

Fractional Order Derivative Nishihara Model of Artificial Frozen Soil

Hui LI^{1,a}, Qiu-Jin ZHANG^{1,b}, Lian-Seng NIU^{1,c}, Ying XU^{1,d}

¹School of Civil Engineering and Architecture, Anhui University of Science and Technology ,
Huainan Anhui 232001, China

^alihui_tumu@163.com, ^bqiujiu_tumu@163.com, ^cnlsi_tumu@163.com, ^dxuying_tumu@163.com

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Abstract. Strength and creep performance is the most important in the frozen soil mechanics, It is an important basic basis on engineering foundation construction in cold region and artificial freezing method in shaft sinking. However, the nature of the frozen soil creep is complex, as well as the restriction of objective conditions, it has not been fully understood today. In order to further study the nature of frozen soil creep, for the creep model, it has other disadvantages. By replacing a Newtonian dashpot in the classical Nishihara model with fractional order derivative dashpot, fractional order derivative Nishihara creep model was constructed to simulate the steady and acceleration stage of creep. By comparing the result of fractional order derivative Nishihara creep model with trial value, the established creep model preferably reflected each stage of creep process of artificial frozen soil. It supplement better nature and theoretical of frozen soil creep.

Introduction

It is proved that reusing the artificial freezing method in shaft sinking, the creep deformation of freezing wall is one of the important reason of freezing pip rupture and freezing wall breaking. So the research on laws of the creep deformation has important significance[1].

Frozen soil is a kind of multiphase gas system material composed of solid particle, ice, liquid water and gas[2]. What's more, the cementation of ice brings the great changes in property and structure of soil. Chen Junhao et al[3] established particle swarm fractional order derivative Kelvin model of frozen soil. By comparing the calculated value of this model with the experimental result, they obtained a mechanic law which the frozen soil can be well simulated by this model. Yao Zhaoming et al[4] put the accelerated component in series to fractional order derivative Burgers model, deducing the genetic fractional order derivative accelerated Burgers creep model. Then by numerical simulation, better simulation results have been achieved.

In this paper, by replacing a Newtonian dashpot in the classical Nishihara model with fractional order derivative dashpot, fractional order derivative Nishihara creep model was constructed to simulate the steady and acceleration stage of creep. what's more, the parameter of the established model was optimized by using particle swarm optimization. By comparing the result of fractional order derivative Nishihara creep model with test value, the established creep model preferably reflected each stage of creep process of artificial frozen soil. It provides a new way for the artificial frozen soil creep calculation.

Fractional Order Derivative Nishihara Creep Model of Frozen Soil

Classical Nishihara Model

The classical Nishihara model is used to describe the relationship of stress and strain of soft soil (such as silty clay and sludge) and soft rock (mudstone and weathering bedrock and so on). This model can be used to consider the mechanical properties of material, such as elasticity, viscoelasticity and elastic-plastic etc. It has a wide filed of application and more mature theoretical derivation[5]. In geotechnical engineering, the five elements mechanical model called Nishihara model. This model consists of a Bingham body and a Kelvin body which are connected in series, see figure 1.

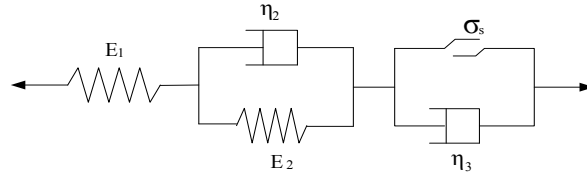


Fig.1 Classic Nishihara Model

The constitutive equation of steady-state Nishihara model:

$$\varepsilon(t) = \frac{\sigma}{E_0} + \frac{\sigma}{E_1} \left[1 - \exp\left(-\frac{E_1}{\eta_1} t\right) \right] \quad (\sigma < \sigma_s) \quad (1)$$

The constitutive equation of accelerated Nishihara model:

$$\varepsilon(t) = \frac{\sigma}{E_0} + \frac{\sigma}{E_1} \left[1 - \exp\left(-\frac{E_1}{\eta_1} t\right) \right] + \frac{\sigma - \sigma_s}{\eta_2} t \quad (\sigma \geq \sigma_s) \quad (2)$$

In the formula, σ and $\varepsilon(t)$ are respectively the total stress and total strain of the model; η_1 and η_2 are viscosity parameters of the material; E_0 and E_1 are elasticity parameters of the material; σ_s is stress threshold of the material; t is time.

