FEM Analysis of Welded Spherical Joints' Stiffness

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Abstract. Welded spherical joints' stiffness has important influences on the strength and stability bearing capacity of reticulated shells. For complication of theoretical analysis and difficulty on experiments, the stiffness of welded spherical joints which is applied widely in the large-span reticulated shells, is analyzed and computed under the different conditions by finite element method with the structural analysis universal program SAP5 in this paper, the expressions of axial flexibility function f and flexural stiffness k_e of welded spherical joints with parameters: joints' diameter and thickness, members' diameter are attained by regression analysis for lots of computing results. Here, orthogonal design and orthogonal multi-item formula are accepted. These expressions can be adopted by the non-linear stability analysis of the large-span space reticulated shells with the welded spherical joints considering the influence of joints' stiffness.

Introduction

Welded spherical joints' stiffness has important influences on the stability bearing capacity of large-span spatial reticulated shells. According to the Current <Design and Construction Regulation of Latticed Shell>, the influence of joints' stiffness can be ignored for the analysis of latticed shell structure, supposing the joints are pinned and the member only bears axial force. It is generally accepted that the joints are completely pinned or rigidly connected in the stability analysis of latticed shell. In fact, the joints of space trusses or reticulated shell have certain axial and flexural rigidity, which are flexible or semi-rigid. Joints' stiffness of reticulated shell is an important factor of shell's stability and its fixing ability is of vital significance to maintaining the stability of reticulated shell. However, joints of reticulated shells constitute the some spatial load system, with the complicated loading situation. Moreover, the geometric dimension, material, technology, precision, welded quality of joints and members all have a direct effect on the joints' rigidity. Therefore, it is difficult to offer analytical solutions to stress and deformation. At present, the determination of joints' stiffness domestically and abroad has been obtained from experiments and is only to bolted spherical joint. However, in terms of large space reticulated structure, there exist many joints, and the dimension of which is the same as the connection of members. If determinations of joints' stiffness are made one after another, it will turn out to be time-consuming and unpractical. In this paper, the structural general program SAP5 is adopted to conduct finite element analysis and calculation of welded spherical joints widely applied in the large space reticulated structure. It focuses on the research of deformation of the boundary between hollow sphere and steel tube; finally the formulas of the axial flexibility coefficient f and flexural stiffness k_e are gotten by regression analysis of orthogonal polynomials.

FEM Analysis

The Mechanical Model

According to the trial result of the whole sphere, when the joints have a single load in one direction, the displacement in the other direction is one order of magnitude greater than that. Hence, it can be overlooked. When it comes to calculation of joints of hollow sphere, simplifying the

multidirectional load to unidirectional one in turn simplifies greatly the analysis. Meanwhile, it satisfies the precision required by engineering. Since the joints of hollow sphere take on symmetry in unidirectional load, the hemisphere is chosen as the mechanical model. The use of slide support and the fixed support respectively as boundary condition of symmetric plane exerts little influence on the calculation result, which conforms to limitations of stress concentration. The trial result of the whole sphere shows that both the circumferential and radial displacements of the boundary are small. Therefore, it is appropriate to make fixed side the boundary condition of symmetric plane.

Element Mesh Subdivision

The calculation in this paper adopts the structural general procedure SAP5. As the comparison of hollow sphere's thickness and diameter is within the range of thin-shell, plate shell element of three-dimensional problem in the program is used. Outside the spherical top element mesh subdivision utilizes the constant-strain triangular element while other parts employ conforming quadrilateral element. On the boundary of sphere joint and steel tube, mesh subdivision is denser. In the calculation of axial flexibility coefficient, axial load has an evenness effect on sphere joint and steel tube. Accordingly, it is a matter of axial symmetry. The paper directly seeks the answer through axially symmetrical problem.

In the calculation of flexural rigidity coefficient, the calculated model is divided into 160 elements and 161 joints, that is 20 equal divisions horizontally and 8 vertically.

