

## Practical Calculation Method of Lateral Displacement for High-rise Structures Considering P- $\Delta$ Effect

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**Abstract.** In order to facilitate solving lateral displacement of column cap for various kinds of high-rise structures, and provide reliable basis for engineering application, practical calculation formulas were derived for lateral displacement of column cap, based on the equilibrium differential equation of the beam, energy principle and equivalent substitution method. Dead load was treated with two conditions: considering and without considering. Equal section and variable section were both considered, as the column cap suffered with vertical and horizontal forces. Three example were given with comparison to present literatures and FEM. Calculation results have shown that the proposed method has not only the succinct formulas but also high precision. It can meet the need of practical calculation works.

### Introduction

In nowadays, research and development of engineering materials and the progress of the construction methods makes more and more high-rise structures in modern civil engineering. For example, the pylons of long span bridges, the piers of bridges over deep-cutting gorge and urban grade separation, various communication and transmission towers, etc. Except suffered vertical loads from column cap by other structures, these structures may also suffered horizontal loads. When the lateral displacement of structure occur in effect under lateral forces, the vertical force will cause additional moment and lateral displacement, so that the total moment and lateral displacement increases, which is called the P- $\Delta$  effect[1]. A simple, practical calculation method of lateral displacement of these structures considering the P- $\Delta$  effect, will not only provide a basis for preliminary design of these structures, but also provide the checking means for finite element method calculation[2-3].

### Formula Derivation

#### Formula for Equal Section Ignoring Dead Load

An equal section column with its lower end fixed, the upper end free. And its dead load ignored. The lateral bending stiffness is  $EI$ . The lateral displacement of the column cap is  $\delta$ , when it suffered with the vertical and horizontal load are  $H$  and  $V$ .

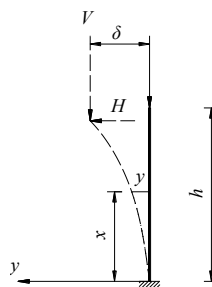


Fig.1 Calculating Diagram of Equal Section Column Ignoring Dead Load

From Fig.1 the equilibrium differential equation of the beam can be expressed as follows

$$EIy'' = V(\delta - y) + H(h - x) \quad (1)$$

Let be  $k = \sqrt{V/EI}$ , the general solution of Eq. (1) can be expressed as follows

$$y = A \cos kx + B \sin kx + \delta + \frac{H}{V}(h - x) \quad (2)$$

$A$  and  $B$  were the undetermined coefficient in Eq. (2), and could be determined by the boundary conditions. Deformation curve equation of the equal section column could be obtained after substituting of the corresponding boundary condition, as follows

$$y = -\left(\delta + \frac{H}{V}h\right) \cos kx + \frac{H}{Vk} \sin kx + \delta + \frac{H}{V}(h - x) \quad (3)$$

Then substituting  $x=h$ , the lateral displacement of column cap could be obtained as follows

$$\delta = \frac{H}{V} \left( \frac{1}{k} \tan kh - h \right) \quad (4)$$

### Formula for Equal Section Considering Dead Load

Assuming that the column formed by the same homogeneous materials, because of the equal section, dead load evenly distributed along the column height, the value is  $q$ , shown in Fig.2.

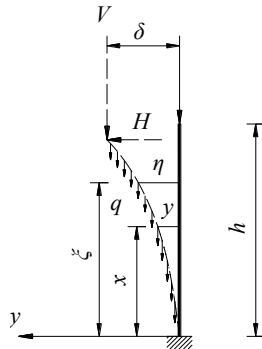


Fig.2 Calculating Diagram of Equal Section Column Considering Dead Load

The equilibrium differential equation of the beam can be expressed as follows

$$EIy'' = V(\delta - y) + H(h - x) + \int_x^h q(\eta - y) d\xi \quad (5)$$

Eq. (5) can be reduced to the solution of Bessel equations after derivation calculus to both sides and variable substitution[4][5]. But the Bessel function is too complex for practical calculation. Therefore, using energy method to simplify the issue. The idea to simplify the issue is as follows: first, the dead load of the column is equivalent to a cap concentrated force, then using Eq. (4) can be solved. Assuming the vertical concentrated force on the column cap  $V=0$ , and the column deformation curve in the action of dead load and the horizontal force on the column cap is as follows

$$y = \delta \left( 1 - \cos \frac{\pi x}{2h} \right) \quad (6)$$

General potential energy of the structure can be written as

$$\Pi = U + V \quad (7)$$

Where

$$U = \frac{\pi^4}{64} \frac{EI}{h^3} \delta^2 \quad (8)$$

$$V = -\frac{\pi^2 - 4}{32} q \delta^2 - H \delta \quad (9)$$

The equilibrium conditions of structure,  $\delta II=0$ , by substitution Eq. (8) and Eq. (9) into Eq. (7), and derivations of  $\delta$ , the lateral displacement of column cap is as follows

$$\delta = \frac{32Hh^3}{\pi^4 EI - (2\pi^2 - 8)qh^3} \quad (10)$$

Then expanding Eq. (4) with  $\tanh$  by Taylor formula, and taking the first three items, there are

$$\delta \approx \frac{Hh^3}{3EI} + \frac{2HVh^5}{15(EI)^2} \quad (11)$$

The first item on the right side of Eq. (11) is the displacement caused by independent action of  $H$ , while the second item is the additional displacement caused by  $V$ . Eq. (10) could be also converted as follows

$$\delta \approx \frac{Hh^3}{3EI} + \frac{32(2\pi^2 - 8)Hqh^6}{\pi^4 EI [\pi^4 EI - (2\pi^2 - 8)qh^3]} \quad (12)$$