Fractional Order Derivative Nishihara Model

Fractional Order Calculus Creep Elements. Using the fractional order derivative creep model theory is to use the Abel body to replace the Newton viscous fluid of Bingham body in classical model theory. See figure 2. The constitutive relation of Abel dashpot is:

$$\sigma(t) = \xi \frac{d^r [\varepsilon(t)]}{dt^r} \quad (0 \leq r \leq 1) \quad (3)$$

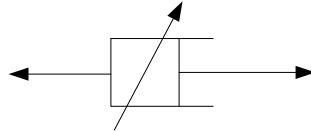


Fig.2 Abel Dashpot

In the formula, ξ is viscosity coefficient. When r equals one, this element is Newton dashpot, representing the ideal fluid; when r equals zero, this element becomes the ideal solid. We can know that the constant coefficients Abel dashpot is a model of simulating material between ideal fluid and ideal solid.

When $\sigma(t)$ is the constant coefficients, that is, to keep the stress under the condition of constant, the element model can be used to describe the creep properties of materials and to fraction integrate type (3) on both sides. According to the operator theory of fractional order calculus Riemann-liouville model, we get:

$$\varepsilon(t) = \frac{\sigma_c}{\xi} \frac{t^r}{\Gamma(1+r)} \quad (0 \leq r \leq 1) \quad (4)$$

This formula is rheological equation which the constant coefficient Abel dashpot describes.

The Equation of Fractional Order Calculus Creep Nishihara Model. Fractional order calculus creep Nishihara components are shown in figure 3.

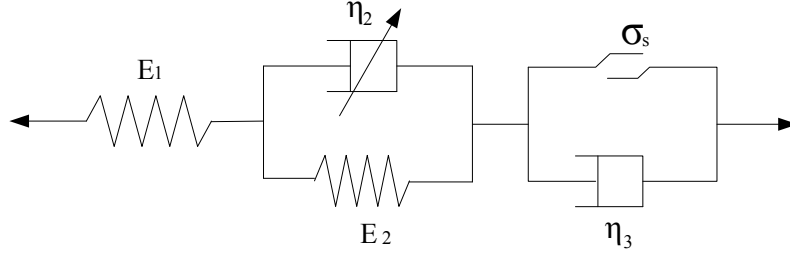


Fig.3 The Fractional Derivative Nishihara Model

The constitutive equation of fractional order derivative steady-state Nishihara model of frozen soil creep model is:

$$\varepsilon(t) = \frac{\sigma_0}{E} \left[1 + \sum_{n=1}^{\infty} (-1)^n \frac{(t/\tau)^n}{\Gamma(1+rn)} \right] \quad (\sigma < \sigma_s) \quad (5)$$

The constitutive equation of fraction order derivative accelerated Nishihara model of frozen soil creep model is:

$$\varepsilon(t) = \frac{\sigma_0}{E} \left[1 + \sum_{n=1}^{\infty} (-1)^n \frac{(t/\tau)^n}{\Gamma(1+rn)} \right] + \frac{\sigma - \sigma_s}{\eta_2} t \quad (\sigma \geq \sigma_s) \quad (6)$$

Fractional Order Derivative Model of Artificial Frozen Soil Creep

Artificial Frozen Soil Creep Test

Representative soil samples is taken form the huainan mining area: clay; the depth selection is 300m~310m; the moisture content of clay is 22%. First, the sample is processed as the standard samples which the size is 50mm (diameter) * 100mm (height). Creep tests were performed in Frozen Soil Research Institute of Anhui University of Science and Technology. The equipment is WDT-100 microcomputer control electro-hydraulic servo frozen soil testing machine which is developed by Frozen Soil Research Institute of Anhui University of Science and Technology. The maximum loading capacity of test equipment is 10 tons; the accuracy is 1%. The test load and test date is automatically controlled and collected by computer according to the given parameters.

The creep test is performed in -5°C , -10°C , -15°C three temperature level respectively. The test load is performed according to $0.3 \sigma_s$, $0.5 \sigma_s$, $0.7 \sigma_s$ (σ_s —uniaxial strength) three load level and so on.

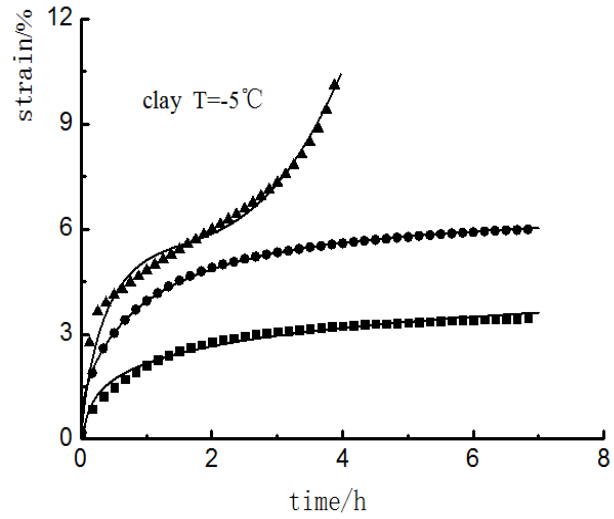
Uniaxial Creep Fractional Order Derivative Model of Frozen Soil

The parameter of fractional order derivative creep model is optimized by particle swarm algorithm. The particle swarm algorithm parameters are shown in table 1. The parameters of fractional order derivative Nishihara creep model are shown in table 2.