Calculation of Nodal Load

In the calculation of flexural rigidity coefficient, bending moment is transformed into equivalent load; the direction of nodal load is determined by the direction of bending moment; size changes with equal proportion from XOZ plane. The force moment produced equals to the known bending moment.

After calculation of joints' vertical displacement on the boundary of sphere joint and steel tube, the conclusion is obtained about welded sphere's axial flexibility and flexural stiffness rigidity f, k_e.

Method of Analysis

In the calculation of axial flexibility coefficient f and flexural rigidity coefficient k_e, considering that f and k_e are related to many factors such as sphere diameter D, pipe diameter d, sphere thickness δ, etc, orthogonal design method is adopted to calculate f and k_e under various conditions. Furthermore, orthogonal polynomial is followed to make regression analysis about the calculation result, with a satisfying regressive formula subsequently. Because of SAP5's worse pre-processing function, the workload of filling in the data file is heavy for spherical shell element with more joints. It only needs to calculate 25 cases by using orthogonal table 25 (56), reducing the workload greatly. The calculation finds that sphere diameter D has an inconspicuous effect on axial flexibility coefficient f and flexural rigidity coefficient k_e, however, the ratio of sphere diameter D and pipe diameter d constitute the main factor. The other important factor is sphere thickness δ. Therefore, in this paper the factors of d/D and δ regress to axial flexibility coefficient f and flexural rigidity coefficient k_e.

Quadratic orthogonal polynomial is applied to the regression of calculation result. The result indicates that precision meets the requirements. Based on the regulations in Current Design and Construction Regulation of Latticed Shell, d = D/27. Thus, d/D is within the range of 0.3 to 0.5. In terms of large span reticulated structure, sphere diameter D is usually above 400 mm and sphere thickness δ over 12 mm. To improve the precision of regressive formula, δ takes 5 levels of 12 mm, 14 mm, 16 mm, 18 mm, 20 mm.

Regression Analysis

Regression of Axial Flexibility Coefficient

The value of axial flexibility coefficient f is shown in Table 1.

Tab. 1 Dates of f (10^{-6} mm/kg)

δ /mm	d/D				
	0.3	0.35	0.4	0.45	0.5
12	3.26	2.60	2.12	1.69	1.41
14	2.61	2.09	1.70	1.39	1.13
16	2.13	1.74	1.43	1.16	0.95
18	1.81	1.44	1.21	0.98	0.82
20	1.55	1.27	1.01	0.84	0.68

The regression formula of axial flexibility coefficient f can be concluded.

$$f(d/D,\delta) = \overline{f} + \phi(d/D) + \phi(\delta) + \Psi(d/D,\delta) \tag{1}$$

Where $\phi(d/D)$ and $\phi(\delta)$ are the effect functions of factors d/D and δ respectively. $\Psi(d/D,\delta)$ is the effect function of d/D and δ interactions. $\overline{f}$ is the average value of axial flexibility coefficient. In the above formula,

$$\phi(d/D) = \sum_{k=1}^{2} \alpha_{ko} P(d/D)$$

$$\phi(\delta) = \sum_{t=1}^{2} \alpha_{ot} P_t(\delta)$$

$$\Psi(d/D,\delta) = \sum_{u=1}^{2}\sum_{v=1}^{2} \alpha_{uv} P_u(d/D) P_v(\delta)$$

Where

$$P_1\left(\frac{d}{D}\right) = \left(\frac{d}{D} + 0.05 - 0.3\right)/0.05 - (5+1)/2 = 20\frac{d}{D} - 8 \quad P_2\left(\frac{d}{D}\right) = (20\frac{d}{D} - 8)^2 - (5^2 - 1)/12 = (20\frac{d}{D} - 8)^2 - 2,$$

$$P_1(\delta) = (\delta + 2 - 12)/2 - (5+1)/2 = \frac{1}{2}\delta - 8, \quad P_2(\delta) = (\frac{1}{2}\delta - 8)^2 - \frac{5^2 - 1}{12} = (\frac{1}{2}\delta - 8)^2 - 2$$

