



$$R = \frac{2e}{\mu_0 S} \quad (2)$$

Electromagnetic induction can be written as

$$B = \frac{\phi}{S \cos \varphi} \quad (3)$$

Where  $\varphi$  is the angle between the magnetic field lines and solenoid vertical surface  $S$ . When  $\varphi = 0$ , we get

$$B = \frac{\phi}{S} \quad (4)$$

Gravity is

$$F = \frac{B^2 S}{\mu_0} \quad (5)$$

In addition, from  $NI = R\phi$ , we can obtain

$$B = \frac{\phi}{S} = \frac{NI}{RS} \quad (6)$$

#### IV. LINEARIZATION AT $e(t) = e_0$

Electromagnetic force  $F$  was determined by the coil current and air gap distance. By assuming that the distance of suspended slider  $M$  is still very small compared to  $e_0$ , so the affect can be ignored. Here the system can be simplified as the following approximation formula of a linear system:

$$F(x, I) = F(x = e_0, I = I_0) + k_1 I(t) + k_2 x(t) \quad (7)$$

$F = Mg$ ,  $g$  is known as the acceleration of gravity, the current expression of block hanging in the distance  $x = e_0$  is

$$I_0 = \frac{2e_0}{N} \sqrt{\frac{Mg}{\mu_0 S}} \quad (8)$$

The coefficients can be written as

$$k_1 = \frac{dF}{dI} = \frac{d}{dI} \left( \frac{\mu_0 S N^2 I^2}{4e_0^2} \right) = \frac{\mu_0 S I_0 N^2}{2e_0^2} \quad (9)$$

$$k_2 = \frac{dF}{dx} = -\frac{dF}{de} = -\frac{d}{de} \left( \frac{\mu_0 S N^2 I^2}{4e_0^2} \right) = \frac{\mu_0 S I_0^2 N^2}{2e_0^3} \quad (10)$$

By Newton's first law, the expression can be obtained

$$\sum F = M\gamma$$

$$k_1 I(t) + k_2 x(t) = M \frac{d^2 x}{dt^2} \quad (11)$$

Because the electromagnetic force can be controlled by the current or voltage, so we can write the transfer function between the displacement  $x(t)$ , current  $I(t)$  and the amplifier output voltage  $U(t)$ .

##### A. electromagnetic-current control

$$k_1 I(t) + k_2 x(t) = M \frac{d^2 x(t)}{dt^2} \quad (12)$$

The Laplace transform of the equation is as follows:

$$k_1 I(s) + k_2 X(s) = Ms^2 X(s) \quad (13)$$

Namely

$$\frac{X(s)}{I(s)} = \frac{k_1}{M} \times \frac{1}{s^2 - \frac{k_2}{M}} \quad (14)$$

##### B. Electromagnetic-voltage control

$$U(t) = rI(t) + \frac{d\phi(t)}{dt} \quad (15)$$

Among them

$$\begin{cases} \phi(t) = L(t)I(t) \\ L(t) = L_0 - \frac{dL}{dx} \end{cases} \quad (16)$$

A voltage control expression is given

$$U(t) = rI(t) + L_0 \frac{dI(t)}{dt} - I_0 \frac{dL(t)}{dx} \times \frac{dx(t)}{dt} \quad (17)$$

Among them

$$L = \frac{N\phi}{I} = \frac{N^2}{R} = \frac{\mu_0 N^2 S}{2e} \quad (18)$$

$$\frac{dL}{dx} = -\frac{dL}{de} = -\frac{\mu_0 N^2 S}{2e} \quad (19)$$

That is

$$U(t) = rI(t) + L_0 \frac{dI(t)}{dt} + \frac{\mu_0 N^2 S I_0}{2e} \times \frac{dx(t)}{dt} \quad (20)$$