For the displacement on the top of column is equal, the approximate relationship between dead load and concentrated force is as follows

$$V = \frac{EIqh}{3.368EI - 0.406qh^3} \quad (13)$$

Thus, the method of calculating lateral displacement of column cap when the column suffering horizontal and vertical load on the cap, and dead load evenly distributed along the column height is as follows: first, take dead load equivalent to a top vertical force by Eq.(13), second, calculating the lateral displacement by Eq.(4). Here, the vertical load is the sum of original vertical force and the equivalent vertical force.

### Formula for Variable Section Considering Dead Load

Except using equal section form, variable section form could also be used in column of high-rise structures in engineering. For mechanical and delight reason, variable section form would be used. Variable cross-section is commonly used with the bending moment of inertia by power index changes continuously and stepped section form, seen in Fig.3.

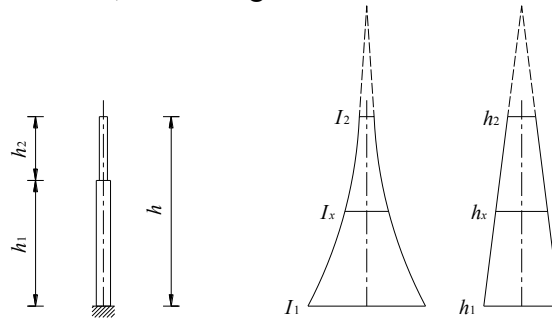


Fig.3 Schematic diagram of Variable Section Column in Engineering

The equilibrium differential equation of variable section column considering dead load of the beam is as follows

$$EI_x y'' = V(\delta - y) + H(h - x) + \int_x^h q_x(\eta - y) d\xi \quad (14)$$

Solving Eq. (14) for an analytical solution in mathematics will be very difficult or even impossible. The solution approach is still to simplify. First of all, to obtain only the action of horizontal force on the cap using conjugate beam method or other methods, displacement is equal to the uniform column with a bending moment of inertia  $I$ . Secondly, simplified uniformly distributed load to  $q$  for the dead load, according to the principle of the equal column total weight. Thus, variable section column can be simplified to a uniform column calculation.

## Examples

### Example 1

An equal cross section of  $600\text{mm} \times 600\text{mm}$  column with height of  $10\text{m}$ , the elastic modulus is  $30\text{GPa}$ , and dead load is ignored. The vertical and horizontal load on the cap of the column is  $V=500\text{kN}$  and  $H=1\text{kN}$ . Solving the lateral displacement of the column cap. The suggested method, present literature methods and FEM calculation results were shown in Tab.1.

Tab.1 Calculation Results for Equal Section Ignoring Dead Load

Lateral Displacement	Literature[1]	Literature[3]	The Suggested Method	FEM
$\delta$	0.0011471	0.0010816	0.0010965	0.0010955

### Example 2

The structure size and material parameters were the same as example 1, considering the dead load, the value is  $25\text{kN/m}^3$ , the calculation results were shown in Tab.2.

Tab.2 Calculation Results for Equal Section Considering Dead Load

Lateral Displacement	Literature[1]	Literature[3]	The Suggested Method	FEM
$\delta$	-	0.001082	0.0011075	0.11081

### Example 3

A steel pylon with height of  $195\text{m}$ , single box and multi-cell, the size across bridge is  $5\text{m}$ , and the size of longitudinal direction is  $6.6\text{m}$  on the top to  $35\text{m}$  at the bottom. Elastic modulus of steel is  $E=206\text{GPa}$ , the vertical and horizontal load on the top is  $V=155144\text{kN}$ ,  $H=100\text{kN}$ , the calculation results were shown in Tab.3.

The equivalent bending stiffness and equivalent weight are  $I=70.55\text{m}^4$  and  $q=350.8\text{kN/m}$ .

Tab.3 Calculation Results for Vertical Section Considering Dead Load

Lateral Displacement	Literature[1]	Literature[3]	The Suggested Method	FEM
$\delta$	-	0.020061	0.021855	0.022354

## Conclusions

Practical calculation formulas were derived for lateral displacement of column cap, based on the equilibrium differential equation of the beam, energy principle and equivalent substitution method. Equal section and variable section were both considered, as the column cap suffered with vertical and horizontal forces. The conclusions are as follows:

(1) For not considering uniform string weight, the suggested method does not introduce any assumptions and equivalent replacement, is a precise method in theory.

(2) For considering the uniform string weight, different from present literature premature introduction of cylinder deformation curve equation, assuming that introduced only considering the weight influence, thus improves the accuracy of calculation.

(3) The suggested method implements all kinds of high-rise structure in the common role of top vertical and horizontal load under the lateral displacement calculation, practical formulas are simple, good precision, and can meet the need of engineering.

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