Put the data in Table of data f into the above formulae and it can be obtained that

$$f(d/D,\delta) = 1.57 - 0.32(20\frac{d}{D} - 8) + 0.04[(20\frac{d}{D} - 8)^2 - 2] - 0.28(\frac{1}{2}\delta - 8) + 0.04$$

$$[(\frac{1}{2}\delta - 8)^2 - 2] + 0.06(20\frac{d}{D} - 8) - 0.009(20\frac{d}{D} - 8)[(\frac{1}{2}\delta - 8)^2 - 2] - 0.009[(20\frac{d}{D} - 8)^2 - 2]$$

$$(\frac{1}{2}\delta - 8) + 0.001[(20\frac{d}{D} - 8)^2 - 2][(\frac{1}{2}\delta - 8)^2 - 2]$$

The significance test is followed. Regression square sum can be inferred from calculation:

$$U = \sum S_\alpha = S_{\alpha_{10}} + S_{\alpha_{20}} + S_{\alpha_{01}} + S_{\alpha_{02}} + S_{\alpha_{11}} + S_{\alpha_{12}} + S_{\alpha_{21}} + S_{\alpha_{22}} = 9.6462$$

Where $S\alpha$ is the variation of regression coefficient α.

The square sum l_{00} of f total variation is

$$l_{00} = \sum_i \sum_j f_{ij}^2 - N\overline{f}^2 = 72.7956 - 25 \times 1.57^2 = 11.1731$$

So surplus sum of squares Q is

$$Q = l_{00} - U = 11.1731 - 9.9462 = 1.5269$$

Because

$$F = \frac{9.6462/8}{1.5269/(25-8-1)} = 12.64 > F_{0.01}(8,16) = 2.57$$

Regression formula is highly significant.

According to the result of significance test, in the obtained f regression formula, $P_1(d/D)P_2(\delta), P_2(d/D)P_1(\delta), P_2(d/D)P_2(\delta)$ are eliminated. After simplification,

$$f(d/D,\delta) = 15.17 - 28.8d/D + 16(d/D)^2 - 0.7\delta + 0.01\delta^2 + 0.06(d/D)\delta \quad (10^{-6} mm/kg) \tag{2}$$

The Regression of Flexural Rigidity Coefficient

The values of flexural rigidity coefficient k_e are shown in Table 2.

Similarly, the regression formula of flexural rigidity coefficient k_e is

$$k_e(d/D) = \overline{k_e} + \phi(d/D) + \phi(\delta) + \Psi(d/D,\delta) \tag{3}$$

The regression coefficients and their variations can be calculated through the same method.

$$k_e = 0.9 - 42.76\frac{d}{D} + 61.2(\frac{d}{D})^2 - 0.15\delta - 0.0005\delta^2 + 3.4\frac{d}{D}\delta \tag{4}$$

The regression formula is also highly significant.

Tab. 2 Dates of $k_e (10^8$ kg·mm/rad)

δ /mm	d/D				
	0.3	0.35	0.4	0.45	0.5
12	4.37	6.19	8.11	10.25	12.81
14	5.76	8.1	10.46	12.95	16.15
16	7.28	10.05	12.83	15.85	19.5
18	8.77	12.01	15.19	18.63	23.28
20	10.26	14.01	17.58	21.43	26.09

Conclusion

The welded hollow spherical joint is a spatial load system. Its rigidity is related to many factors such as load condition, the dimensions of geometry and member, material, technology, precision and so on. Obviously, without considering the quality of weld seam and weld, the most important factors that determine the rigidity of joints lie in the sphere diameter D, pipe diameter d and sphere thickness δ. The calculation result in this paper suggests that axial the flexibility coefficient f and flexural rigidity coefficient k_e take on a regular change in accordance with the sphere diameter D, pipe diameter d and sphere thickness δ. The given regression formulas boast high significance. They can be applied to rigidity and stability analyses of large span reticulated structure of the welded hollow spherical joints which are influenced by joints' rigidity.

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