  $\omega_x$  around x axis always exist. At this time the rotor rotation around x axis called precession. At this time angular velocity  $\omega_x$  is called precession angular velocity. Further study found that, the precession angular speed  $\omega_x$  is proportional to rotor rotation rate  $\Omega$  and torque  $M_y$ , and is inversely proportional to the inertia of rotor.

To sum up, when the gyro rotor on high speed rotation has implicated rotation, because of the coriolis acceleration, the reaction force of each particle on rotor generate the reaction force moment, that is the properties of mechanical essence.

## Summary

The gyro rotor is simplified into a hollow disk, the each particle of rotor motion with high-speed in a circular around the axis z, at same time follow the rotor to rotate around the coordinate axis y. That is the each particle of rotor motion with high-speed in a circular around the axis z, at same time follow the rotor to rotate around the coordinate axis y. On the basis of the simplified model, application of knowledge of theoretical mechanics and Newton's law is easy to analyze the stress state of each particle on the rotor. And then reveals the mechanical essence of gyro nutation and precession characteristics, avoid the problem that "inertia" is really or fiction force. Make the mechanical properties of gyroscope essence of clear, easy to understand.

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