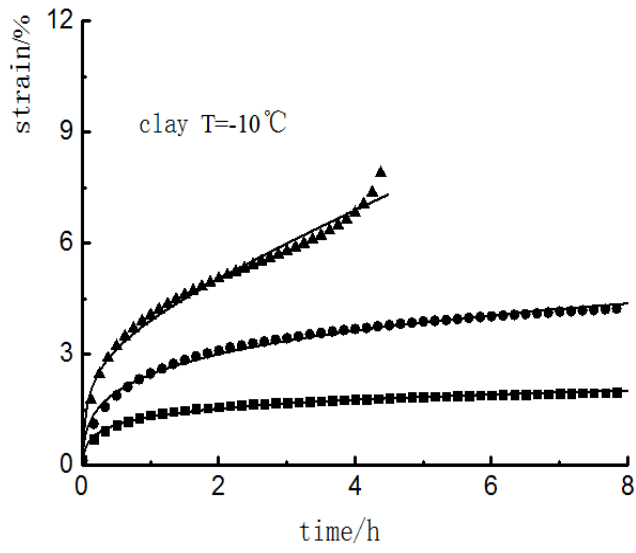
Tab.1 Particle Swarm Parameters

The population size	The near population size	C_1	C_2	The maximum acceleration
70	14	2.9	2.9	14

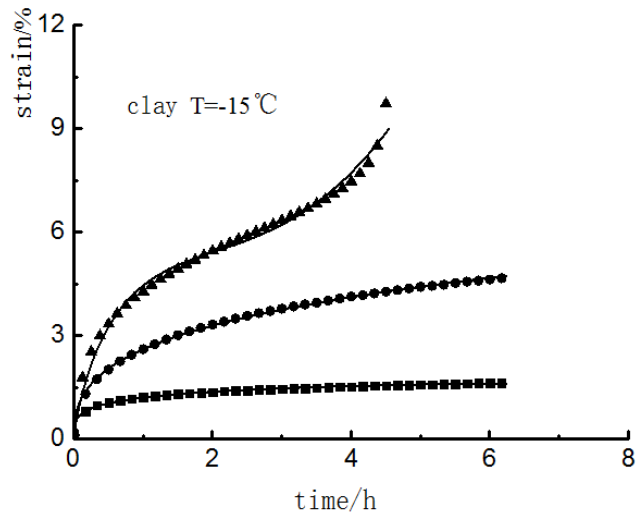
The model calculation values and test values of particle swarm optimization fractional order derivative Nishihara model are shown in figure 4.



a



b



c

Fig.4 Test Values and the Model Calculation Values

1- σ =1.05 MPa, test value; 2- σ =1.75 MPa, test value; 3- σ =2.45 MPa, test value; 4- σ =1.5 MPa, test value; 5- σ =2.5 MPa, test value; 6- σ =3.5 MPa, test value; 7- σ =2.1 MPa, test value; 8- σ =3.5 MPa, test value; 9- σ =4.9 MPa, test value; Note: ▲ ●

■ are the corresponding numerical values.

Tab.2 The Parameters of Particle Swarm Fractional Order Derivative Nishihara Model

The temperature	The loading coefficient	σ	$E_1[MPa]$	η_2	r	$\sigma_s[MPa]$
-5	0.3	1.05	0.06	—	0.1116	—
	0.5	1.75	0.29	—	0.71	—
-10	0.3	1.5	0.24	—	0.0067	—
	0.5	2.5	1.15	—	0.0196	—
-15	0.3	2.1	0.65	—	0.3117	—
	0.5	3.5	0.18	—	0.3901	—
-5	0.7	2.45	0.000645	0.000631	1.0493	2.38
-10	0.7	3.5	0.11855	1.0638	0.00810	3.25
-15	0.7	4.9	0.01117	0.00546	1.138	4.74

Conclusions

(1)The fractional order derivative Nishihara model is established, which is based on fractional order derivative and integer order derivative. It has the characteristics of simple and useful method, less model parameters and high precision;

(2)Through comparative analysis of the fitting result and numerical simulation value of fractional order derivative Nishihara model, found that the creep characteristic of frozen soil can be well reflected by fractional order derivative Nishihara model which the theory of fractional calculus and integer order calculus are unified.

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