So

$$U(t) = rI(t) + L_0 \frac{dI(t)}{dt} + k_1 \frac{dx(t)}{dt} \quad (21)$$

By the Laplace transform, we get

$$U(s) = rI(s) + L_0 sI(s) + k_1 sX(s) \quad (22)$$

We can get the following the transfer function between the displacement and the electromagnetic control voltage:

$$\frac{X(s)}{U(s)} = \frac{k_1}{k_1^2 s + (r + L_0 s)(Ms^2 - k_2)} \quad (23)$$

Let  $g = 10m/s^2$ ,  $e_0 = 5 \times 10^{-3} m$ ,  $N = 1000$ ,  $M = 1kg$ ,  $\mu_0 = 4\pi \times 10^{-7} H/m$ ,  $S = 4 \times 10^{-4} m^2$ , it can be calculated

$$\begin{cases} I_0 = \frac{2e_0}{N} \sqrt{\frac{Mg}{\mu_0 S}} \approx 1.4A \\ k_1 = \frac{\mu_0 SI_0 N^2}{2e_0^2} \approx 14.07 \\ k_3 = \frac{\mu_0 SI_0^2 N^2}{2e_0^3} \approx 3941 \\ L_0 = \frac{\mu_0 N^2 S}{2e_0} \approx 50mH \end{cases} \quad (24)$$

The transfer function formula  $\frac{X(s)}{U(s)}$  can be written as:

$$G(s) = \frac{X(s)}{U(s)} = \frac{14}{0.05s^3 + 2s^2 - 1.05s - 7882} \quad (25)$$

## V. DESIGN OF PI CONTROLLER

Because the coil inductance  $L$  is equal to the constant  $L_0$ , the coil current was produced by the voltage  $V_m(t)$  conversion when the coefficient  $k$  equals 0.04,:

$$G_1(s) = \frac{I(s)}{V(s)} = \frac{1}{r + \tau s} = \frac{0.5}{1 + 0.025s} \quad (26)$$

To make  $I_0$  is equal to 1.4A, we assumed  $V_{ref} = 56mV$  as a reference. The closed-loop transfer function is as follows:

$$\frac{V_m(s)}{V_{ref}(s)} = \frac{kKA}{1 + kKA} \times \frac{1}{1 + \frac{\tau}{1 + kKA} s} \quad (27)$$

PI compensator as follows was used:

$$C(s) = A + \frac{I}{s}$$

So, the transfer function of the control system can be written as

$$\frac{V_m(s)}{V_{ref}(s)} = \frac{1 + \frac{A}{I} s}{1 + \frac{(1 + kKA)}{kKI} s + \frac{\tau}{kKI} s^2}$$

Assuming that the angular frequency is 500 rad/s and the damping coefficient is  $\xi = 0.7$ , we can get the parameters of the compensator

$$\begin{cases} A = 825 \\ I = 312500 \end{cases} \quad (28)$$

The step response of the control system was shown as Fig .2.

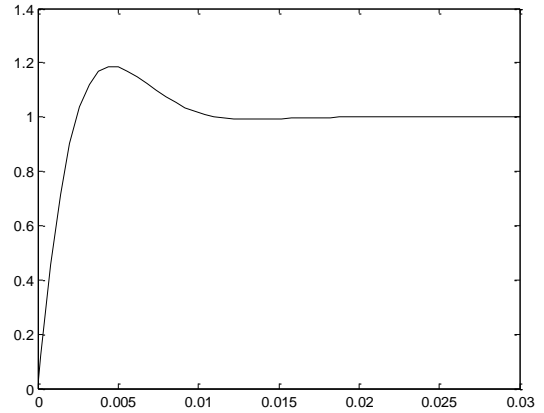


Figure 2. Step response of the control system

## VI. DESIGN OF FUZZY CONTROLLER

### A. Fuzzy rules

We defined three inputs with trimf triangle membership, where neg expressed as a negative, zero indicates zero, pos expressed as positive. And the output is also defined as five triangular memberships, where GN indicates a large negative value, N represents a small negative value, Z is zero, P is a small positive value, GP is a large positive value. The Fuzzy rules are shown in Table 1.

TABLE I. FUZZY RULES

$V_C$ $e(t)$			
	neg	zero	pos
neg	GN	N	Z
zero	N	Z	P
pos	Z	P	GP

### B. Fuzzy clarifications

After fuzzy inference, the output is the fuzzy set that is a combination of the conclusion many fuzzy control rules. The membership functions are basically piecewise and irregular shape fuzzy value, The purpose of clarification is to make them clear value that is mapped to a representative value. Usually we uses the center of area method "Centroid".

### C. Simulations of fuzzy control

The simulation structure of fuzzy control is as shown in Fig .3.

