Normalized Hysteretic Energy Demand of SDOF System Subjected to Earthquake Excitation

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Abstract. Currently, the methods of energy-based seismic design have been widely developed. As cumulative response, hysteretic energy can be regarded as an index directly to estimate the earthquake induced structural damage, so it is important for researchers to analyze hysteretic energy demand of SDOF systems. For obtaining the mean demand of hysteretic energy, hysteretic energy is normalized as the ratio of hysteretic energy to square of peak ground acceleration, on the basis of which the spectra of normalized hysteretic energy of constant ductility factors is established. The earthquake motion records of three kinds of soil sites are selected as excitations of the SDOF system to analyze the regularities of mean demands of normalized hysteretic energy. The influence factors of hysteretic energy demand such as period, ductility factor and soil site are analyzed. Analytic results show that the spectra have the typical spectral pattern features for different soil sites, and the normalized hysteretic energy demand is affected obviously by ductility factor and soil site.

Introduction

The current seismic design method allows the structure to undergo inelastic deformation of structures, so the demand of displacement has been regarded as the important index to evaluate the performance of structures. But, as we known, the cumulative damage of structures under earthquake excitations can not be interpreted as deformation demands, but rather as cumulative energy.

After pioneering works of Housner, the energy-based seismic design (EBSD) has been developed rapidly. Arroyo and Ordaz[1] proposed an EBSD procedure to estimate hysteretic energy demands of structures. Prasanth, Ghosh and Collins[2] used the procedure of modal pushover analysis to estimate hysteretic energy of structures. Benavent-Climent[3] proposed an EBSD procedure for retrofitting existing framed structures. Habibi, Chan and Albermani[4] proposed an EBSD procedure for retrofitting structures with passive energy dissipation systems. In addition, in order to analyze the trends of earthquake energy demand, some studies have been developed rapidly, such as earthquake input energy spectra[5, 6], hysteretic energy spectra[7], momentary absorbed energy spectra[8], inelastic cyclic demand spectra[9] etc.

Generally, hysteretic energy can be regarded as an index to estimate the earthquake damage of structures. So, this research is to develop the expressions for computing hysteretic energy demands.

Fundamental

The energy balance equation of the SDOF system subjected to earthquake excitations $\ddot{u}_g(t)$ can be expressed as:

$$\int_0^t m\ddot{x}(t)\dot{x}(t)dt + \int_0^t c\dot{x}(t)\dot{x}(t)dt + \int_0^t f(t)\dot{x}(t)dt = -\int_0^t m\ddot{u}_g(t)\dot{x}(t)dt \tag{1}$$

in which $x(t)$ is instantaneous displacement of the system, m, c and $f(t)$ are mass, damping coefficient and restoring forces of the system, respectively. The restoring force property of the system is defined as ideal elastic-plastic. The yield forces of the system is determined by $f_{yie} = k \cdot x_{yie}$, in which k and x_{yie} are elastic

stiffness and yield displacement. The relation that $u(t)$ equals $x(t)/x_{yie}$ is defined, and the Eq.1 can be deduced as:

$$\int_0^t \ddot{u}(t)\dot{u}(t)dt + 2\xi\omega\int_0^t \dot{u}(t)\dot{u}(t)dt + \omega^2\int_0^t \frac{f(t)}{f_{yie}}\dot{u}(t)dt = -\omega^2\int_0^t \frac{\ddot{u}_g(t)}{f_{yie}}\dot{u}(t)dt \tag{2}$$

where ω and ξ are natural vibration frequency and damping ratio of the system, respectively, in which ω equals $(k/m)^{0.5}$. The peak of elastic force of the system is expressed as $f_{e,max}=m\cdot\beta\cdot\max|\ddot{u}_g(t)|$, in which β is elastic amplification coefficient spectra. The strength reduction factors is expressed as $R=f_{e,max}/f_{yie}$. The $f_{e,max}$ is substituted into the relationship of $R=f_{e,max}/f_{yie}$, and the yield displacement of the system can be expressed this $x_{yie}= (m\cdot\beta\cdot\max|\ddot{u}_g(t)|)/(\omega^2\cdot R)$. Both sides of the Eq. 2 is multiplied by f_{yie} , and the above mentioned equation of xyie is substituted into this adjusted equations, and then the following equation is applied:

$$\frac{\beta^2\left(\max|\ddot{u}_g|\right)^2}{\omega^4 R^2}\int_0^t \ddot{u}(t)\dot{u}(t)dt + \frac{2\xi\beta^2\left(\max|\ddot{u}_g|\right)^2}{\omega^3 R^2}\int_0^t \dot{u}(t)\dot{u}(t)dt + \frac{\beta^2\left(\max|\ddot{u}_g|\right)^2}{\omega^2 R^2}\int_0^t \frac{f(t)}{f_{yie}}\dot{u}(t)dt$$

$$= -\frac{\beta\max(|\ddot{u}_g|)}{\omega^2 R}\int_0^t \ddot{u}_g(t)\dot{u}(t)dt \tag{3}$$

For establishing the 'mean' energy spectra, each energy component of Eq.3 is divided by $(\max|\ddot{u}_g(t)|)^2$, and then the Eq.3 can be deduced as:

$$\frac{\beta^2}{\omega^4 R^2}\int_0^t \ddot{u}(t)\dot{u}(t)dt + \frac{2\xi\beta^2}{\omega^3 R^2}\int_0^t \dot{u}(t)\dot{u}(t)dt + \frac{\beta^2}{\omega^2 R^2}\int_0^t \frac{f(t)}{f_{yie}}\dot{u}(t)dt = -\frac{\beta}{\omega^2 R}\int_0^t \frac{\ddot{u}_g(t)}{\max(|\ddot{u}_g(t)|)}\dot{u}(t)dt \tag{4}$$

The Eq.4 may be simplified as:

$$e_K(t) + e_D(t) + e_H(t) = e_I(t) \tag{5}$$

in which $e_K(t)$, $e_D(t)$, $e_H(t)$ and $e_I(t)$ are normalized kinetic energy, viscous damping energy, hysteretic energy and input energy. The relationship between normalized energy demand $e(t)$ and corresponding energy demand $E(t)$ of SDOF system is $E(t)=e(t)/(\max|\ddot{u}_g(t)|)^2$. The Eq.4 or the Eq.5 can be regarded as the equation for solving the demand of normalized hysteretic energy. In addition, the ductility factor μ of the system can be expressed as $\mu= \max|x(t)|/x_{yie}$ for equation(1), so $\mu=\max|u(t)|$ is given for Eq.4.

Analysis of Normalized Hysteretic Energy Demand

For analyzing the normalized hysteretic energy demand, the earthquake motion records for hard soil site, intermediate soil site and soft soil site are selected[10]. The mean amplification coefficient β spectra based on the selected earthquake motion records are illustrated as Fig.1.

The regularities of normalized hysteretic energy demand can be obtained by analyzing the relationship between normalized hysteretic energy and natural period of SDOF system which can be called normalized hysteretic energy spectra. The normalized hysteretic energy spectra of constant ductility factors for the three soil sites are illustrated in Fig. 2, where the ductility factors μ are respectively assumed as 1.5, 2, 3, 4 and 6. As shown in Fig. 2, the influence of ductility factor on hysteretic energy demand is obviously, and the general trend is that hysteretic energy demands increase gradually with the increase of ductility factors. Furthermore, the spectral curves of the three soil sites are all composed of upward stage, peak platform stage and downward stage. The period ranges of different curve stages are affected obviously by ductility factor, so, considering ductility factor μ equals 3, the period ranges of different curve stages are given as: (1) As for hard soil site, the periods are from about 0 sec to 0.5 sec for upward stage, from about 0.5 sec to 0.8 sec for peak platform stage and from about 0.8 sec to 5.0 sec for downward stage. (2) As for intermediate soil site, the period ranges for the three curve stages are about 0~0.6sec, 0.6~2.2sec and 2.2~5.0sec, respectively. (3) As for soft soil site, the corresponding period

ranges are about 0~3.3sec, 3.3~5.5sec and 5.5~7.0sec, respectively. The downward stages of hard soil site is more sharply than that of intermediate soil site and soft soil site.
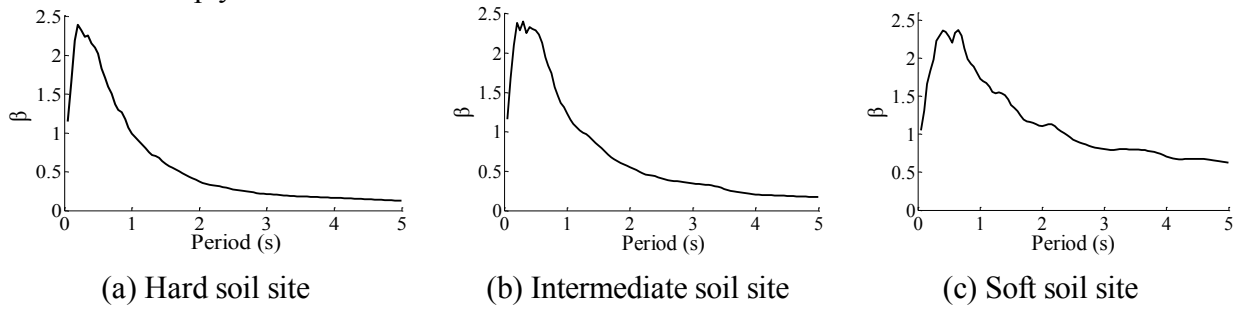