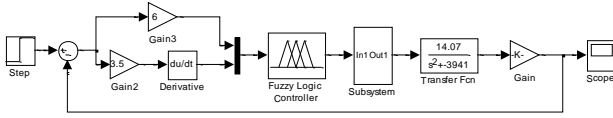


Figure 3. The simulation structure of fuzzy control

The simulation results with 056 mv step signal are shown in Fig .4.

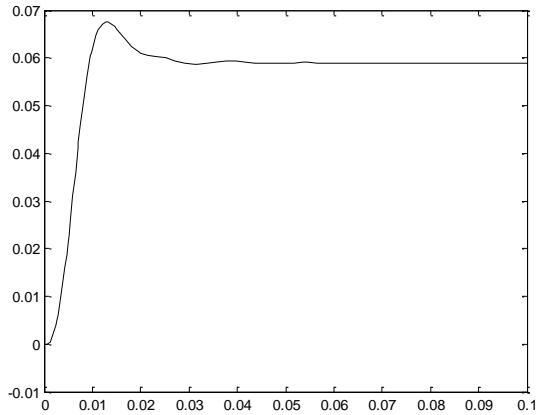


Figure 4. Step response with fuzzy control

We can know from the analysis of Fig .4 that there is smaller steady-state error, compared with the PID controller. However, the fuzzy control system has the advantage of rapid response time, small overshoot and adjustment time.

### VII. BANG-BANG CONTROL

The system with fuzzy control can obtain fast response time, small overshoot and adjustment time, but due to the effect of current loop PI controller, the system has steady-

state error. The system with PID control can eliminate the steady-state error, but with slow response time, and big overshoot and adjustment time. If combining the two controllers, the system will achieve satisfactory control effect. The Bang-bang control structure is shown in Fig .5.

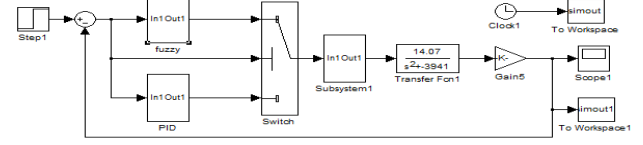


Figure 5. Bang-bang control system

Due to the fast response of fuzzy controller and no steady-state error of PID controller, we let the fuzzy controller work at the beginning, when the steady state error reaches a certain value, it can automatically be switched to the PID control. So the system can achieve the fast response speed and no steady-state error.

Obviously, it is very important to select the appropriate deviation to switch. If deviation is too large, the function of the fuzzy controller is too small or even it can't switch to PID control, the system could not reach the requirements of rapid response. If the deviation is too small, the PID controller action time is too late, the Switch will switch repeatedly in the small scope and the system will oscillatory on a small scale.

By revising setting parameters, we finally let the deviation value  $e = 0.001$ , namely when the system error is less than or equal to 0.001 PID controller acts. The step response with Bang-bang control is shown in Fig .6.

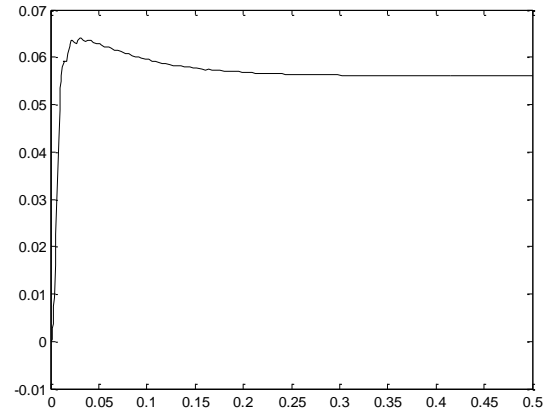


Figure 6. Step response with Bang-bang control

As shown in Fig .6, under the control of the fuzzy controller, the system rapidly responses, when the deviation is 0.001 the PID controller begins to control. Finally the steady-state error is zero and overshoot is smaller

### VIII. CONCLUSIONS

In this paper the electromagnetic suspension system was investigated First of all, according to the Newtonian mechanics principle the forced state of the system was analyzed. The differential equation and the transfer function of the controlled system were deduced based on the system's law of electromagnetic induction. We designed the PID controller and fuzzy controller

respectively analyzed the control effect. Finally applying Bang-bang control combined the fuzzy control method with PID control method in order to achieve the optimal control of the system.

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