(a) Hard soil site (b) Intermediate soil site (c) Soft soil site

Fig. 1 The Mean Amplification Coefficient β Spectra Based on the Selected Earthquake Motion Records

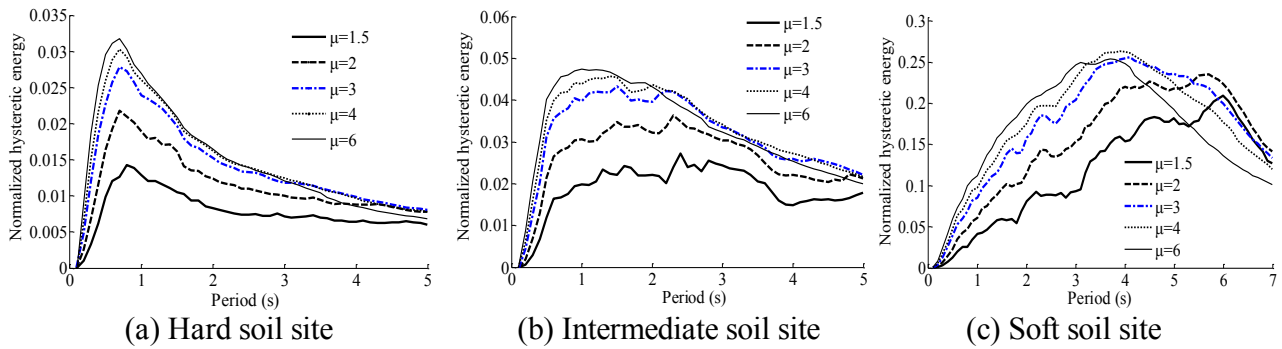(a) Hard soil site (b) Intermediate soil site (c) Soft soil site

Fig. 2 Normalized Hysteretic Energy Demand of SDOF System Subjected to Earthquake Excitations

Conclusion

The mean normalized hysteretic energy spectra are calculated based on the selected earthquake motion records. The influences of ductility factor and soil site on the demand of normalized hysteretic energy are analyzed. Some conclusions are given as follows:

(1) The normalized hysteretic energy spectra have the typical spectral pattern features for different soil sites. The spectral curves of the three kinds of soil sites are all composed of upward stage, peak platform stage and downward stage, and the control natural periods of each stage are affected obviously by ductility factors.

(2) The influence of ductility factor on normalized hysteretic energy demand is obviously, and the general trend is that energy demands increase gradually with the increase of ductility factors.

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References

[1]D. Arroyo, M. Ordaz, On the estimation of hysteretic energy demands for SDOF systems, Earthquake Engineering and Structural Dynamics, 36(2007), 2365-2382.

[2]T. Prasanth, S. Ghosh, K. R. Collins, Estimation of hysteretic energy demand using concepts of modal pushover analysis, Earthquake Engineering and Structural Dynamics, 37(2008), 975-990.

[3]A. Benavent-Climent, A energy-based method for seismic retrofit of existing frames using hysteretic dampers, Soil Dynamics and Earthquake Engineering, 31(2011), 1385-1396.

[4]A. Habibi, R. W. K. Chan and F. Albermani, Energy-based design method for seismic retrofitting with passive energy dissipation systems, Engineering Structures, 46(2013), 77-86.

[5]G. G. Amiri, G. A. Darzi and J. V. Amiri, Design elastic input energy spectra based on Iranian earthquakes, Canadian Journal of Civil Engineering, 35(2008), 635-646.

[6]A. Benavent-Climent, F. Lopez-Almanse, D. A. Bravo-Gonzalez, Design energy input spectra for moderate-to-high seismicity regions based on Colombian earthquakes, Soil Dynamics and Earthquake Engineering, 30(2011), 1129-1148.

[7]R. Riddell, J. E. Garcia, Hysteretic energy spectrum and damage control, Earthquake Engineering and Structural Dynamics, 30(2001), 1791-1816.

[8]Y. Hagiwara, Momentary energy absorption and effective loading cycles of structures during earthquakes, The 15th World Conference on Earthquake Engineering, 2000, Auckland, New Zealand, January.

[9]S. K. Kunnanth, H. Chai, Cumulative damage-based inelastic cyclic demand spectrum, Earthquake Engineering and Structural Dynamics, 33(2004), 499-520.

[10]F. Wang, Studies on performance-based seismic design methods of structures subjected to multi-dimensional earthquake excitations. PHD Dissertation, Dalian University of Technology, Dalian, China, 2007.